A Power Resolution for Nonsinusoidal and Unbalanced Systems-
Part I: Literature Overview and Motivation

M. E. Balci and M. H. Hocaoglu

Abstract-- **This study presents an overview on the apparent power definitions and power resolutions widely known in the literature. It is shown from the literature overview that there is a need for a power resolution, which can be used as a tool for the direct provision of the optimum compensation capacitor's power in unbalanced & nonsinusoidal systems. In addition to that, it is also mentioned that the modern power resolutions should be useful to detect the harmonic producing loads.**

Index Terms—**Apparent power definitions, power resolutions, nonsinusoidal conditions, unbalanced systems, optimum capacitive compensation, harmonic source detection.**

I. Introduction

By the proliferation of a.c. in the transmission and distribution systems, apparent power was defined as the product of voltage and current rms values to size the system equipment and to be a measure for the system efficiency. Historically, the current of the system was divided into two parts: These are; active current, which transports the net energy from source to the load, and reactive current that is the remaining current component when the active part is subtracted from the total current. According to this resolution, apparent power was expressed as the vector sum of active and reactive powers, which flows due to the active and reactive currents, respectively. Sequently, the ratio of active and apparent powers is named as the power factor, and be utilised to measure the efficiency of the power systems. In addition, conventionally, classical single-phase apparent power is directly extended to three-phase systems by treating each phase individually. Thus, arithmetic apparent power, which is calculated as the arithmetic sum of each phase's apparent power, and vector apparent power, which is calculated as the vector sum of total active and total reactive powers of the system, were constituted for the three-phase systems [1], [2]. Nevertheless, due to the fact that the classical apparent power and its resolution are defined under sinusoidal and balanced conditions, they did not attain their goals in the case of nonsinusoidal and/or unbalanced conditions. Consequently, a number of apparent power definitions and their resolutions have been proposed for nonsinusoidal single-phase and nonsinusoidal & unbalanced three-phase systems to fulfil the gap left out in the classical apparent power concept.

This study figures out that the current status of the literature on the frequency domain power theory and presents the qualitative and quantitative analysis on the widely known apparent power definitions and their resolutions with respect to the optimum capacitive compensation and the detection of the harmonic producing loads.

II. Optimum Capacitive Compensation for Nonsinusoidal Single-Phase Systems

For single-phase systems, there are three distinct definition of the apparent power; namely: classical apparent power [1] and the apparent powers defined by Wilcynski [3] and Ghassemi [4], [5]. The classical apparent power definition gives information on the losses of the system and provides a tool for the effective utilisation of the system. On the other hand, the definitions proposed by Wilcynski and Ghassemi consider the oscillations of instantaneous and complex instantaneous powers, respectively. Thus, both definitions are more relevant for the sizing of the system and its equipment. Due to the fact that the losses are more significant than the sizing for the proper utilisation of the system, classical apparent power is still recommended [2] and enforced by standards [6].

In addition, it is clear that the reactive powers placed in the resolutions of Wiclynski's apparent power and Ghassemi's apparent power, of which expressions are given in Table I (appendix), does not provide the useful information on the power of the optimum compensation capacitor due to the fact that both apparent powers do not consider system efficiency.

The resolutions of classical apparent power, proposed by Budeanu, Fryze, Kimbark, Shepherd & Zand, Sharon, Depenbrock, Kusters & Moore, Czarnecki and IEEE std. 1459-2010, are widely recognized and employed for various purposes in the literature [2], [7].

Budeanu's power resolution, first proposed power resolution for nonsinusoidal systems, has the reactive power component, which is calculated as the sum of individual harmonic reactive powers.

In second one, Fryze's power resolution, the reactive power is defined as the vector difference between classical apparent

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M. E. Balci is with Balikesir University, Balikesir, TURKEY (e-mail: mbalci@balikesir.edu.tr).

M. H. Hocaoglu is with Gebze Institute of Technology, Kocaeli, TURKEY (e-mail: hocaoglu@gyte.edu.tr).

power and active power.

In another power resolution, the power resolution of Shepherd and Zand, the reactive power is defined as the product of the voltage's rms value and the total rms value of the individual harmonic reactive currents.

The reactive power has the same calculation in the power resolutions proposed by Sharon and Shepherd & Zand.

Depenbrock's power resolution has the reactive power component, which is the product of the voltage's rms value and the fundamental harmonic reactive current's rms value.

Kusters and Moore expressed the reactive current as in phase with the derivative of voltage for the inductive loads. On the other hand, the reactive current was expressed as in phase with the integral of voltage when the load is capacitive. Thus, they calculated the reactive power by production of the rms values of voltage and the reactive current for their resolution.

In Czarnecki's power resolution, the reactive power is calculated by production of the voltage's rms value and the total rms value of the individual harmonic currents, which are drawn by n^{th} harmonic susceptance of the load.

In addition to the above mentioned reactive power definitions, fundamental harmonic reactive power is placed in Kimbark's power resolution and IEEE std. 1459-2010 power resolution.

For nonsinusoidal conditions a close examination of the reactive powers placed in these classical apparent power resolutions, of which expressions given in Table I, reveals some points [7]:

- Fundamental harmonic reactive power, Budeanu's reactive power and Depenbrock's reactive power do not give any useful information on the power of optimum compensation capacitor, which maximizes power factor.
- The reactive powers defined in the resolutions of Fryze, Shepherd & Zand, Sharon and Czarnecki could be used to find the power of optimum compensation capacitor with extra mathematical efforts.
- Finally, only Kusters & Moore's reactive power can be handled for the direct provision of optimum compensation capacitor's power when the terminal voltage is almost constant during the compensation and there is not any resonance in the system.

III. Optimum Capacitive Compensation for Nonsinusoidal & Unbalanced Three-Phase Systems

For nonsinusoidal and unbalanced systems apparent power definitions and power resolutions are briefly summarized in this section. The reactive power components of the resolutions are also quantitatively analyzed with respect to the optimum capacitive compensation.

A. Apparent power definitions and power resolutions

For nonsinusoidal and unbalanced conditions, the earliest apparent power found in the literature is that the vector apparent power. It is defined as the vector sum of Budeanu's power components calculated for each phase [1], [2], [8]. On the other hand, arithmetic apparent power is still recommended as a measure for the effectiveness of nonsinusoidal and unbalanced systems [2]. Both apparent powers were defined by treating each phase individually. However, this assumption is only valid for balanced and sinusoidal three-phase systems. Thus, they are not proportional or linearly related to line loss and they could not give true information on the system efficiency in nonsinusoidal and unbalanced conditions. Accordingly in [9], by treating the system as a single unit Buchollz defined apparent power as the product of the collective rms values of three-phase voltages and currents. In a follow up study [10], he extended his definition to m-line systems. At the present time, the extended version of this definition is encouraged by DIN std. 40110 [6].

In another important standard on the power measurement, IEEE std. 1459-2010 [2], the apparent power is calculated by the product of the equivalent rms values of three-phase voltages and currents. For nonsinusoidal and unbalanced conditions, [11] and [12] pointed out that the IEEE std. 1459 apparent power definition gives the maximal active power, which can be transferred under ideal conditions (sinusoidal and balanced) with the same voltage and current rms values. On the other hand, in the same studies, it is also mentioned that Bucholz's apparent power is the maximal active power, which can be transferred for the given voltage waveform and the given current rms value. In addition, two main apparent powers proposed by Bucholz and IEEE std. 1459 give the same results for the systems without zero sequence voltages.

In the literature, for three-phase systems mainly there are five power resolutions: These are; the resolutions of the Vector and IEEE apparent powers [2] and three different resolutions of Bucholz's apparent power proposed by Czarnecki [13], DIN std. 40110 [6] and Ari & Stankovich [14].

The major properties of the reactive powers placed in these resolutions can be expressed as;

- Vector apparent power has the reactive power, which is the sum of Budeanu's reactive powers calculated for each phase.
- IEEE std. 1459-2010 resolution contains fundamental harmonic-positive sequence reactive power.
- Czarnecki's resolution, which is proposed for nonsinusoidal three-phase system with balanced voltages and unbalanced currents, has the reactive power component, which is product of the collective rms values of three phase voltages and reactive currents drawn by n^{th} harmonic susceptance of the load impedance's balanced part.
- DIN std. 40110 power resolution contains reactive power component, which is caused by the parts of line currents orthogonal to the line-to-virtual star point voltages.
- Ari & Stankovich defined temporal and circulating reactive powers in their resolution. Temporal reactive power is caused by the oscillation of the instantaneous power in other words unbalance of the load. Circulating reactive power is the vector difference among apparent, active and temporal reactive powers.

In addition to the studies mentioned above, it should be noted that Ghassemi expanded his apparent power definition and resolution proposed for the single-phase systems to the three-phase systems by treating each phase individually [4]. However, his power resolution is not useful for the measurement and improvement of the system efficiency in three-phase systems as in single-phase systems.

B. Analysis

For nonsinusoidal and unbalanced systems, it is clear that the reactive power components of Vector and IEEE std. 1459-2010 apparent power resolutions and Czarnecki's power resolution can not be used for the direct determination of the optimum compensation capacitors bank, which minimizes line loss or maximizes system efficiency, as in nonsinusoidal single-phase systems. Accordingly, the reactive powers placed in DIN std. 40110 resolution and the resolution proposed by Ari & Stankovich should be evaluated with respect to optimum capacitive compensation. The evaluation is done by using a balanced capacitors bank due to the fact that the capacitor banks are produced as a delta or star connection of the identical capacitors for their practical operation in three-phase and three-line systems.

For the evaluation, a three-phase and three-line case, which has the voltages and currents given in Fig. 1 and Fig. 2, are used.

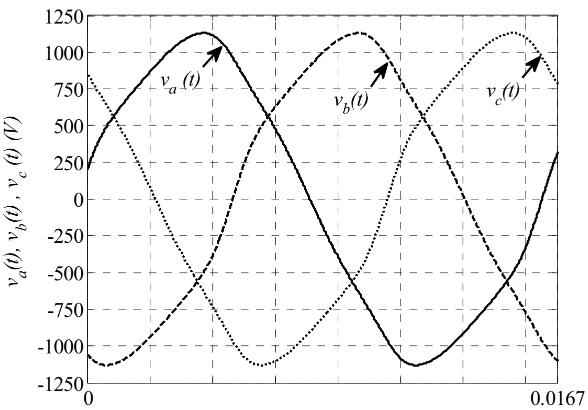

Fig. 1: The wave shapes of the phase-to-neutral point terminal voltages.

Phase-to-neutral voltages of the test case, given in Fig. 1, have balanced and nonsinusoidal wave shapes with *THD* values measured as 5%. Line (a, b and c phases) currents of the test case, given in Fig. 2, have unbalanced and nonsinusoidal wave shapes with I_1^- / I_1^+ measured as 26.35 % and *THD* values measured as 14 %, 22 % and 16 % for a, b and c phases. In the test system, to find the maximum power factor value, which can be obtained via a balanced capacitors bank, the variations of the power factor, active power and the apparent power during the increment of the power, S_C, of the balanced capacitors bank are plotted in Fig. 3. Note that, in this analysis, the apparent power at the terminal, the power of the balanced capacitors bank and power factor are calculated with respect to Bucholltz's apparent power ($S = V_\Sigma I_\Sigma = \sqrt{\sum_m V_{m0}^2}\sqrt{\sum_m I_m^2}$) currently placed in DIN std. 40110. Furthermore, IEEE std. 1459-2010 and DIN std. 40110 apparent powers and the power factors, which are calculated according to both apparent powers, have same numerical values for the test case due to the fact that the voltages do not have zero sequence components.

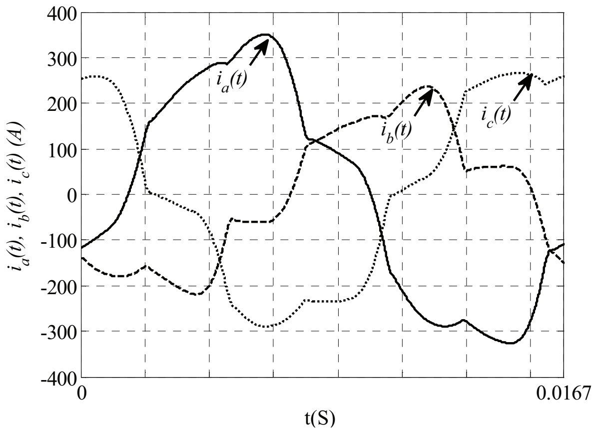

Fig. 2: The wave shapes of the line currents.

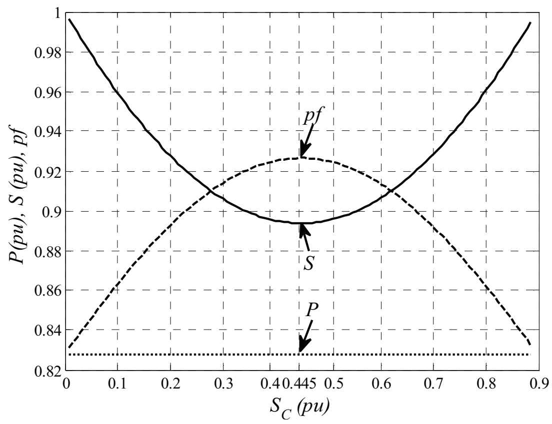

Fig. 3: The variations of the power factor, active power and apparent power during the increment of the power of the balanced capacitors bank.

It is shown from Fig. 3 that power factor (pf) is improved from 0.828 to 0.926 by using the optimum balanced capacitor bank, of which power (S_C) is 0.445 pu.

For the system without compensation and with the optimum balanced capacitive compensation, the reactive power ($Q_{tot\Sigma\perp}$) of the DIN std. 40110 resolution is measured as 0.531 pu and 0.288 pu, respectively. Again for the system without compensation, circulating reactive power (Q_x) of the Ari & Stankovich's resolution is measured as 0.529 pu. In addition to that, for the optimum balanced capacitive compensation case, Q_x is measured as 0.275 pu.

It is seen from the analysis that the reactive powers of both resolutions do not give the direct provision of the optimum balanced capacitors bank.

IV. Harmonic Source Detection

Several harmonic source detection and quantification methods exist in the literature [15]. However, a few of them use active and nonactive powers in other words based on the power resolutions [16]-[20]. According to the method based on active power (Active Power Direction, APD, method), load side is considered as the main harmonic source for a harmonic number when the respective harmonic active power has a negative sign; otherwise, main harmonic source is the utility side for the respective harmonic number [16]. However, it is shown that this method may not be reliable for some cases [21].

The method, given in [17], compares three different nonactive powers, which are fundamental harmonic reactive, Sharon's reactive and Fryze's reactive powers, for the detection of the main harmonic source at PCC. According to the method, in the case of a nonsinusoidal supply voltage and a linear load, Sharon's reactive power should be considerably closer to the fundamental harmonic reactive power than to Fryze's reactive power. On the contrary, in the case of a sinusoidal supply voltage and a nonlinear load, Sharon's reactive power should be considerably closer to Fryze's reactive power than to the fundamental harmonic reactive power. For the case consists of a nonsinusoidal supply voltage and a nonlinear load, Sharon's reactive power can be assumed as an intermediate value between fundamental harmonic reactive power and Fryze's reactive power. When supply voltage is sinusoidal and load is linear, all three reactive powers give the same numerical values.

In [18], Sharon's reactive power is expressed from voltage, current and power components defined in IEEE std. 1459 resolution. Therefore, the aforementioned method can entirely be implemented in the time domain. On the other hand, in [17] and [18], the method is applied to the three-phase four-wire systems by considering each of the nonactive powers as the sum of the respective phase quantities. However, in both nonsinusoidal and unbalanced three-phase systems, the method was not enough for the direct separation of unbalance and harmonic distortion effects when this extension is used. Consequently, in [19], three nonactive power quantities, which are conceptually similar to those of the previous case of the method, are derived from the resolution of IEEE std. 1459-2010 three-phase apparent power definition.

Finally, in one of the most recent studies on the subject [20], by using the harmonic numbers in consumer and utility responsibilities determined according to APD method IEEE std. 1459-2010 power components are divided into three groups: the powers measured at fundamental frequency, frequencies associated with the utility side distortion and frequencies associated with consumer distortion. Thus, several power factors were resulted from the new proposal by concerning the responsibility of consumer and utility sides for the harmonic distortion.

V. Conclusion

From the literature overview presented in this study, it can be concluded that there is a need for a power resolution, which can be used as a tool for the direct determination of the power of optimum balanced capacitive compensator in unbalanced & nonsinusoidal systems.

It can also be mentioned that proposing a power resolution, which can be employed to detect harmonic producing loads, will be a contribution to the power theory.

VI. References

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VII. Appendix

Table I: The expressions of the reactive powers defined in the widely known power resolutions proposed for nonsinusoidal single-phase systems.

	Reactive Power Expressions
Budeanu's Resolution	$Q_b = \sum_{n\in N^+} Q_n = \sum_{n\in N^+} V_n I_n sin\varphi_n$
Fryze's Resolution	$i_{ac}(t) = \frac{P}{V^2} v(t),\ \ i_{fre}(t) = i(t) - i_{ac}(t),\ \ Q_f = VI_{fre}$
Kimbark's Resolution	$Q_k = V_1 I_1 sin\varphi_1$
Shepherd & Zand's Resolution	$I_{Im} = \sqrt{\sum_{n\in n_1} I_n^2 \sin^2\varphi_n},\ V = \sqrt{\sum_{n\in n_1\cup n_2} V_n^2},\ S_{Im} = VI_{Im} = \sqrt{\sum_{n\in n_1\cup n_2} V_n^2}\, I_{Im}$ *n_1 and n_2 denote that the common harmonic numbers of voltage & current and the harmonic numbers which exist in voltage not in current, respectively.*
Sharon's Resolution	S_{Im}
Depenbrock's Resolution	$v_g(t) = \sqrt{2}V_1 sin(\omega_1 t + \theta_1),\ \ i_q(t) = \frac{\frac{1}{T}\int_0^T i(t) v_g(t - \frac{T}{4}) dt}{V_g^2} v_g(t - \frac{T}{4}),\ \ Q_q = VI_q$
Kusters & Moore's Resolution	$i_{qc}(t) = \frac{\frac{1}{T}\int_0^T \overset{\circ}{v}(t) i(t) dt}{\overset{\circ}{V}{}^2} \overset{\circ}{v}(t),\ Q_{kus} = VI_{qc}$ *where* $\overset{\circ}{v}(t) = \frac{dv}{dt}$ *for inductive loads*
Czarnecki's Resolution	$I_R = \sqrt{\sum_{n\in n_1} B_n^2 V_n^2},\ \ Q_R = VI_R$
IEEE std. 1459-2010's Resolution	$Q_1 = V_1 I_1 sin\varphi_1$
The Resolution of Ghassemi's Apparent Power	$Q_G = Im\left(\frac{1}{T}\int_0^T \overline{p}(t) dt\right)$ *where* $\overline{p}(t) = \overline{v}(t) i(t)$ *and* $\overline{v}(t) = \sum_n \sqrt{2} V_n e^{j(\omega_n t + \theta_n)}$
The Resolution of Wilcynski's Apparent Power	$Q_W = \sqrt{\frac{1}{T}\int_0^T p_{\text{var}}(t)^2 dt}$ *where* $p_{\text{var}}(t)$ *and active power are two orthogonal parts of* $p(t)$.

Table II: The expressions of the reactive powers defined in the widely known power resolutions proposed for nonsinusoidal and unbalanced three-phase systems.

	Reactive Power Expressions
Vector Apparent Power's Resolution	$Q_V = \sum_{m=a,b,c} Q_m, \quad Q_m = \sum_n V_{mn} I_{mn} \sin\varphi_{mn}$ *V_{mn} and I_{mn} are n^{th} harmonic rms values of the voltage measured between m^{th} line and neutral point and the current measured at m^{th} line.* *φ_{mn} is the phase angle difference between n^{th} harmonic of m^{th} line-to-neutral voltage and m^{th} line current.*
Czarnecki's Resolution	$I_{\Sigma R} = \sqrt{\sum_n \left[B_{ne}^2 V_{\Sigma n}^2\right]}, \; Q_R = V_\Sigma I_{\Sigma R}$ *where* $V_{\Sigma n}^2 = \sum_{m=a,b,c} V_{mn}^2, \; V_\Sigma = \sqrt{\sum_n V_{\Sigma n}^2}$ *and* $B_{ne} = \dfrac{\left(\sum_{m=a,b,c} V_{mn} I_{mn} \sin\varphi_{mn}\right)}{V_{\Sigma n}^2}$
DIN std. 40110 Resolution	$Q_{tot\,\Sigma\perp} = V_\Sigma \sqrt{\sum_m \left[I_m^2 - G_m^2 V_{m0}^2\right]}$ *where* $v_{m0}(t) = v_{m\rho}(t) - \frac{1}{N_m}\sum_m v_{m\rho}(t)$, $V_\Sigma = \sqrt{\sum_m V_{m0}^2}$ *and* $G_m = P_m / V_{m0}^2$ *ρ denotes any refence line for the voltage measurement and it is generally selected as neutral line in the three-phase and four-line systems. N_m is the number of the total lines. $v_{mo}(t)$ is the voltage between m^{th} line and virtual neutral point (0).*
Ari & Stankovich's Resolution	*for* $v_\Sigma(t) = \begin{bmatrix} v_{a0}(t) \\ v_{b0}(t) \\ \vdots \\ v_{m0}(t) \end{bmatrix}, i_\Sigma(t) = \begin{bmatrix} i_a(t) \\ i_b(t) \\ \vdots \\ i_m(t) \end{bmatrix}, \; v_{m0}(t) = v_{m\rho}(t) - \frac{1}{N_m}\sum_m v_{m\rho}(t)$ $i_{\Sigma ac}(t) = \frac{P}{V_\Sigma^2} v_\Sigma(t), \; i_{\Sigma y}(t) = i_{\Sigma p}(t) - i_{\Sigma ac}(t), \; i_{\Sigma x}(t) = i_\Sigma(t) - i_{\Sigma p}(t), \; i_{\Sigma p}(t) = \frac{i_\Sigma^{Tr}(t) v_\Sigma(t)}{v_\Sigma^{Tr}(t) v_\Sigma(t)} v_\Sigma(t) = \frac{p(t)}{v_\Sigma^{Tr}(t) v_\Sigma(t)} v_\Sigma(t)$ $I_{\Sigma y} = \sqrt{\frac{1}{T}\int_0^T i_{\Sigma y}^{Tr}(t) i_{\Sigma y}(t)\,dt}, \; I_{\Sigma x} = \sqrt{\frac{1}{T}\int_0^T i_{\Sigma x}^{Tr}(t) i_{\Sigma x}(t)\,dt}, \; V_\Sigma = \sqrt{\sum_m V_{m0}^2}$ $Q_y = V_\Sigma I_{\Sigma y}$ (Temporal reactive power) and $Q_x = V_\Sigma I_{\Sigma x}$ (Circulating reactive power)
IEEE std. 1459-2010 Resolution	$Q_1^+ = 3V_1^+ I_1^+ \sin\varphi_1^+$
The Resolution of Ghassemi's Apparent Power	$Q_G = \mathrm{Im}\left(\frac{1}{T}\int_0^T \sum_{m=a,b,c} \overline{p}_m(t)\,dt\right)$