A Power Resolution for Nonsinusoidal and Unbalanced Systems-Part II: Theoretical Background

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***Abstract*—It is seen from the study [1] that there is a need for a power resolution, which can be utilized for the direct provision of the optimum compensation capacitor's power and give some useful information on the detection of the harmonic producing loads, in the literature. Accordingly, for nosinusoidal and unbalanced systems, a power resolution, which achieves these goals, is proposed in this paper.**

***Index Terms*-- Apparent power definitions, power resolutions, nonsinusoidal conditions, unbalanced systems, optimum capacitive compensation, harmonic source detection.**

I. Background

In [1], the literature on the frequency-domain power theory is rigorously investigated and it is reported that a power resolution, which can be used as a tool for the direct provision of optimum compensation capacitor's power and the detection of harmonic producing loads in nonsinusoidal and unbalanced systems, will be a contribution to the literature.

In this paper, for nonsinusoidal single-phase systems and three-phase and three-line systems, which contain nonsinusoidal & balanced voltages and nonsinusoidal & unbalanced currents, a power resolution is proposed by considering the above mentioned goals. It should be reminded that the single-phase case of the proposed resolution is first interpreted and analysed for compensation in [2].

II. Proposed Power Resolution for Unbalanced & Nonsinusoidal Three-Phase and Three-Line Systems

In this section, a single-phase power resolution proposed in [2] is extended to three-phase and three-line systems, which consist of nonsinusoidal & balanced voltages and nonsinusoidal & unbalanced currents. Accordingly, in the first step, by expressing line-to-neutral voltages and line currents as;

$$\vec{v}(t)=\begin{bmatrix}\sum_n \sqrt{2}V_n sin\left(\omega_n t+\theta_{a,n}\right)\\ \sum_n \sqrt{2}V_n sin\left(\omega_n t+\theta_{b,n}\right)\\ \sum_n \sqrt{2}V_n sin\left(\omega_n t+\theta_{c,n}\right)\end{bmatrix},\ \vec{i}(t)=\begin{bmatrix}\sum_n \sqrt{2}I_{a,n} sin\left(\omega_n t+\delta_{a,n}\right)\\ \sum_n \sqrt{2}I_{b,n} sin\left(\omega_n t+\delta_{b,n}\right)\\ \sum_n \sqrt{2}I_{c,n} sin\left(\omega_n t+\delta_{c,n}\right)\end{bmatrix} \quad (1)$$

the balanced and unbalanced parts of the line currents can be separated as below:

$$\vec{i}(t)=\vec{i}_B(t)+\vec{i}_u(t) \quad (2)$$

To find the expression of balanced current component ($\vec{i}_B(t)$), the powers ($U_{l,n}$), which are drawn due to the n^{th} harmonic line currents in phase with the n^{th} harmonic of respective line-to-neutral voltages, and the powers ($Q_{l,n}$) , which are drawn due to the n^{th} harmonic line currents in quadrature with the n^{th} harmonic of respective line-to-neutral voltages, are calculated:

$$U_{l,n}=V_n I_{l,n} cos\left(\theta_{l,n}-\delta_{l,n}\right) \qquad (l=a,b,c) \quad (3)$$

$$Q_{l,n}=V_n I_{l,n} sin\left(\theta_{l,n}-\delta_{l,n}\right) \qquad (l=a,b,c) \quad (4)$$

And then, these powers are shared to each phase equally; thus, fictitious n^{th} harmonic balanced active (P_n) and reactive (Q_n) powers are found to be:

$$P_n=\frac{1}{3}\left(U_{a,n}+U_{b,n}+U_{c,n}\right) \quad (5)$$

$$Q_n=\frac{1}{3}\left(Q_{a,n}+Q_{b,n}+Q_{c,n}\right) \quad (6)$$

For n^{th} harmonic, balanced active and balanced reactive powers of each line, given in (5) and (6), are drawn by the balanced part of n^{th} harmonic load impedance. Thus, for each phase of the load, n^{th} harmonic balanced conductance ($G_{B,n}$) and n^{th} harmonic balanced susceptance ($B_{B,n}$) can be calculated as;

$$G_{B,n}=\frac{P_n}{V_n^2} \quad (7)$$

$$B_{B,n}=\frac{Q_n}{V_n^2} \quad (8)$$

By using n^{th} harmonic balanced conductance and susceptance, the balanced current component can be expressed as;

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$$\vec{i}_B(t) = \vec{i}_{B,x}(t) + \vec{i}_{B,y}(t)$$

$$= \begin{bmatrix} \sum_n \sqrt{2} G_{B,n} V_n \, sin(\omega_n t + \theta_{a,n}) \\ \sum_n \sqrt{2} G_{B,n} V_n \, sin(\omega_n t + \theta_{b,n}) \\ \sum_n \sqrt{2} G_{B,n} V_n \, sin(\omega_n t + \theta_{c,n}) \end{bmatrix} + \begin{bmatrix} \sum_n \sqrt{2} B_{B,n} V_n \, sin\left(\omega_n t + \theta_{a,n} - \frac{\pi}{2}\right) \\ \sum_n \sqrt{2} B_{B,n} V_n \, sin\left(\omega_n t + \theta_{b,n} - \frac{\pi}{2}\right) \\ \sum_n \sqrt{2} B_{B,n} V_n \, sin\left(\omega_n t + \theta_{c,n} - \frac{\pi}{2}\right) \end{bmatrix} \quad (9)$$

On the other hand, to find an expression for unbalanced current component ($\vec{i}_u(t)$), the conductance ($G_{l,n}$) and the susceptance ($B_{l,n}$) of the n^{th} harmonic load impedance for each line are calculated:

$$G_{l,n} = \frac{U_{l,n}}{V_n^2} \quad (10)$$

$$B_{l,n} = \frac{Q_{l,n}}{V_n^2} \quad (11)$$

And then, for each line, $G_{l,n}^u$ and $B_{l,n}^u$ can be found as;

$$G_{l,n}^u = G_{l,n} - G_{B,n} \quad (12)$$

$$B_{l,n}^u = B_{l,n} - B_{B,n} \quad (13)$$

Thus, by using $G_{l,n}^u$ and $B_{l,n}^u$, the unbalanced current component can be expressed as;

$$\vec{i}_u(t) = \vec{i}_{up}(t) + \vec{i}_{uq}(t)$$

$$= \begin{bmatrix} \sum_n \sqrt{2} G_{a,n}^u V_n \, sin(\omega_n t + \theta_{a,n}) \\ \sum_n \sqrt{2} G_{b,n}^u V_n \, sin(\omega_n t + \theta_{b,n}) \\ \sum_n \sqrt{2} G_{c,n}^u V_n \, sin(\omega_n t + \theta_{c,n}) \end{bmatrix} + \begin{bmatrix} \sum_n \sqrt{2} B_{a,n}^u V_n \, sin\left(\omega_n t + \theta_{a,n} - \frac{\pi}{2}\right) \\ \sum_n \sqrt{2} B_{b,n}^u V_n \, sin\left(\omega_n t + \theta_{b,n} - \frac{\pi}{2}\right) \\ \sum_n \sqrt{2} B_{c,n}^u V_n \, sin\left(\omega_n t + \theta_{c,n} - \frac{\pi}{2}\right) \end{bmatrix} \quad (14)$$

Up to now, the current is separated into three components: balanced ($\vec{i}_B(t)$), unbalanced in phase ($\vec{i}_{up}(t)$) and unbalanced quadrature ($\vec{i}_{uq}(t)$) currents. Note that, due to the fact that the sum of three-phase voltages (star point voltage) is zero the harmonic number (n) of the voltage expressions given in (1) is not zero *(dc)* and triplen numbers. As a result, the currents also do not have these harmonic numbers. That is a reasonable assumption for the most practical cases of the three-phase and three-line power systems.

In the second step, balanced current is decomposed into four orthogonal components namely; active, reactive, scattered conductance and scattered susceptance currents by treating each phase individually. This decomposition is valid due to the fact that voltage is balanced and the decomposed current is the balanced current component. Therefore, by means of the same methodology presented in [2], the balanced current will be separated to active, reactive, scattered conductance and scattered susceptance currents here. It should be underlined that scattered conductance and scattered susceptance currents was previously named as nonlinearity conductance and nonlinearity susceptance currents in [2].

Accordingly, for each phase of the load, equivalent conductance is calculated as;

$$G_e = \frac{\sum_n P_n}{\sum_n V_n^2} \quad (15)$$

and active current is defined:

$$\vec{i}_{ac}(t) = \begin{bmatrix} \sum_n \sqrt{2} G_e V_n \, sin(\omega_n t + \theta_{a,n}) \\ \sum_n \sqrt{2} G_e V_n \, sin(\omega_n t + \theta_{b,n}) \\ \sum_n \sqrt{2} G_e V_n \, sin(\omega_n t + \theta_{c,n}) \end{bmatrix} \quad (16)$$

And then, for each phase of the load, by calculating n^{th} harmonic equivalent susceptance as;

$$B_{e,n} = nB_{e,1} = n\frac{1}{Xc_1} = \frac{n\sum_n nQ_n}{\sum_n n^2 V_n^2} \quad for\ Xc_1 = \sum_n n^2 V_n^2 \Big/ \sum_n nQ_n \quad (17)$$

reactive current can be expressed as:

$$\vec{i}_r(t) = \begin{bmatrix} \sum_n \sqrt{2} B_{e,n} V_n \, sin\left(\omega_n t + \theta_{a,n} - \frac{\pi}{2}\right) \\ \sum_n \sqrt{2} B_{e,n} V_n \, sin\left(\omega_n t + \theta_{b,n} - \frac{\pi}{2}\right) \\ \sum_n \sqrt{2} B_{e,n} V_n \, sin\left(\omega_n t + \theta_{c,n} - \frac{\pi}{2}\right) \end{bmatrix} \quad (18)$$

Sequently, using (16) and (18), the expression of the balanced current is transformed to (19):

$$\vec{i}_B(t) = \begin{bmatrix} \sum_n \sqrt{2} G_e V_n \, sin(\omega_n t + \theta_{a,n}) \\ \sum_n \sqrt{2} G_e V_n \, sin(\omega_n t + \theta_{b,n}) \\ \sum_n \sqrt{2} G_e V_n \, sin(\omega_n t + \theta_{c,n}) \end{bmatrix} + \begin{bmatrix} \sum_n \sqrt{2} B_{e,n} V_n \, sin\left(\omega_n t + \theta_{a,n} - \frac{\pi}{2}\right) \\ \sum_n \sqrt{2} B_{e,n} V_n \, sin\left(\omega_n t + \theta_{b,n} - \frac{\pi}{2}\right) \\ \sum_n \sqrt{2} B_{e,n} V_n \, sin\left(\omega_n t + \theta_{c,n} - \frac{\pi}{2}\right) \end{bmatrix} + \vec{i}_s(t) \quad (19)$$

By equating the right hand sides of (9) and (19), the scattered current ($\vec{i}_s(t)$) can be expressed as:

$$\vec{i}_s(t) = \begin{bmatrix} \sum_n \sqrt{2} (G_{B,n} - G_e) V_n \, sin(\omega_n t + \theta_{a,n}) \\ \sum_n \sqrt{2} (G_{B,n} - G_e) V_n \, sin(\omega_n t + \theta_{b,n}) \\ \sum_n \sqrt{2} (G_{B,n} - G_e) V_n \, sin(\omega_n t + \theta_{c,n}) \end{bmatrix} + \begin{bmatrix} \sum_n \sqrt{2} (B_{B,n} - B_{e,n}) V_n \, sin\left(\omega_n t + \theta_{a,n} - \frac{\pi}{2}\right) \\ \sum_n \sqrt{2} (B_{B,n} - B_{e,n}) V_n \, sin\left(\omega_n t + \theta_{b,n} - \frac{\pi}{2}\right) \\ \sum_n \sqrt{2} (B_{B,n} - B_{e,n}) V_n \, sin\left(\omega_n t + \theta_{c,n} - \frac{\pi}{2}\right) \end{bmatrix} \quad (20)$$

And lastly, the parts related to conductance and susceptance of $\vec{i}_s(t)$ can be named: scattered conductance current;

$$\vec{i}_{sc}(t) = \begin{bmatrix} \sum_n \sqrt{2} (G_{B,n} - G_e) V_n \, sin(\omega_n t + \theta_{a,n}) \\ \sum_n \sqrt{2} (G_{B,n} - G_e) V_n \, sin(\omega_n t + \theta_{b,n}) \\ \sum_n \sqrt{2} (G_{B,n} - G_e) V_n \, sin(\omega_n t + \theta_{c,n}) \end{bmatrix} \quad (21)$$

and scattered susceptance current;

$$\vec{i}_{ss}(t)=\begin{bmatrix}\sum_n \sqrt{2}\left(B_{B,n}-B_{e,n}\right)V_n \sin\left(\omega_n t+\theta_{a,n}-\frac{\pi}{2}\right)\\ \sum_n \sqrt{2}\left(B_{B,n}-B_{e,n}\right)V_n \sin\left(\omega_n t+\theta_{b,n}-\frac{\pi}{2}\right)\\ \sum_n \sqrt{2}\left(B_{B,n}-B_{e,n}\right)V_n \sin\left(\omega_n t+\theta_{c,n}-\frac{\pi}{2}\right)\end{bmatrix} \quad (22)$$

As a result, the total current can be expressed as;

$$\vec{i}(t)=\vec{i}_{ac}(t)+\vec{i}_r(t)+\vec{i}_{sc}(t)+\vec{i}_{ss}(t)+\vec{i}_{up}(t)+\vec{i}_{uq}(t) \quad (23)$$

By using the collective current rms value defined in Bucholz's apparent power definition [4], [5];

$$I_\Sigma=\sqrt{\frac{1}{T}\int_0^T\left(\vec{i}(t)\right)^{Tr}\cdot\vec{i}(t)dt}=\sqrt{\sum_l I_l^2} \quad (24)$$

the rms values of these current components can be calculated in harmonic domain as;
active current's rms value,

$$I_{ac}=\sqrt{\frac{1}{T}\int_0^T\left(\vec{i}_{ac}(t)\right)^{Tr}\cdot\vec{i}_{ac}(t)dt}=\sqrt{3G_e^2\sum_n V_n^2} \quad (25)$$

reactive current's rms value,

$$I_r=\sqrt{\frac{1}{T}\int_0^T\left(\vec{i}_r(t)\right)^{Tr}\cdot\vec{i}_r(t)dt}=\sqrt{3\sum_n B_{e,n}^2V_n^2} \quad (26)$$

scattered conductance current's rms value,

$$I_{sc}=\sqrt{\frac{1}{T}\int_0^T\left(\vec{i}_{sc}(t)\right)^{Tr}\cdot\vec{i}_{sc}(t)dt}=\sqrt{3\sum_n\left(G_{B,n}-G_e\right)^2V_n^2} \quad (27)$$

scattered susceptance current's rms value,

$$I_{ss}=\sqrt{\frac{1}{T}\int_0^T\left(\vec{i}_{ss}(t)\right)^{Tr}\cdot\vec{i}_{ss}(t)dt}=\sqrt{3\sum_n\left(B_{B,n}-B_{e,n}\right)^2V_n^2} \quad (28)$$

unbalanced in phase current's rms value,

$$\begin{aligned}I_{up}&=\sqrt{\frac{1}{T}\int_0^T\left(\vec{i}_{up}(t)\right)^{Tr}\cdot\vec{i}_{up}(t)dt}\\&=\sqrt{\sum_n\left[\left(G_{a,n}^u\right)^2+\left(G_{b,n}^u\right)^2+\left(G_{c,n}^u\right)^2\right]V_n^2}\end{aligned} \quad (29)$$

and unbalanced quadrature current's rms value,

$$\begin{aligned}I_{uq}&=\sqrt{\frac{1}{T}\int_0^T\left(\vec{i}_{uq}(t)\right)^{Tr}\cdot\vec{i}_{uq}(t)dt}\\&=\sqrt{\sum_n\left[\left(B_{a,n}^u\right)^2+\left(B_{b,n}^u\right)^2+\left(B_{c,n}^u\right)^2\right]V_n^2}\end{aligned} \quad (30)$$

Due to the fact that the proposed current components are orthogonal, the square of the collective rms value of total current can be expressed as:

$$I_\Sigma^2=I_{ac}^2+I_r^2+I_{sc}^2+I_{ss}^2+I_{up}^2+I_{uq}^2 \quad (31)$$

Due to the fact that the n^{th} harmonic of conductance based current components are in phase with the n^{th} harmonic of voltage and the n^{th} harmonic of susceptance based current components are in quadrature with the n^{th} harmonic of voltage, all combinations between conductance based currents and susceptance based currents are orthogonal. The rest of orthogonality proofs are provided in the parts from A.1 to A.6 of appendix using Gramm-Schmidt orthogonality condition [3]. Finally, if both sides of (31) is multiplied by the square of collective voltage rms ($V_\Sigma^2=\sum_l V_l^2$), the resolution of Bucholz's apparent power can be obtained as;

$$\begin{aligned}S^2&=V_\Sigma^2I_\Sigma^2=V_\Sigma^2I_{ac}^2+V_\Sigma^2I_r^2+V_\Sigma^2I_{sc}^2+V_\Sigma^2I_{ss}^2+V_\Sigma^2I_{up}^2+V_\Sigma^2I_{uq}^2\\&=P^2+Q_r^2+D_{sc}^2+D_{ss}^2+D_{up}^2+D_{uq}^2\end{aligned} \quad (32)$$

In (32), power components are named as active (P), reactive (Q_r), scattered conductance (D_{sc}), scattered susceptance (D_{ss}), unbalanced in phase (D_{up}) and unbalanced quadrature (D_{uq}) powers.

It is apparent for the systems without zero sequence voltages that the apparent powers of Bucholz and IEEE std. 1459-2010 [6] have the same numerical values [7], [8], therefore, proposed power resolution also decomposes IEEE std. 1459-2010 apparent power. The contributions of the proposed power resolution to frequency-domain power theory are underlined below:

A. Providing a tool for the direct determination of optimum balanced capacitors bank's power

Proposed resolution contains the reactive power component (Q_r) that gives the power of optimum balanced capacitive compensator under the load terminal voltage.

When the proposed resolution is compared with Czarnecki's power resolution [9], [10], which is similar to the proposed power resolution, some features should be underlined:

- The reactive power component of Czarnecki's power resolution is divided into two power components, namely; reactive power (completely compensable with a balanced capacitors bank) and scattered susceptance power in the proposed resolution.
- And also, the unbalanced power component of Czarnecki's power resolution is decomposed into unbalanced in phase and unbalanced quadrature powers in the proposed resolution. His unbalanced power is defined as the vector subtraction of apparent power and other power components and it has no particular expression in harmonic-domain. On the other hand, in the proposed resolution, the unbalanced power can be calculated by means of particular expression as follows;

$$D_u^2=D_{up}^2+D_{uq}^2=V_\Sigma^2\left(I_{up}^2+I_{uq}^2\right) \quad (33)$$

We think this issue is a remarkable contribution for the practical measurement of unbalanced power.

B. Providing a tool for the detection of harmonic producing loads

From the outlines of the proposed power resolution, it can be qualitatively concluded that two power components could be used to detect the harmonic producing loads: These are; scattered conductance power (D_{sc}), which occurs by the difference between n[th] harmonic balanced conductance and equivalent conductance, and scattered susceptance power (D_{ss}), which occurs by the difference between n[th] harmonic balanced susceptance and n[th] harmonic equivalent susceptance. However, scattered susceptance power is quiet sensitive to the source side harmonic distortion, thus; its usage for the detection of the harmonic producing loads will be problematic. On the other hand, scattered conductance power (D_{sc}) of the proposed resolution has the cases underlined below;

- **Sinusoidal Voltage (or Voltage with Negligible THD_V):** For a linear load, one can see that apparent power is very close to the fundamental harmonic apparent power due to the fact that the voltage and current harmonics are negligible. As a result, for a linear load under negligible voltage distortion, scattered conductance power has negligible value. On the contrary, for a nonlinear (or harmonic producing) load and the same voltage distortion case, the load balanced conductances calculated for the harmonics a part from fundamental harmonic have the considerable values due to the fact that the load injects current harmonics, which is extremely higher than the respective voltage harmonics. Therefore, the rms value of the scattered conductance current and scattered conductance power have considerable values.
- **Nonsinusoidal Voltage (with the reasonable THD_V value):** In IEEE std. 519-1992 [11], the limit of THD_V is determined as 5% at the bus voltages lower than 69kV, 2.5% at the bus voltages between 69kV and 161kV and 1.5% at the bus voltages higher than 161kV. On the other hand, in IEC 61000-3-6 [12], it is determined as 8% at LV, 6.5% at MV and 3% at HV. However, [13] shows that 5% is the caution level of THD_V for the consumers. Thus, the reasonable THD_V of the PCC voltage can be assumed as 5%. For this voltage distortion level, it is clear that a linear load's n[th] harmonic balanced conductances are very small if there is no any resonance condition in the system. Thus, for a linear load under the voltage with the reasonable THD_V the rms value of the scattred conductance current and scattered conductance power will be very small. For a harmonic producing load under the same voltage distortion level it is feasible that its current still have some harmonic components, which is extremely larger than respective harmonic components of the voltage, in other words it behaves as considerably large conductances for the respective harmonic numbers. Consequently, the rms values of the scattered conductance current and the scattered conductance power drawn by the harmonic producing load still have considerable values for this supply voltage conditions.

According to the manners mentioned above, it can be concluded whether the load has a non-harmonic producing characteristic or not.

III. Conclusion

In this paper, a power resolution is proposed for nonsinusoidal and unbalanced systems. The motivation of the proposed resolution is to provide the direct determination of the power of optimum balanced capacitive compensator and to be used for detection of the harmonic producing loads.

The evaluation results will be given in the part 3 of the study [14].

IV. Appendix

The components of balanced current are orthogonal as in single-phase resolution. Thus, the orthogonality of active & scattered conductance and reactive & scattered susceptance currents given in [2] is again detailed using the single-phase n[th] harmonic voltage and current notations ($V_n\angle\theta_n$ and $I_n\angle\delta_n$) in A.1 and A.2.

A.1 Proof of the orthogonalty between active and scattered conductance currents is detailed below:

The product of active and scattered conductance currents of the single-phase resolution is integrated for a period:

$$\int_{-\frac{T}{2}}^{\frac{T}{2}} i_{ac}(t)\cdot i_{sc}(t)\,dt = TG_e\left[\sum_{n\in N}(G_n - G_e)V_n^2\right] \tag{E.1}$$

By rearranging (E.1); (E.2) is found:

$$\int_{-\frac{T}{2}}^{\frac{T}{2}} i_{ac}(t)\cdot i_{sc}(t)\,dt = TG_e\left[\sum_{n\in N}G_nV_n^2 - G_e\sum_{n\in N}V_n^2\right] \tag{E.2}$$

And then, if $P=\sum_{n\in N}G_nV_n^2$ and $P=G_e\sum_{n\in N}V_n^2$ are substituted in (E.2), the orthogonality is proven as:

$$\int_{-\frac{T}{2}}^{\frac{T}{2}} i_{ac}(t)\cdot i_{sc}(t)\,dt = TG_e\left[P-P\right]=0 \tag{E.3}$$

A.2 Proof of the orthogonalty between reactive and scattered susceptance currents is detailed below:

The product of reactive and scattered susceptance currents of the single-phase resolution is integrated for a period:

$$\int_{-\frac{T}{2}}^{\frac{T}{2}} i_r(t)\cdot i_{ss}(t)\,dt = T\sum_{n\in N^+}(B_n - B_{e,n})B_{e,n}V_n^2 \tag{E.4}$$

By substituting $B_{e,n}=nB_{e,1}=\dfrac{n\sum_{n\in N^+}nV_nI_n\sin(\theta_n\text{-}\delta_n)}{\sum_{n\in N^+}n^2V_n^2}$ and $B_n=\dfrac{V_nI_n\sin(\theta_n-\delta_n)}{V_n^2}$ in (E.4); (E.5) is found:

$$\int_{-\frac{T}{2}}^{\frac{T}{2}} i_r(t)\cdot i_{ss}(t)\,dt = = TB_{e,1}\sum_{n\in N^+}\left[\left(\frac{V_nI_n\sin(\theta_n\text{-}\delta_n)}{V_n^2} - \frac{n\sum_{n\in N^+}nV_nI_n\sin(\theta_n\text{-}\delta_n)}{\sum_{n\in N^+}n^2V_n^2}\right)nV_n^2\right] \tag{E.5}$$

By rearranging (E.5); the orthogonality is proven as:

$$\int_{-\frac{T}{2}}^{\frac{T}{2}} i_r(t)\cdot i_{ss}(t)\,dt =$$

$$= TB_{e,1}\left[\sum_{n\in N^+} nV_nI_n sin(\theta_n\text{-}\delta_n) - \frac{\sum_{n\in N^+} nV_nI_n sin(\theta_n\text{-}\delta_n)}{\sum_{n\in N^+} n^2V_n^2}\sum_{n\in N^+} n^2V_n^2\right] \quad \text{(E.6)}$$

$$= TB_{e,1}\left[\sum_{n\in N^+} nV_nI_n sin(\theta_n\text{-}\delta_n) - \sum_{n\in N^+} nV_nI_n sin(\theta_n\text{-}\delta_n)\right] = 0$$

A.3 Proof of orthogonality between active and unbalanced in phase currents is detailed below:

The product of active and unbalanced in phase currents is integrated for a period:

$$\int_{-\frac{T}{2}}^{\frac{T}{2}} \left(\vec{i}_{up}(t)\right)^{Tr}\cdot \vec{i}_{ac}(t)\,dt = T\sum_n G_eG^u_{a,n}V_n^2 + T\sum_n G_eG^u_{b,n}V_n^2 + T\sum_n G_eG^u_{c,n}V_n^2 \quad \text{(E.7)}$$

By substituting the expressions of $G_{l,n}{}^u$, given in (12), in (E.7) and rearranging the respective equation; (E.8) is found:

$$= T\sum_n G_e\left(G_{a,n}-G_{B,n}\right)V_n^2 + T\sum_n G_e\left(G_{b,n}-G_{B,n}\right)V_n^2 +$$
$$+T\sum_n G_e\left(G_{c,n}-G_{B,n}\right)V_n^2 \quad \text{(E.8)}$$
$$= TG_e\left[\sum_n\left(G_{a,n}+G_{b,n}+G_{c,n}\right)V_n^2 - 3\sum_n G_{B,n}V_n^2\right]$$

And then, by substituting the expressions of $G_{B,n}$ and $G_{l,n}$, given in (7) and (10), in (E.8); (E.8) is transformed to (E.9):

$$= TG_e\left[\sum_n\left(U_{a,n}+U_{b,n}+U_{c,n}\right) - 3\sum_n P_n\right] \quad \text{(E.9)}$$

Finally, by substituting the expression of P_n, given in (5), in (E.9); the orthogonality is proven as:

$$= TG_e\left[\sum_n\left(U_{a,n}+U_{b,n}+U_{c,n}\right) - \sum_n\left(U_{a,n}+U_{b,n}+U_{c,n}\right)\right] = 0 \quad \text{(E.10)}$$

A.4 Proof of orthogonality between scattered conductance and unbalanced in phase currents is detailed below:

The product of scattered conductance and unbalanced in phase currents is integrated for a period:

$$\int_{-\frac{T}{2}}^{\frac{T}{2}} \left(\vec{i}_{up}(t)\right)^{Tr}\cdot \vec{i}_{sc}(t)\,dt =$$
$$= T\sum_n\left(G_{B,n}-G_e\right)G^u_{a,n}V_n^2 + T\sum_n\left(G_{B,n}-G_e\right)G^u_{b,n}V_n^2 + T\sum_n\left(G_{B,n}-G_e\right)G^u_{c,n}V_n^2$$
$$\text{(E.11)}$$

By substituting the expression of $G_{l,n}{}^u$, given in (12), in (E.11) and rearranging respective equation; (E.12) is found:

$$= T\sum_n\left(G_{B,n}-G_e\right)\left(G_{a,n}-G_{B,n}\right)V_n^2 + T\sum_n\left(G_{B,n}-G_e\right)\left(G_{b,n}-G_{B,n}\right)V_n^2 +$$
$$+T\sum_n\left(G_{B,n}-G_e\right)\left(G_{c,n}-G_{B,n}\right)V_n^2$$
$$= T\sum_n\left[\left(G_{B,n}-G_e\right)\left(G_{a,n}+G_{b,n}+G_{c,n}-3G_{B,n}\right)V_n^2\right]$$
$$\text{(E.12)}$$

And then, by substituting the expressions of $G_{B,n}$ and $G_{l,n}$, given in (7) and (10), in (E.12); (E.12) is transformed to (E.13):

$$= T\sum_n\left[\left(\frac{P_n}{V_n^2}-G_e\right)\left(U_{a,n}+U_{b,n}+U_{c,n}-3P_n\right)\right] \quad \text{(E.13)}$$

Finally, by substituting the expression of P_n, given in (5), in (E.13); the orthogonality is proven as:

$$= T\sum_n\left[\left(\frac{\frac{1}{3}\left(U_{a,n}+U_{b,n}+U_{c,n}\right)}{V_n^2}-G_e\right)\left(U_{a,n}+U_{b,n}+U_{c,n}-U_{a,n}-U_{b,n}-U_{c,n}\right)\right]$$
$$= 0$$
$$\text{(E.14)}$$

A.5 Proof of orthogonality between reactive and unbalanced quadrature currents is detailed below:

First, the product of reactive and unbalanced quadrature currents is integrated for a period:

$$\int_{-\frac{T}{2}}^{\frac{T}{2}} \left(\vec{i}_{uq}(t)\right)^{Tr}\cdot \vec{i}_{r}(t)\,dt =$$
$$= T\sum_n B_{e,n}B^u_{a,n}V_n^2 + T\sum_n B_{e,n}B^u_{b,n}V_n^2 + T\sum_n B_{e,n}B^u_{c,n}V_n^2 \quad \text{(E.15)}$$

By substituting the expressions of $B_{l,n}{}^u$, given in (13), in (E.15) and rearranging respective equation; (E.16) is found:

$$= T\sum_n B_{e,n}\left(B_{a,n}-B_{B,n}\right)V_n^2 + T\sum_n B_{e,n}\left(B_{b,n}-B_{B,n}\right)V_n^2 +$$
$$+T\sum_n B_{e,n}\left(B_{c,n}-B_{B,n}\right)V_n^2 \quad \text{(E.16)}$$
$$= T\sum_n\left[B_{e,n}\left(B_{a,n}+B_{b,n}+B_{c,n}-3B_{B,n}\right)V_n^2\right]$$

And then, by substituting the expressions of $B_{B,n}$ and $B_{l,n}$, given in (8) and (11), in (E.16); (E.16) is transformed to (E.17):

$$= T\sum_n\left[B_{e,n}\left(Q_{a,n}+Q_{b,n}+Q_{c,n}-3Q_n\right)\right] \quad \text{(E.17)}$$

Finally, by substituting the expression of Q_n, given in (6), in (E.17); the orthogonality is proven as:

$$= T\sum_n\left[B_{e,n}\left(Q_{a,n}+Q_{b,n}+Q_{c,n}-Q_{a,n}-Q_{b,n}-Q_{c,n}\right)\right] = 0 \quad \text{(E.18)}$$

A.6 Proof of orthogonality between scattered susceptance and unbalanced quadrature currents is detailed below:

First, the product of scattered susceptance and unbalanced quadrature currents is integrated for a period:

$$\int_{-\frac{T}{2}}^{\frac{T}{2}} \left(\vec{i}_{uq}(t)\right)^{Tr}\cdot \vec{i}_{ss}(t)\,dt =$$
$$= T\sum_n\left(B_{B,n}-B_{e,n}\right)B^u_{a,n}V_n^2 + T\sum_n\left(B_{B,n}-B_{e,n}\right)B^u_{b,n}V_n^2 + \quad \text{(E.19)}$$
$$+T\sum_n\left(B_{B,n}-B_{e,n}\right)B^u_{c,n}V_n^2$$

By substituting the expressions of $B_{l,n}{}^u$, given in (13), in (E.19) and rearranging respective equation; (E.20) is found:

$$
\begin{aligned}
&= T\sum_n \left(B_{B,n} - B_{e,n}\right)\left(B_{a,n} - B_{B,n}\right)V_n^2 + T\sum_n \left(B_{B,n} - B_{e,n}\right)\left(B_{b,n} - B_{B,n}\right)V_n^2 \\
&+ T\sum_n \left(B_{B,n} - B_{e,n}\right)\left(B_{c,n} - B_{B,n}\right)V_n^2 \\
&= T\sum_n \left[\left(B_{B,n} - B_{e,n}\right)\left(B_{a,n} + B_{b,n} + B_{c,n} - 3B_{B,n}\right)V_n^2\right]
\end{aligned}
\tag{E.20}
$$

And then, by substituting the expressions of $B_{B,n}$ and $B_{l,n}$, given in (8) and (11), in (E.20); (E.20) is transformed to (E.21):

$$
= T\sum_n \left[\left(\frac{Q_n}{V_n^2} - B_{e,n}\right)\left(Q_{a,n} + Q_{b,n} + Q_{c,n} - 3Q_n\right)\right] \tag{E.21}
$$

Finally, by substituting the expression of Q_n, given in (6), in (E.21); the orthogonality is proven as:

$$
\begin{aligned}
&= T\sum_n \left[\left(\frac{\frac{1}{3}\left(Q_{a,n} + Q_{b,n} + Q_{c,n}\right)}{V_n^2} - B_{e,n}\right)\left(Q_{a,n} + Q_{b,n} + Q_{c,n} - Q_{a,n} - Q_{b,n} - Q_{c,n}\right)\right] \\
&= 0
\end{aligned}
\tag{E.22}
$$

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