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Regression control chart for determination of Young's modulus: A case study

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The fundamental problem while obtaining initial modulus (Young's modulus) of concrete in experimental studies is to determine the region on stress-strain curve that will be used in calculations of initial tangent. In the present paper, an approach that depends on regression control chart - that is one of the statistical process control (SPC) tools - for determining the initial tangent and Young's modulus is proposed. The aim of this study is to determine Young's modulus from a particular region on stress-strain curve, which is determined by monitoring the experimental data obtained from an experimental set-up, with regression control chart. According to the current standards, Young's modulus of concrete and steel materials is given individually. There is no established young's modulus exists for reinforced concrete sections. For the displacement calculations, the elasticity modulus is used which is calculated by considering only the compressive strength of concrete. However, steel reinforcements are contained here too. In the relevant literature, there exists no study that uses statistical process control charts to determine the Young's modulus of data obtained from experimental experience with reinforced concrete specimens. The results demonstrate that the Young's modulus values calculated by proposed method correspond with values calculated by analytical method.

Key words: Reinforced concrete columns, stress-strain models, Young's modulus, regression control chart.

INTRODUCTION

The reinforced concrete (RC) can be defined as the combination of concrete and steel reinforcement bars in a way in which they will operate together and make up the deficiencies of one another. A RC column is a structural member used primarily to support compressive loads; it is usually subjected to the axial force and bending moment (Kwak and Kwak, 2010). It is necessary to determine the

stress-strain characteristics of the steel and concrete to calculate the displacements under axial force. This is quite easy for the steel; however, this is not the case for the concrete. The modulus of elasticity of concrete is a parameter required in structural analysis to determine the strain distributions and displacements, particularly when the design of the structure is based upon elasticity considerations (Topcu, 2005). The instantaneous deformations of a concrete under load can be defined properly by stress-strain curve (Popovics, 1970). A typical stress-strain curve is given in Figure 1.

The stress-strain curve of concrete pursues nearly linear for low stress values. With the effect of increasing load the linearity is deteriorated and the curve turns into to a parabola. After the peak point that represents the compressive strength, the deformation continues unless the stress increases. The Young's modulus of the concrete has been studied for many years. Attempts have been made to determine the Young's modulus via

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Notation: σ, Standard deviation of population; σ_e, Standard deviation of residuals; ε, Random error term; β_0, Intercept of regression equation; β_1, Slope of regression equation; t, Time (or observation number); e_t, Residual at time t.
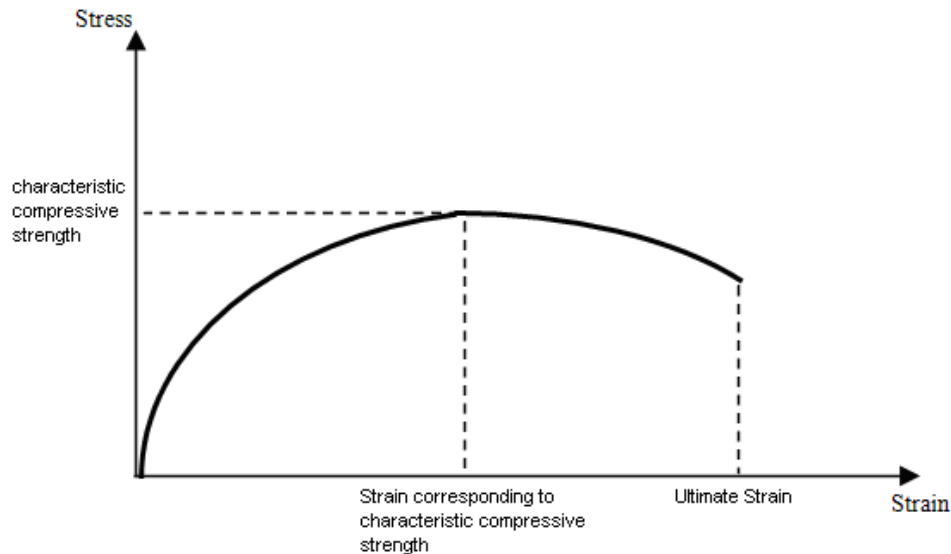
Figure 1. A typical stress-strain curve.

many methods. These various methods caused different values of the Young's modulus of the concrete to be obtained.

Young's modulus is defined as a function of concrete compressive strength in the different regulations those are still in force in different countries. For example, in Turkey it is defined as $E_{cj}=10270\sqrt{f_{cj}}+140000$ (Turkish Standards Institute - TS-500); where, E_{cj} is the jth day concrete elasticity modulus (Young's modulus) (kgf/cm^2), f_{cj} is the jth day concrete cylinder compressive strength (kgf/cm^2).

It is difficult to determine the Young's modulus of concrete whose stress-strain curve exhibits a nonlinear behavior. Because the factors, that affect the compressive strength of the concrete and stress-strain, also affects the Young's modulus. So the stress-strain curve of concrete is affected from numerous components and because of this reason, it is impossible to determine a unique curve that represents all situations. However, mathematical models that represent the stress-strain relation of concrete are needed to understand the behavior of the problems and get solutions. Because of this necessity, many researchers suggested different concrete models.

The Young's modulus can be defined as the slope of the tangent that is plotted from the starting point of stress-strain curve. If the concrete is under low stress values, realistic results can be obtained by using initial tangent. The fundamental problem while obtaining Young's modulus of concrete in experimental studies is to determine the region on stress-strain curve that will be used in calculations of initial tangent. In the present paper, regression control chart is used to determine the region (which will be used for determining the young

modulus) on stress-strain curve that is obtained from experimental data.

Control charts are SPC tools used to determine whether a process is in-control. Since the first control chart proposed by Shewhart in 1931, lots of charts have been developed and then improved to use for different process data. In its basic form, a Shewhart control chart compares process observations with a pair of control limits. On the other hand, if independent process data exhibit an underlying trend due to systemic causes, usually control charts based on ordinary least squares (OLS) regression are used for monitoring and control. A device useful for monitoring and analyzing processes with trend is the regression control chart. Rather than using standard Shewhart charts, practitioners typically implement regression based control charts to monitor a process with systemic trend (Utley and May, 2008). A regression based control chart which is the combination of the conventional control chart and regression analysis was first proposed by Mandel in 1969. This chart is designed to control a varying (rather than a constant) average, and assumes that the values of the dependent variable are linearly (causally) related with the values of the independent variable.

Various studies have demonstrated the control chart applications on RC and concrete. Novokshchenov et al. (1997) applied Shewhart $\bar{X}$ and R (range) chart to monitor the production process of concrete blocks in a concrete block factory in Kuwait. Sohn et al. (2000) focused on applying control chart to vibration-based damage diagnose. They applied Shewhart $\bar{X}$ control chart to the autoregressive (AR) coefficients of the AR model fitted to the measured response time histories of a structure which have been chosen as damage sensitive

feature and successfully detected the anomalies associated with the damaged structures. The same SPC tool has also been used by Fugate et al. (2001) to monitor the residuals defined as the differences between the AR models predicted response histories and the measured ones. Hourani and Inyang (2006) studied on distribution deviation of large aggregates from uniformity in waste containment concrete and they used the nonconformity quality control chart for evaluating aggregate distribution deviation. Wang and Ong (2007) presented a structural health monitoring (SHM) scheme for RC frames subjected to progressive damage based on Hotelling's T^2 control chart. In the following year (in 2008) they presented an innovative structural health monitoring (SHM) scheme based on time series analysis and multivariate statistical process control (MSPC) techniques. One year later (in 2009), they presented a novel structural damage detection scheme using autoregressive-model-incorporating multivariate exponentially weighted moving average (MEWMA) control chart.

Although, we made a comprehensive review on major journals published in 1980 to 2011 year range, there appears to be no study that uses control charts for determining the appropriate region on stress-strain curve for calculations of Young's modulus. However, since the Mandel's regression control chart was developed for determining unnatural pattern behavior of trending process observations, it has not been used for this scope. This observation has been the motivation for the present work on determining the optimal point where the linearly trending values are disturbed on stress-strain curves. We determined Young's modulus from a particular region on stress-strain curve that is determined by monitoring the experimental data obtained from an experimental set-up with regression control chart. The stress-strain curves that are determined from the experimental data obtained from reinforced concrete columns are compared with the confined concrete model proposed by Mander et al. (1988a and 1988b).

MATERIALS AND METHODS

Basic concepts of control chart methodology

SPC is a tool used to measure the variability of a process and determine its capability to produce a particular part. A major objective of SPC is to quickly detect the occurrence of assignable causes of process shifts. The control chart, which is a graphical SPC tool used to determine whether a process is in-control, is an on-line process control technique widely used for this purpose. Control charts may also be used to estimate the parameters of a process. In its basic form, a control chart compares process observations with a pair of control limits. A control chart shows the amount and nature of variations in the process over time. It also enables pattern interpretation and detection of changes in the process (Ross, 1999). The control chart method for variables is a means of visualizing the variations that occur in the central tendency and dispersion of a set of observations (Besterfield et al.,

2003).

A typical control chart plots the averages of measurements of a quality characteristic in samples taken from the process versus time (or the sample number). The chart has a center line (CL) and upper and lower control limits (UCL and LCL). The center line represents where this process characteristic should fall if there are no unusual sources of variability present. The control limits are determined from some simple statistical considerations. Classically, control charts are applied to the output variable(s) in a system. A control chart may indicate an out-of-control condition either (1) when one or more points fall beyond the control limits or (2) when the plotted points exhibit some nonrandom pattern of behavior. If the points are truly random, we should expect a more even distribution of them above and below the center line.

Regardless of the distribution of the quality characteristic, it is standard practice to determine the control limits as a multiple of the standard deviation of the statistic plotted on the chart. The multiple usually chosen is 3; hence, 3-sigma (3σ) limits are customarily employed on control charts, regardless of the type of chart employed. We typically justify the use of 3σ control limits on the basis that they give good results in practice. Moreover, in many cases, the true distribution of the quality characteristic is not known well enough to compute exact probability limits.

There is a close connection between control charts and hypothesis testing. A point plotting within the control limits is equivalent to failing to reject the hypothesis of statistical control, and a point plotting outside the control limits is equivalent to rejecting the hypothesis of statistical control (Montgomery, 1997). In another words the aim of quality monitoring is to test the null hypothesis $H_0 : s = 0$ (in-control state of the process) against the alternative hypothesis $H_1 : s \neq 0$ (out-of-control state of the process), where s represents the mean shift (Pacella and Semeraro, 2007).

There are a wide variety of control charts that are developed for use in different processes. And each of them has different characteristics and structure. So many different kinds of control charts developed from the first creation of the control chart and are improved to solve different kind of quality problems. Each kind of control chart has a corresponding method of determining the center line and control limits. A regression based control chart which is the combination of the conventional control chart and regression analysis is designed to control a varying (rather than a constant) average, and assumes that the values of the dependent variable are linearly (causally) related with the values of the independent variable.

In the present paper, regression control chart is used to determine the assignable cause on stress-strain curve for its linearly trended region. When an assignable cause is observed the regression control chart produces its signal. After the first signal that is produced by the chart is observed, monitoring the stress-strain curve is stopped and the observed stress-strain values on the curve from the initial point to the signal are selected to calculate the parameters of initial tangent and determine Young's modulus.

Regression analysis and regression control chart

It is necessary to explain the basic concepts of linear regression before discussing construction steps of conventional regression control chart. In many problems two or more variables are inherently related, and it is necessary to explore the nature of this relationship. Regression analysis is a statistical technique for modeling and investigating the relationship between two or more variables (Montgomery and Runger, 1999). In statistics, linear regression is a regression method that models the relationship

between a dependent variable (y), independent variable (x) and a random term ε :

$$y_t = \beta_0 + \beta_1 x_{t1} + \beta_2 x_{t2} + ... + \beta_m x_{tm} + \varepsilon_t \qquad t = 1, 2, ..., N \qquad (1)$$

Where the error ε_t is assumed to be an independently and identically distributed normal variable with a mean of zero and constant variance σ^2. The first subscript (t) denotes the index of the observation and the second subscript (m) denotes the index of the input quality characteristic (Shu et al., 2004).

Simple linear regression that is given in Equation (2) considers a single regressor or predictor x and a dependent or response variable y. We assume that each observation, y, can be described by this model (Montgomery and Runger, 1999). In simple linear regression, we want to find the straight line for which the differences (or residuals) between the actual values of y_t and the predicted values $\hat{y}_t$ from the fitted model are as small as possible. A mathematical technique which determines the values of β_0 and β_1 other than those determined by the least squares method would lead to a greater sum of squared differences between the actual and predicted values of y (Levine et al., 1995).

The estimates of β_0 and β_1 should result in a line that is (in some sense) a "best fit" to the data. The German scientist Karl Gauss proposed estimating the parameters β_0 and β_1 in Eq. (4) and Eq. (5) in order to minimize the sum of the squares of the vertical deviations in estimated regression model (Montgomery and Runger, 1999):

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t, \qquad t = 1, 2, ..., N \qquad (2)$$

In this study, we use observation number as the independent variable. So the regression equation estimated as:

$$\hat{y}_t = \hat{\beta}_0 + \hat{\beta}_1 t \qquad (3)$$

We construct the regression control chart in the following three steps:

Step 1: Fit a simple linear regression model to data
The center line of the regression control chart is a regression line. So it is needed to estimate the parameters β_1 (slope) and β_0 (intercept) by:

$$\hat{\beta}_1 = \frac{\sum_{t=1}^{N}(y_t - \bar{y})(t - \bar{t})}{\sum_{t=1}^{N}(t - \bar{t})^2} \qquad (4)$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{t} \qquad (5)$$

Where N represents the sample size, y_t is the tth observation, $\bar{y}$ the mean value of y_t, $t=1,2,...,N$ and $\bar{t}$ the mean value of t ,

$$\bar{y} = (1/N)\sum_{t=1}^{N} y_t \text{ and } \bar{t} = (1/N)\sum_{t=1}^{N} t \text{ (Montgomery and Runger, 1999).}$$

Step 2: Calculate the standard deviation of residuals { e_t }:

$$\hat{\sigma}_e = \sqrt{\frac{\sum_{t=1}^{N}(e_t - \bar{e})^2}{N-1}} \qquad (6)$$

Where { e_t } is the difference between expected and observed values of y_t for the given sample data. Note that we use the notation ε_t to represent the residuals of population, while e_t represents the residuals of sample data;

$$e_t = y_t - \hat{y}_t \qquad (7)$$

and $\bar{e}$ is the sample mean of { e_t }

$$\bar{e} = \frac{\sum_{t=1}^{N} e_t}{N} \qquad (8)$$

Step 3: Calculate control limits and the center line.

Control limits and the centre line of the regression control chart are regression lines. Then the center line (CL), upper control limit (UCL), and lower control limit (LCL) of the regression control chart for the 3-standard deviations from the center line are given below in Equations 9 to 11 respectively (Montgomery, 1997):

$$UCL = \hat{\beta}_0 + \hat{\beta}_1 t + 3\sigma_e \qquad (9)$$

$$CL = \hat{\beta}_0 + \hat{\beta}_1 t \qquad (10)$$

$$LCL = \hat{\beta}_0 + \hat{\beta}_1 t - 3\sigma_e \qquad (11)$$

Subsequently, the proposed procedure for determining the initial tangent and Young's modulus that depends on regression control chart is described.

Proposed procedure for regression control chart aided determination of young's modulus

A regression based control chart which is the combination of the conventional control chart and regression analysis is designed to control the varying average of stress values at the linearly trended region of stress-strain curve of experimental data. Regression control chart is applied to the stress values obtained from the experiments. Note that the stress values from strictly trending part of trending region of the curve have to be selected to construct the regression control chart. For each experiment, the first stress value that fall beyond the control limits is recorded. This means the chart produced a signal for out-of-control condition. By using the recorded stress values, parameters of each initial tangent equation
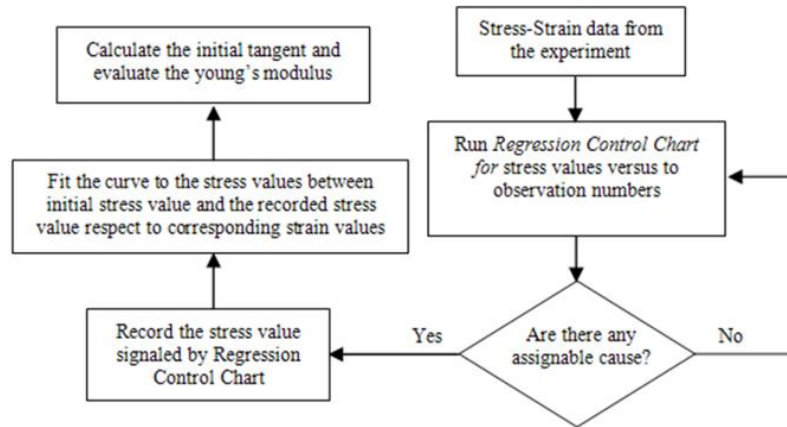
Figure 2. Regression control chart aided determination of Young's modulus.

are calculated and then we were able to calculate the Young's modulus values for each experiment. Figure 2 displays the proposed process for regression control chart aided determination procedure of Young's modulus.

Testing apparatus

In order to test the column specimens under axial pressure, a loading framework with a capacity of 1000 kN axial pressure was prepared at the Construction Laboratory of the Department of Civil Engineering, Balikesir University. The static analyses for the loading frame were conducted through Sap2000 program and the carrying system was dimensioned.

The hydraulic components of the loading frame with a capacity of 1000 kN axial pressure consists of a cylinder with 24 cm diameter, a pump, an oil sump, an electric motor, a three-way speed adjustment valve which regulates the loading speed and pilot-exciter check valve which enables one to fix the load at a desired spot. A moveable head was mounted on the testing apparatus in order to enable the load to be axially applied during the experiment. A 16-channel data transmitter and the data collection software package TDG CODA were used to record load and displacement data. Each channel received eight readings per second. Load values obtained with the help of Loadcell which had a capacity of 2000 kN and the displacement values were received from three spots with the help of LVDTs, which had a capacity of 25 mm. Figure 3 displays the experimental set-up.

RESULTS

Experimental results

To perform the experiments, 6 cube and 6 cylindrical specimens were casted. Also 7 units of 15 ×15 ×100 cm column specimens with 1% reinforcement ratio were casted simultaneously with these reference cube and cylindrical specimens. The strength values of the 6 cube and 6 cylindrical specimens that are used in this study are listed in Table 1.

Figure 4 displays the reinforcement layout of the RC column specimens. 7 specimens were tested at the 28th

day under axial pressure and the stress-strain curves of specimens are determined from experiments.

Figure 5 displays the appearance of a specimen at the end of the experiment. When the stress-strain curves of specimens are inspected with a widen image for the nearly linear region, different curve equations and consequently different Young's modulus values are obtained for different stress values. Graphical expression of this statement is displayed in Figure 6. These Young's modulus values are listed in Table 2. It is clearly observed that the curve equations and thereby the initial tangents obtained from the identical experimental data show disparities for different stress values even at the linearly trending region of the curve. In this study, regression control chart is used to determine the ideal region of stress-strain curve to determine the initial tangent.

The specimens are also investigated analytically by Response-2000 reinforced concrete section analysis package software. Concrete model is selected as Mander confined concrete model (Mander et al., 1988a and 1988b) and reinforcement model is selected as ASTM A615 model (ASTM A615/A615M-03A, 2003). The analytically obtained stress-strain curve for the specimens and the stress-strain curves obtained by experiments are displayed in Figure 7 (S1 represents the specimen 1, S2 represents the specimen 2 and so on) and widen image of Figure 7 is displayed in Figure 8. The analytically calculated Young's modulus value by Mander confined concrete model is 26333.9 MPa. It is clearly observed from the figure that the curves obtained from experiments and curve obtained analytically are nearly identical.

Using regression control chart for monitoring experimental data

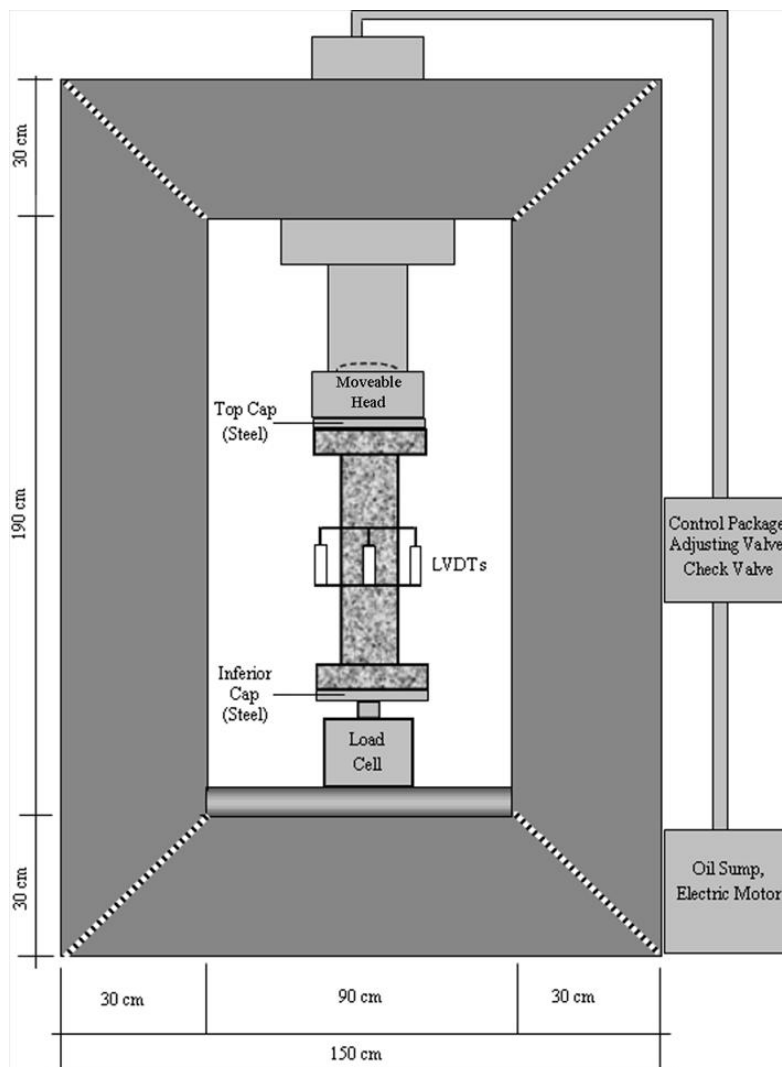Seven identical columns were tested under axial load and

Figure 3. Experimental set-up.

Table 1. Specifications of the test specimens.

Specimen no.	28th Day cube strength		28th Day cylinder strength	
	Load (kN)	Stress (MPa)	Load (kN)	Stress (MPa)
1	9729	43.24	4238	23.98
2	9814	43.62	4421	25.02
3	9427	41.90	4692	26.55
4	7780	34.58	4717	26.69
5	7961	35.38	4225	23.91
6	7655	34.02	4682	26.50

the relation between the stress and strain was obtained.

Analytic stress-strain relation of the axially loaded columns were obtained through a computer program and compared with experimental results.

Figures 9 and 10 display the regression control charts for sample data obtained from experiments. In this figures while straight lines represents the upper control limit (UCL), lower control limit (LCL), and the center line (CL);
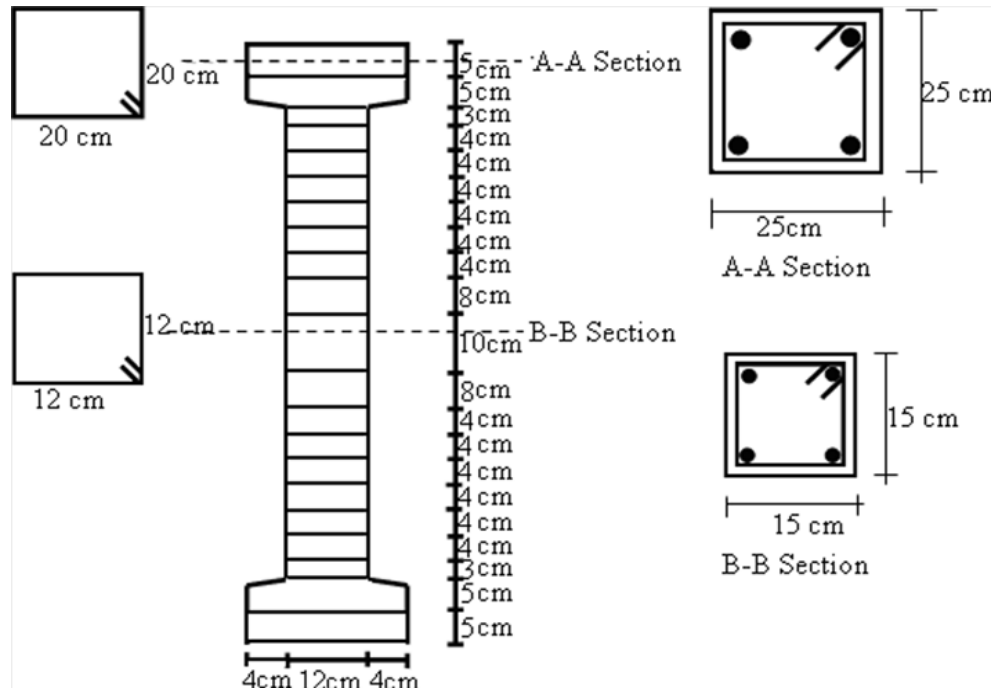
Figure 4. The reinforcement layout for RC column specimens.

Figure 5. Appearance of a specimen at the end of the experiment.

dashed line represents the stress values obtained from experiments. It is clearly observed from Figure 9 that the regression control chart indicates an out-of-control signal at the 4901st observation that corresponds to stress value of 30.36 MPa. This means that the first 4900 observations must be taken (except the 4901st observation with stress value of 30.36 MPa) to evaluate the Young's modulus from stress-stain curve. Similarly, it is observed from Figure 10 that the regression control chart indicates an out-of-control signal at the 1411th observation that corresponds to stress values of 22.85 MPa. Table 3 summarizes the Young's modulus values

calculated by proposed regression control chart based procedure. Experimental results listed in Table 3 indicate that the mean of Young's modulus values of specimens (25349.6 MPa) is close to the value calculated by Mander model (26333.9 MPa).

DISCUSSION

Stress values obtained from experiments are monitored with regression control chart. The stress value that corresponds with the out-of-control signal produced by
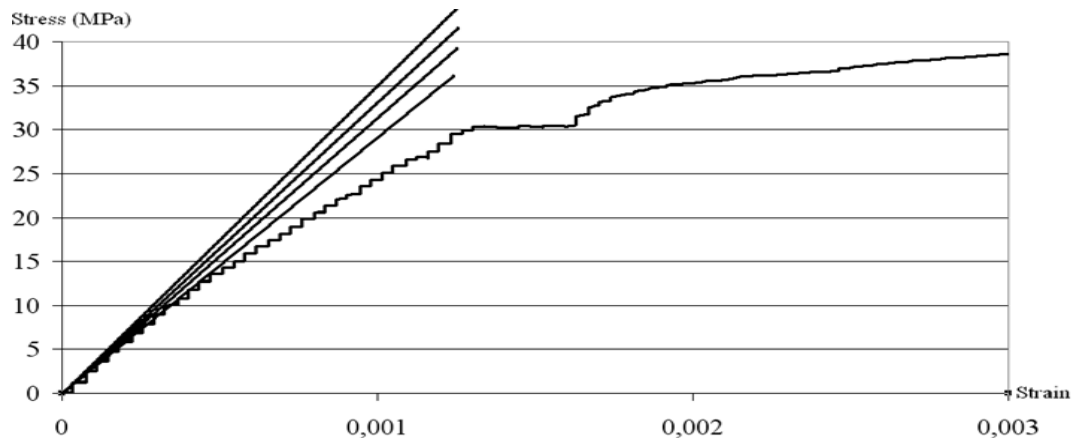
Figure 6. Different regions on stress-strain curve that will be possibly used in calculations of initial tangent.

Table 2. Young's modulus values determined from different stress levels for the first specimen (S1).

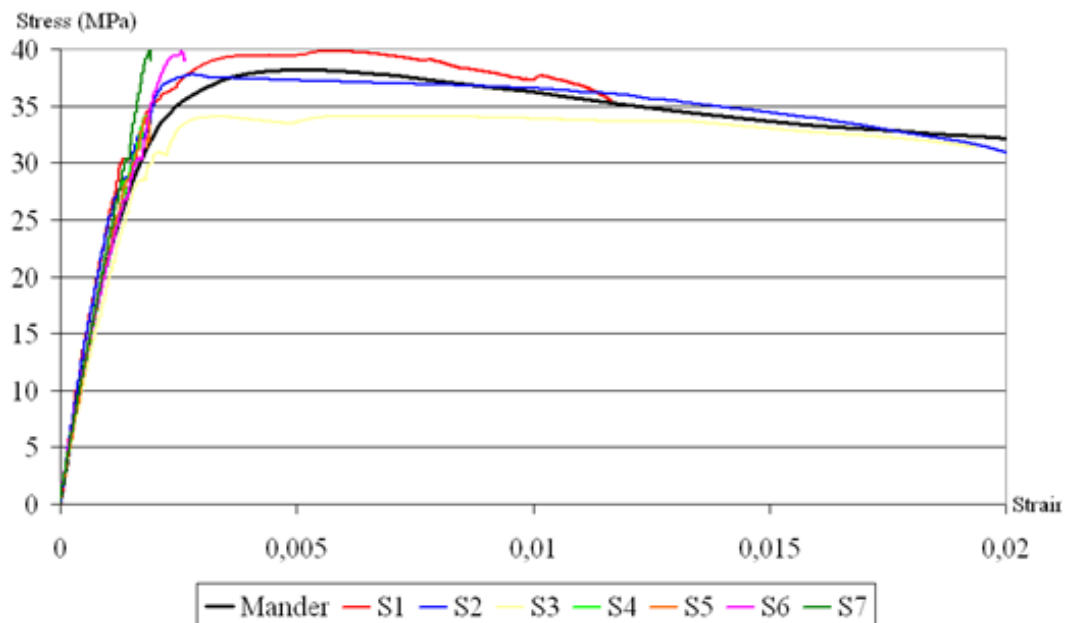| Stress (MPa)	Young's modulus(MPa)
30.00	30320.0
25.00	31627.1
20.00	33467.9
15.00	34578.8

Figure 7. Analytically and experimentally obtained stress-strain curves.

the chart, which means a value that falls beyond the control limits of the chart, is recorded. The stress values from initial data up to the recorded one are chosen as the particular region for calculations of Young's modulus. The
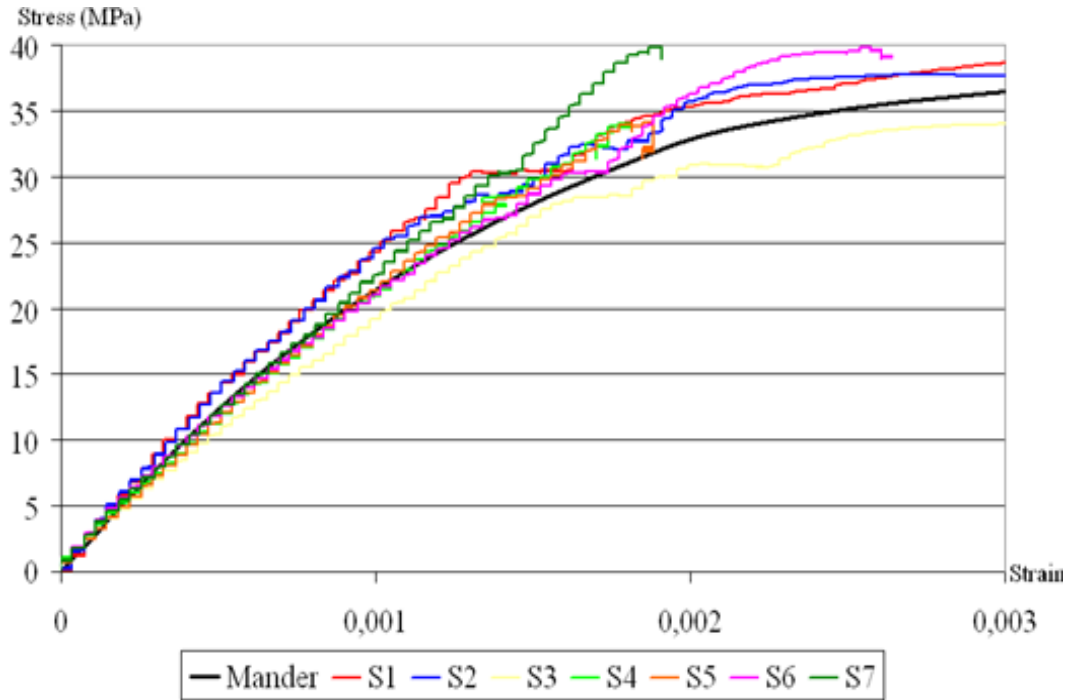
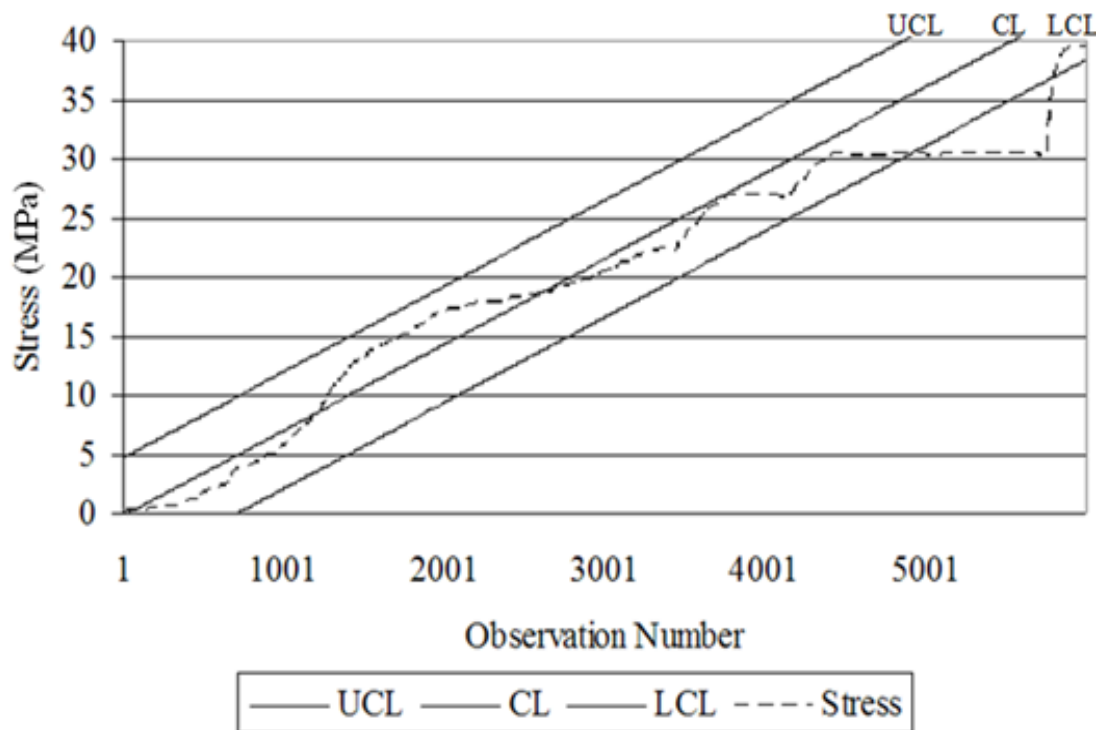**Figure 8.** Analytically and experimentally obtained stress-strain curves with widen Image.

Figure 9. Regression control chart for sample data obtained from the first experiment (S1).

results demonstrate that mean of the Young's modulus values calculated by proposed method corresponds with the value calculated by analytical method. Also, the proposed method is easy to computerize and effective to
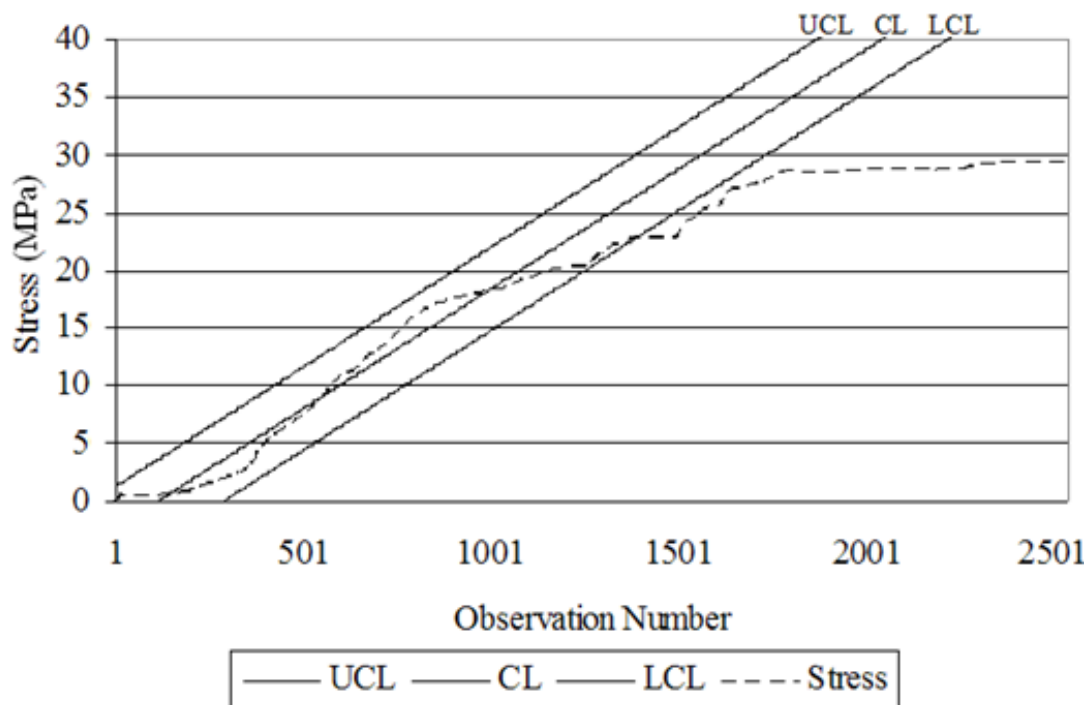
Figure 10. Regression control chart for sample data obtained from the second experiment (S2).

Table 3. Young's modulus values calculated by proposed regression control chart based procedure.

Experiment	Stress (MPa)	Young's modulus (MPa)
S1	30.36	31423.2
S2	22.85	31347.3
S3	28.80	20561.4
S4	16.49	22189.4
S5	17.05	23524.6
S6	30.35	24577.4
S7	29.85	23823.9
Mean of experiments		25349.6

determine the point that the stress-strain curve turned into second order polynomial from the linearity.

Conclusions

Experimental studies on the reinforced concrete members have been performed for many years. For the data obtained from experimental studies, it is difficult to refer a unique curve equation for particular regions such in analytical models. Because of this reason, the experimental researches are performed by using best fitted curve equation for the experimental data. When the stress-strain curves of specimens are inspected with a widen image for the nearly linear region, different curve equations and consequently different Young's modulus values are obtained for different stress values. In the present paper, to make right choice for the proper region on stress-strain curve that will be used in determining the Young's modulus of experimental data, we proposed a regression control chart depended method. Proposed method is effective to determine the point that the stress-strain curve turned into second order polynomial from the linearity.

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