An Educational Virtual Laboratory for Sliding Mode and PID Control of Inverted Pendulum

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Abstract—**This paper presents a new tool which teaches sliding mode control (SMC) and proportional-integral-derivative (PID) control to undergraduate and graduate students without laboratory. This educational virtual laboratory tool contains the control of inverted pendulum on the cart. This system is typical example of nonlinear and under-actuated systems. This system is also well known in control engineering for practice of various control theories. At first, the nonlinear dynamic equations of the inverted pendulum are obtained. Then a virtual laboratory tool is designed for SMC and PID. After that the simulation results are analyzed.**

Key words—**Inverted pendulum, educational virtual laboratory, sliding mode control, PID control.**

I. INTRODUCTION

Recently, most control strategies are applied in the linear and nonlinear control systems. The control of inverted pendulum is significant in the control applications. Nonlinear and under-actuated inverted pendulum on the cart problem, especially, is well known as an example system. Therefore, the control of inverted pendulum systems are commonly studied in the control areas. [1–4].

Sliding mode control (SMC) is robust control for nonlinear and linear feedback control method, which has been developed and applied to nonlinear feedback control systems for the last three decades. The SMC approach has been widely used for control design problem. [5,6]. SMC is one of the effective nonlinear robust control approaches since it provides desired system dynamics with an invariance property to uncertainties once the system dynamics are controlled in the sliding mode [7-10]. A SMC is designed so that the system trajectories move onto a sliding surface in a finite time and tends to an equilibrium point along this surface [11].

LabVIEW simplifies the scientific computation, process control, research, industrial application and measurement applications. Because *LabVIEW* has the flexibility of a programming language combined with built-in tools designed specifically for test, measurement, and control [12-13]. This connectivity to I/O has also enabled *LabVIEW* to be used for control applications. *LabVIEW* is an ideal choice used for pragmatic teaching and learning for it supports and serves a wide variety of needs for test, measurement, control and automation applications [14]. The utilization of virtual tools for teaching and learning in engineering has been well accepted by many workers including some work as mentioned in [15, 16].

At the present time, some experiment sets are expensive, and most experiment sets cannot also enable to be changed their parameters. For example inverted pendulum systems, dc and ac motors etc. Therefore the virtual laboratory is quite significant in order to observe the effects of control parameters. In particular, to well understand the systems which haven't experiment sets is difficult in teaching. Additionally in the education of these sets, to be understood the effects of the controller parameters on the system by the students are difficult since the students cannot observe the parameters effect on the system. In the literature, there are studies about an educational tool for a lot of systems [17-19].

The inverted pendulum system consists of the rod on the cart for test bed in this study. Firstly its mathematical model is obtained. Then an educational virtual laboratory tool is designed for SMC and PID control of inverted pendulum. While initial conditions of the pendulum angle and cart position are zero, PID and sliding mode control is applied to the inverted pendulum system and the cart is driven by the controller. Thus, the desired angle of inverted pendulum is obtained by the designed educational tool. With this tool, the effect of parameters can be observed in the control system. SMC and PID virtual laboratory tool is arranged for the education of undergraduate and graduate students.

II. DYNAMIC MODEL OF INVERTED PENDULUM SYSTEM

Figure 1 shows inverted pendulum on the movable cart.

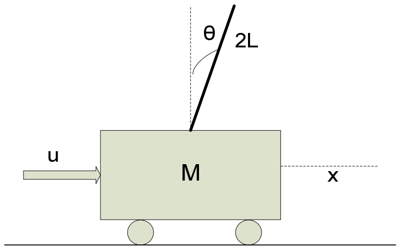


Figure 1. Inverted pendulum cart system

The dynamic model of the inverted pendulum cart system can be deduced by Newton's method as follows.

$$D[\ddot{x} \quad \ddot{\theta}]^T = Eu + F \tag{1}$$

The matrixes D, E, and F can be represented as follows.

$$D = \begin{bmatrix} h_1 & -h_2 \\ -h_2 & h_3 \end{bmatrix}, E = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, F = \begin{bmatrix} A \\ B \end{bmatrix}, \tag{2}$$

here

$$h_1 = mL^2 + J; \quad h_2 = -mL\cos\theta; \quad h_3 = m + M$$
$$A = mLg\sin\theta - a_1\frac{d\theta}{dt} \tag{3}$$
$$B = mL\sin\theta\left(\frac{d\theta}{dt}\right)^2 - a_2\frac{dx}{dt}$$

Where m is the mass of the pendulum, M is mass of the cart, L is the distance from centre of the gravity of the pendulum, x is horizontal displacement of the cart, g is gravitational acceleration, θ is the pendulum angular displacement, a_1 is the viscous friction coefficient of cart, a_2 is the viscous friction coefficient of pendulum, J is inertia moment of the pendulum, u is the horizontal control force on the cart.

Let $x_1 = \theta,\ x_2 = \dot{\theta},\ x_3 = x,\ x_4 = \dot{x}$ According to the canonical form of a class of under-actuated systems, we can transform the above mathematical model of the system into the following state space expression, briefly.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f_1(X) + b_1(X)u \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= f_2(X) + b_2(X)u \end{aligned} \tag{4}$$

Where $X = (x_1, x_2, x_3, x_4)^T$ is state variable vector; $f_i(X)$ and $b_i(X)$ are the nonlinear functions of the state variables. They are abbreviated as f_i and b_i in the following description, which are given as follows.

$$\frac{d^2\theta}{dt^2} = \frac{h_3A + h_2(B+u)}{h_1h_3 - h_2h_2}, \quad \frac{d^2x}{dt^2} = \frac{h_2A + h_1(B+u)}{h_1h_3 - h_2h_2} \tag{5}$$

Then we have

$$f_1 = \frac{h_3A + h_2B}{h_1h_3 - h_2h_2}, \qquad b_1 = \frac{h_2u}{h_1h_3 - h_2h_2} \tag{6}$$

$$f_2 = \frac{h_2A + h_1B}{h_1h_3 - h_2h_2}, \qquad b_2 = \frac{h_1u}{h_1h_3 - h_2h_2} \tag{7}$$

III. SLIDING MODE CONTROL

To derive the sliding-mode control law which forces the motion of the states to be along the sliding surface s=0; a positive definite Lyapunov function is defined as $V(t) = \frac{1}{2}s^2$. If its derivative value $\dot{V}$ is negative definite, then the system is stable and its system trajectory will approach the sliding surface on till converging toward the origin. This is a well-known sliding-mode condition $\dot{V} = s.\dot{s} < 0$

Consider the design of a sliding mode controller for the following system:

$$\ddot{x} = f(x, \dot{x}, t) + B(t, x)u \tag{8}$$

s is called the switching function because the control action switches its sign on the two sides of the switching surface s=0; s is defined as

$$s(x) = ce + \dot{e} \tag{9}$$

where $e = x - x_d$ and x_d is the desired state. *c* is a positive constant. sign(*s*) is a sign function defined as

$$\text{sign}(s) = \begin{cases} 1 & s > 0 \\ 0 & s = 0 \\ -1 & s < 0 \end{cases} \tag{10}$$

If the control law meets equation (2), the control strategy adopted here will guarantee that a system trajectory moves toward and stays on the sliding surface s=0 from any initial condition.

IV. SMC AND PID CONTROL OF INVERTED PENDULUM

The desired angle of inverted pendulum is obtained using SMC and PID controller. SMC and PID control structures are given by subsection.

A. Sliding mode control

Sliding surface is defined for SMC as follows.

$$s = cx_1 + x_2 \tag{11}$$

here *c* is positive constant and x_1, x_2 as follows .

$$x_1 = \theta_d - \theta; \qquad x_2 = \frac{dx_1}{dt} \tag{12}$$

The sliding surface and its derivative is obtained as follows by combination equation (11) and (12)

$$\begin{aligned} s &= c(\theta_d - \theta) + \frac{d(\theta_d - \theta)}{dt} \\ \dot{s} &= c\left(\frac{d\theta_d}{dt} - \dot{\theta}\right) + \frac{d^2(\theta_d - \theta)}{dt^2} \end{aligned} \tag{13}$$

Either state in equation (14) must be provided for SMC [20].

$$lim_{s\to 0^+}(\dot{s} < 0)\ ve\ lim_{s\to 0^-}(\dot{s} < 0) \tag{14}$$

If equation (13) is provided, the trajectory reaches as stable and exponentially to sliding surface. If equation (14) is provided, the closed loop system is stable as asymptotic.

The control force is defined as $u = U_0.\text{sign}(s)$. A saturation function is used in place of sign function for eliminating chattering problem

B. PID control

Figure 2 shows block diagram of inverted pendulum with classic *PID* controller.

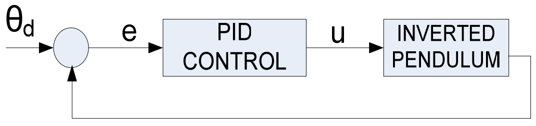


Figure 2. PID controller

The generated control force by PID controller is $u = e(P + I/s + Ds)$. Here $e = \theta_d - \theta$.

V. THE DESIGNED EDUCATIONAL TOOL

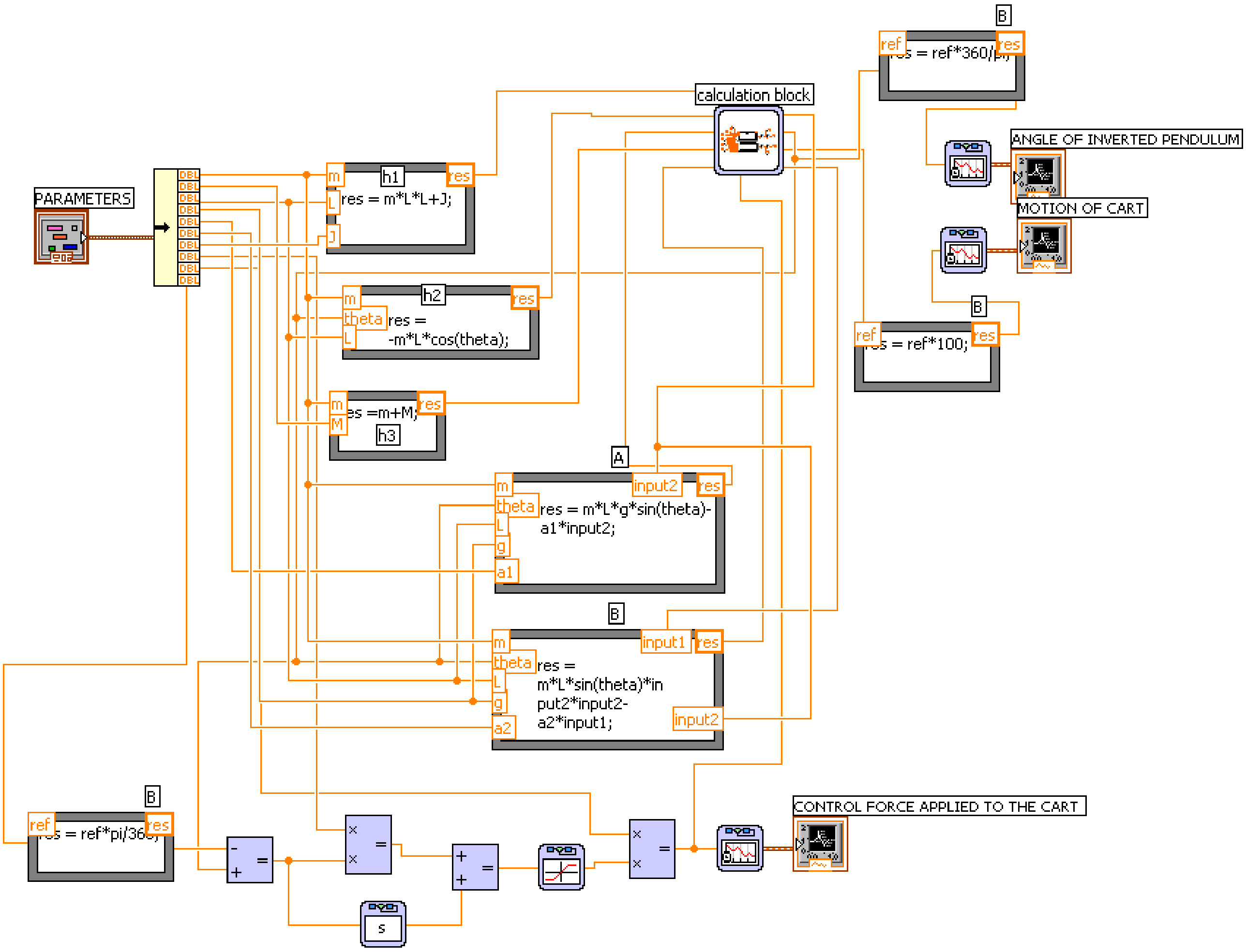


Figure 3. SMC and PID control block diagram

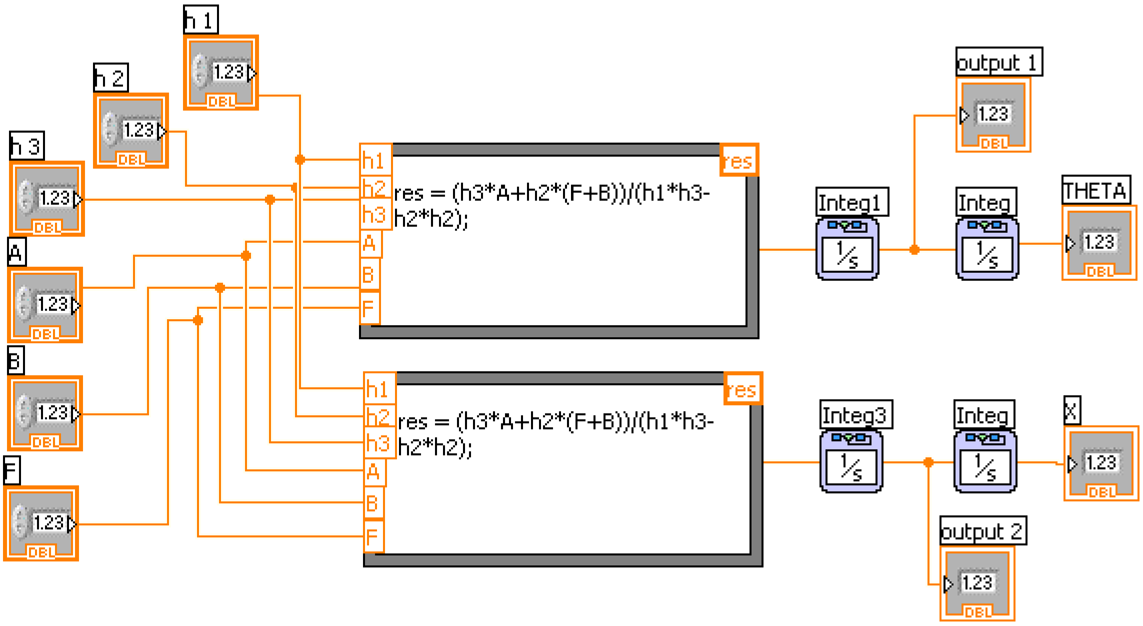
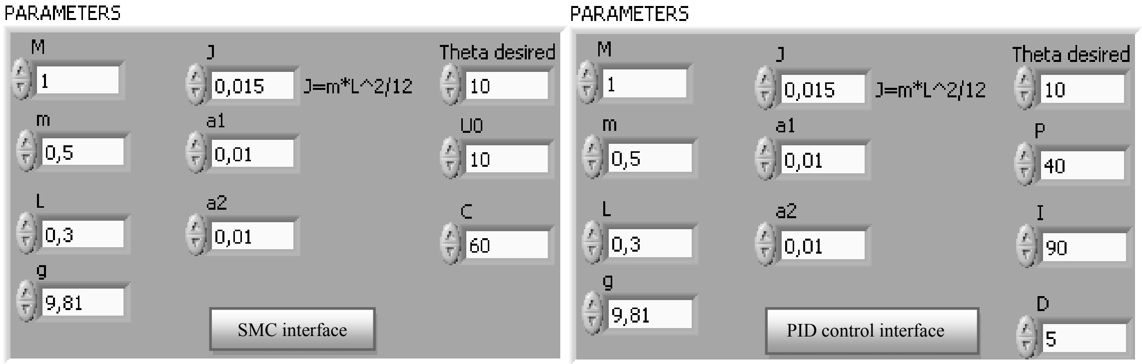


Figure 4. Calculation block diagram in figure 3


PARAMETERS
M 1
J 0,015 J=m*L^2/12
Theta desired 10
m 0,5
a1 0,01
U0 10
L 0,3
a2 0,01
C 60
g 9,81

PARAMETERS
M 1
J 0,015 J=m*L^2/12
Theta desired 10
m 0,5
a1 0,01
P 40
L 0,3
a2 0,01
I 90
g 9,81
D 5

Figure 5. SMC and PID control screen

θ_d (theta desired) is obtained using *LabVIEW* program. This program is designed as an educational tool. Figure 3 shows *LabVIEW* block diagram for the designed educational virtual laboratory. Figure 4 shows the calculation block subsystem is given in figure 4.

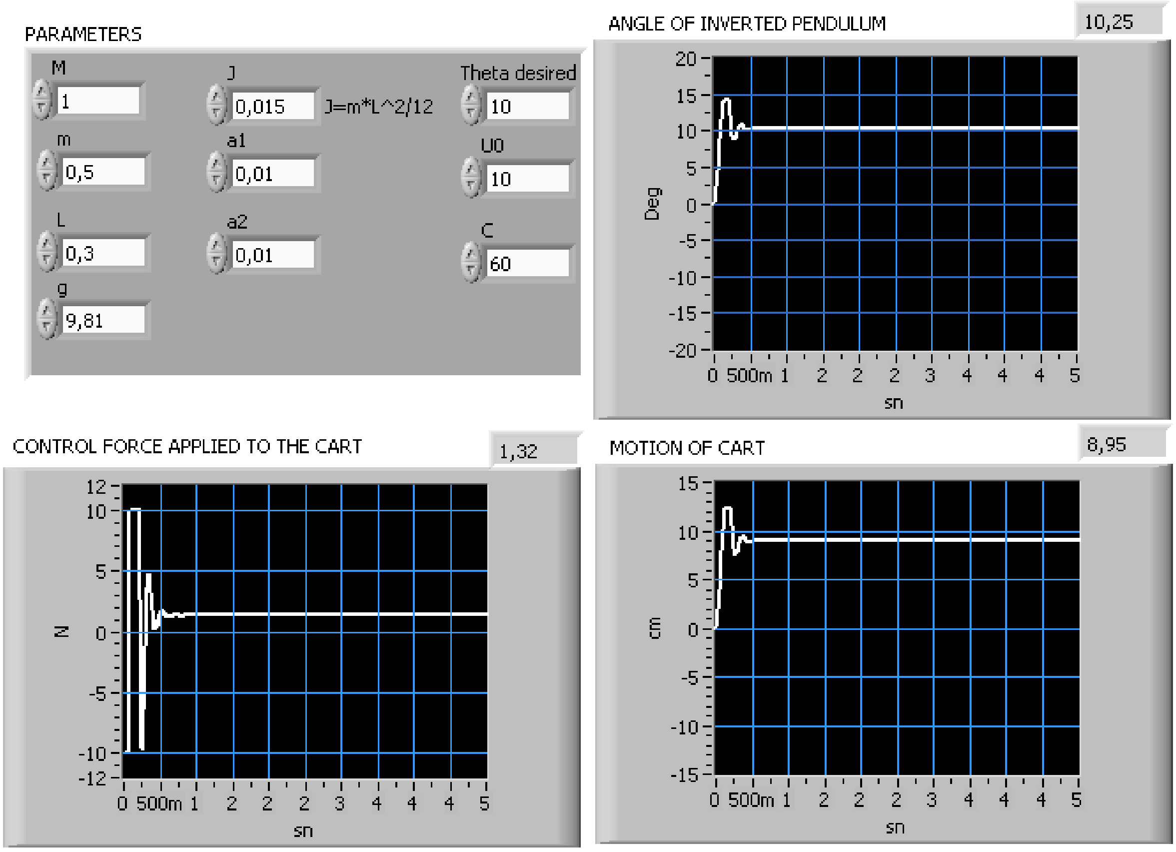
Figure 5 shows inputs screen for SMC and PID controller. This tool has a lot of advantages. All of the parameters of inverted pendulum system can be changed by the tool. Furthermore the parameters of controller can be changed by the tool interface. Thus, the effects of inverted pendulum system and controller parameters on the system can be observed without expensive laboratory.

As shown in figure 5, the tool has inverted pendulum and controller parameters. Where we can change M, m, L, g, a_1, a_2, J, θ_d, U_0 and c. Hence we can observe their effects on angle of the inverted pendulum and the settling time.

VI. THE EDUCATIONAL TOOL EXAMPLE

In this section the educational tool example is given for different parameter. The chosen inverted pendulum parameters are given in table I and III. And the chosen SMC and PID control parameters are given in table II and IV.

Initial conditions are θ=0 and x=0. The tool results are shown in figure 6, 7, 8 and 9 for SMC and PID control of the inverted pendulum. Where θ, x, u outputs are given as degree, centimeter, newton respectively.

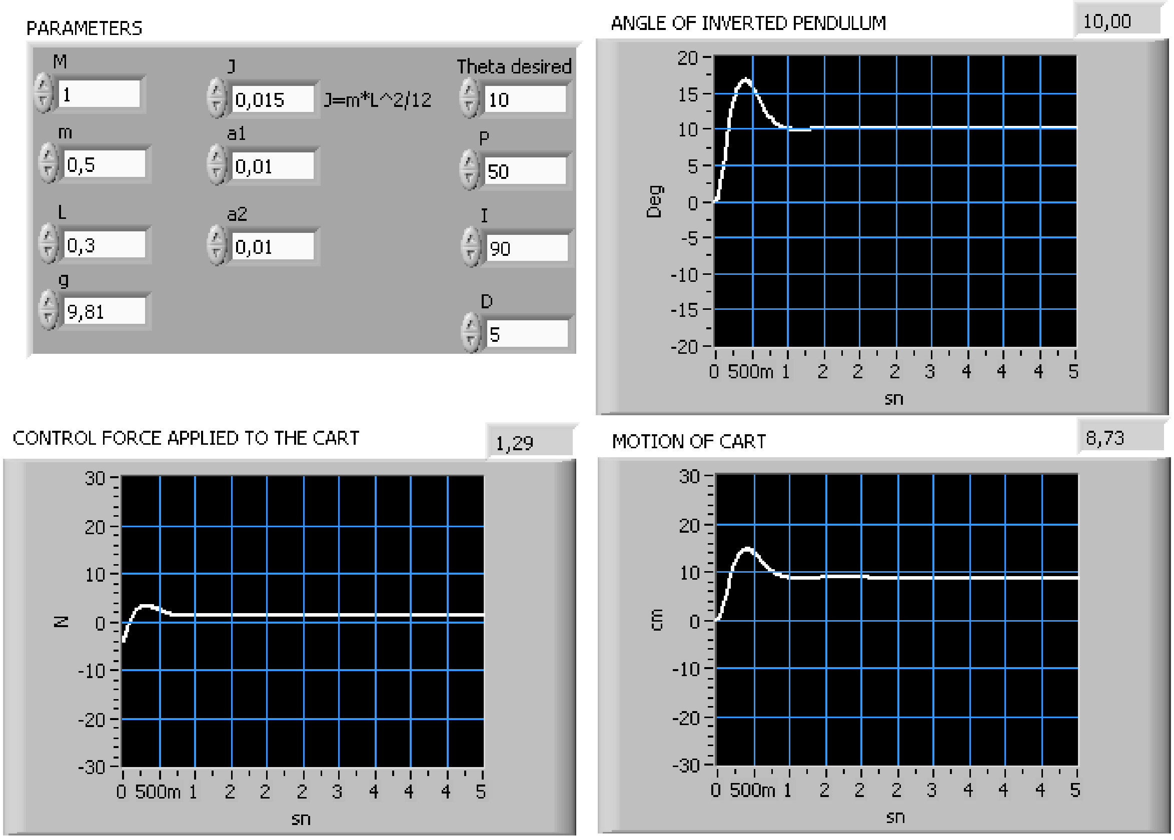

PARAMETERS
M 1
J 0,015 J=m*L^2/12
Theta desired 10
m 0,5
a1 0,01
U0 10
L 0,3
a2 0,01
C 60
g 9,81

ANGLE OF INVERTED PENDULUM 10,25

CONTROL FORCE APPLIED TO THE CART 1,32

MOTION OF CART 8,95

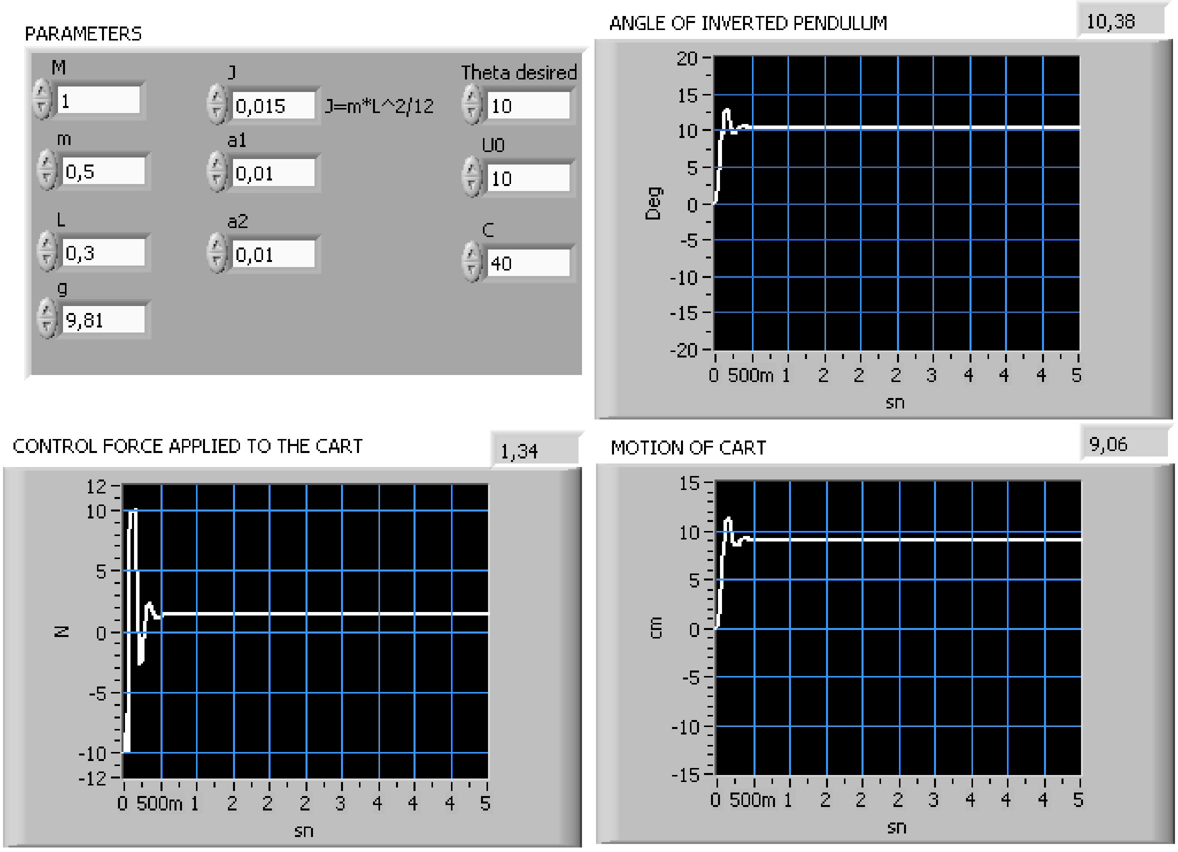
Figure 6. The system outputs with SMC for the parameters table I and II



PARAMETERS

M: 1
m: 0,5
L: 0,3
g: 9,81
J: 0,015 J=m*L^2/12
a1: 0,01
a2: 0,01
Theta desired: 10
P: 50
I: 90
D: 5

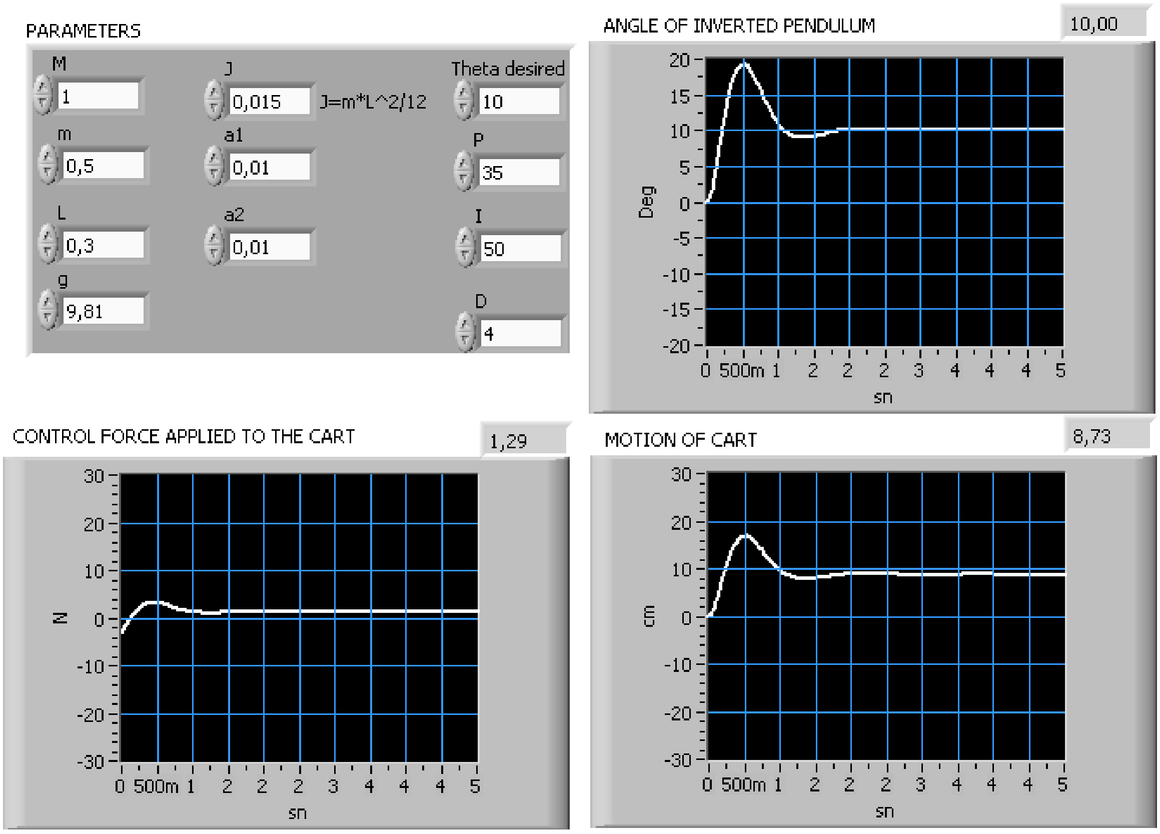
Figure 7. The system outputs with PID controller for the parameters table I and II



PARAMETERS

M: 1
m: 0,5
L: 0,3
g: 9,81
J: 0,015 J=m*L^2/12
a1: 0,01
a2: 0,01
Theta desired: 10
U0: 10
C: 40

Figure 8. The system outputs with SMC for the parameters in table III and IV



PARAMETERS

M: 1

J: 0,015 J=m*L^2/12

Theta desired: 10

m: 0,5

a1: 0,01

P: 35

L: 0,3

a2: 0,01

I: 50

g: 9,81

D: 4

ANGLE OF INVERTED PENDULUM: 10,00

CONTROL FORCE APPLIED TO THE CART: 1,29

MOTION OF CART: 8,73

Figure 9. The system outputs with PID controller for the parameters table III and IV

Figure 6 and 7 show graphs of θ, x, u for first parameter values in table I and II. Figure 8 and 9 show the results for second parameter values in table III and IV. As shown in figure 6, 7, 8 and 9, θ_d can be obtained by changing the controller and system parameters in the tool.

This educational virtual laboratory tool shows each parameter effect on inverted pendulum system.

PID controller coefficients are quite important for the system. Therefore, the effects of P, I and D coefficients on inverted pendulum system can be observed and analyzed by designed the educational tool packet. Similarly the choice of c coefficient is crucial for SMC. c must be determined very well due to the fact that it affects system stability. Changing the coefficients of two controllers by the designed tool's button, inverted pendulum outputs are observed as shown figure 6, 7, 8 and 9. So the outputs of PID and sliding mode controller can be compared. In addition; changing inverted pendulum parameters, pendulum outputs can be observed.

Consider the problem of teaching sliding mode controller. During the lecture session, the lecturer has a limited amount of time in which to explain and illustrate the results of sliding mode control. Often the students will ask for understanding the effectiveness of the parameter changes on the sliding mode and PID controller. A request that may not be practical given the time constrains. The module on sliding mode and PID control provides a practical solution to this works.

TABLE I.

FIRST PARAMETER VALUES OF INVERTED PENDULUM

M	1 kg
m	0.5 kg
L	0.3 m
g	9.81
a_1	0.01 N/m/sec
a_2	0.01 N/m/sec
J	0.015 kg.m^2
θ_{des}	10^0

TABLE II.

FIRST PARAMETER VALUES OF PID AND SMC

c	60
U_0	10
P	50
I	90
D	5

TABLE III.

SECOND PARAMETER VALUES OF INVERTED PENDULUM

M	1 kg
m	0.5 kg
L	0.3 m
g	9.81
a_1	0.01 N/m/sec
a_2	0.01 N/m/sec
J	0.015 kg.m^2
θ_{des}	10^0

TABLE IV.

SECOND PARAMETER VALUES OF PID AND SMC

c	40
U_0	10
P	35
I	50
D	4

VII. CONCLUSION

In this paper, an educational virtual laboratory tool is designed for SMC and PID control of inverted pendulum. The tool enables users to change parameters of inverted pendulum and controller coefficients. The user interface shows the outputs as graphical. This tool helps students to improve through their understanding for SMC and PID control of inverted pendulum.

The virtual laboratory is useful for paperless laboratory with reports written. Virtually experimenting is possible. The main objective behind our virtual laboratory development project was to allow undergraduate and graduate students without any previous programming experience about sliding mode control of inverted pendulum system.

The students can understand the differences between PID and sliding mode controller to applied the inverted pendulum system by using the designed virtual laboratory tool.

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