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A research on proof perceptions and attitudes towards proof and proving: some implications for elementary mathematics prospective teachers

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Abstract

The study was conducted with elementary mathematics prospective teachers in 2011-2012 academic years in the education faculty of Necatibey. In this research, the prospective teachers' proof perceptions, the effect of proof perceptions to the process of proving; whether their attitudes towards proof and proving differentiate concerning their proof perceptions were studied. It was found that the types of proof perceptions of the prospective teachers had an effect on the process of proving and their attitudes towards proof and proving could change concerning their proof perception. A typed prospective teachers got higher attitude scores towards proof and proving than the others.

Keywords: Proof perceptions; formal-informal proof; attitudes towards proof and proving; elementary mathematics prospective teachers.

1. Introduction

There are serious difficulties encountered by the elementary mathematics prospective teachers in learning to do formal mathematical proofs. Mathematical proof provides a warrant for mathematical knowledge and it is an essential activity in doing and understanding mathematics. Proving is a tool in mathematics learning. Proving qualifications have an important role on acquisition of gain of Analysis, Abstract Mathematics, and General Mathematics courses in Mathematics education department. For this reason, it is important to determine proof perceptions of the elementary mathematics prospective teachers.

In this study, the answers for the following questions were searched:1.What are the proof perceptions of prospective teachers?,2. How do prospective teachers' proof perceptions affect their process of proving?,3. Do the attitudes towards proof and proving of prospective teachers differentiate concerned their proof perceptions?

2. Method

The study was carried out with 67 elementary mathematics prospective teachers of 4th grade in the spring term of 2010–2011 academic years. Study group was chosen randomly. Both quantitative and qualitative research methods

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were used in the study. The attitudes towards proof and proving of the prospective teachers who attended to the study were determined by using the scale developed by Üzel and Özdemir (2009). Cronbach Alpha's reliability coefficient of the whole scale was found as .897; Cronbach Alpha's reliability coefficient of attitudes towards proving (factor1) and general aspect to proof (factor 2) were respectively found as .87 and .89. Quantitative data were analyzed by using statistical methods. "Proof preference questionnaire" consisted of two parts, was prepared by researchers in order to determine proof perceptions and proving process of the prospective teachers. In the first part, prospective teachers were wanted to prove the given theorems by using proving methods. In the second part there are 4 theorems and related proofs. Prospective teachers were asked to decide which of the 'proofs' they find the most convincing, not convincing and incorrect and give brief reasons for their choices.

3. Findings

3.1. Determination of proof perceptions of the prospective teachers

Proof perceptions of prospective teachers were determined based on the types clarified in Almeida's study by analyzing the choices (Almeida, 2000). The types are clasified as fallows,

Type A: The student accepts the need for working with formal proof and rejects informal proof, but he/she is unable to live up the demands of rigueur in his/her own proof practices.

Type B: The student accepts the need for formal proof but provisionally uses informal proof practices till she/he becomes adapted on formal ones.

Type C: The student accepts intuitive and empirical arguments as proof. He/she views formal proof in terms of passing examinations.

Type D: The student accepts the need for formal proof but generally he/she is only able to view this as a symbol manipulation. The lack of understanding leads to disaffection in proof.

It was determined that 8(%12) of the prospective teachers were A typed. The prospective teachers in this type find formal proof necessary, they refuse informal proof. They preferred formal proofs suitable for this view in proof choice questionnaire and they evaluated them as "the most convincing". They have chosen informal proofs as "unconvincing" or "wrong". A typed prospective teachers found the proofs based on inductive and deductive more convincing, they indicated that they could not understand the proofs based on informal ways and they found them complicated. The view of an A typed prospective teacher supports this finding: *"I preferred formal proof because the other proofs did not satisfy me, I could not understand because of the usage of shapes and diagrams. I found the proof based on inductive more clear. If I proved this theorem, I would prefer formal way."* It was determined that 37(%55) of the prospective teachers were B typed. These typed prospective teachers found formal proof necessary; they saw informal proofs as a tool until they were adapted to formal proofs. Prospective teachers found 3rd proof which was given for the first theorem (The sum of the angles of a plane triangle is 180^0) as the most convincing. In this proof; "the sum of angles in a straight line about a point is 180^0" and "alternate angles are equal" assumptions/results were used. In the 2nd theorem, they found both proving by means of setting value and the proof constructed with drawing a graph as the most convincing. It was found in the 3rd theorem that the proof based on inductive was the most convincing. When the prospective teachers' views were analyzed; this proof is either a conception resulting from axioms and the results which have been proved before or an inference which validates for all values. It was determined that if there was a theorem, the proof was actually similar to generalization. It was concluded that these views affected choices of the prospective teachers.It is determined that 6(%9) of the prospective teachers were C typed. They found formal proof difficult because of the technical difficulties; these typed students preferred informal proof and relied on their intuitions and experiences mostly. In the 2nd theorem, it was seen that the prospective teachers generally preferred informal proof. A prospective teacher in this type indicated that "I find the 2nd proof the most convincing because it is more understandable, I understand visual proofs more easily." Another prospective teacher in C type indicated that he found the proof based on inductive suspicious in the 3rd theorem and he found formal proof the most convincing. It was determined that 16 (%24) of the prospective teachers were D typed. Prospective teachers in this type admitted the necessity of formal proof but they saw formal proof as a symbolic manipulation. One of the D typed prospective teachers who preferred informal proof

ordered the reasons of his choices: "I understand the proofs which are constructed by using diagrams and shapes better because I slog on understanding the proofs which are constructed by using the symbols. I memorize these typed proofs. I find informal proofs more simple and understandable. According to me formal proof is complicated and abstract."

3.2 Effects of prospective teachers' proof perceptions to their processes of proving

The prospective teachers were wanted to prove three theorems in order to examine their process of proving. In the light of the obtained data, the proving processes of the students related to their proof perception types were analyzed descriptively. The frequency distribution accordance with the ways(formal and informal) of the prospective teachers prefer while proving theorems considering with their perception types and wrong proving situations of theorems were determined. When we considered the distribution of proving preferences of theorems of the prospective teachers who participated in the study, it was indicated that %25 of them make the proof incorrect. This obtained result showed in line with the results of the study made by Powers et al. (2010). When the processes of proving were studied, A typed prospective teachers preferred formal ways while proving but none of the teachers preferred informal way. Moreover, it was seen that they largely preferred proving ways based on inductive and deductive, their priorities were concept and descriptions while proving. Obtained findings supported that formal proof was seen necessary by A typed students but they refused informal proof. It was seen that, B typed prospective teachers preferred formal proof way rather than informal ones but, some of B typed prospective teachers preferred informal proof ways unlike A typed ones. It was indicated that these typed prospective teachers also used some kind of informal ways while reaching formal proof and they preferred to reach to the conclusion by evaluating previously proven results and axioms. Additionally, it was seen that they used mathematical language and symbols truer than prospective teachers in other perception types. It was indicated that C typed prospective teachers mostly preferred informal proof ways when it was compared with formal proof ways. It was stated that this typed prospective teachers generally used shapes and diagrams in the process of proving and they preferred generalizing instead of being overwhelm with mathematical language and symbols. It is seen that some prospective teachers also try to generalize by considering a specific example of the given theorem. It was shown that D typed prospective teachers preferred informal and formal proof ways but they could not reach conclusion because of misusing the mathematical language and symbols. Instead of applying different ways, they tried to prove the proofs by constructing the proofs on a prepared method. The findings concerning the differences of the proving processes of the prospective teachers and the finding concerning the study made by Almedia (2003) showed parallelism. It was seen that proof perception types affected the proving processes.

3.2. The prospective teachers' attitudes towards proof and proving and their proof perceptions

MANOVA analysis in program SPSS 12.0 was used to see whether the attitudes of prospective teachers towards proof and proving considering with their proof perceptions changed or not. Kolmogorow-Smirnow ($p>.05$) test was utilized to control the situation of normal distribution. After obtaining the descriptive statistic about scores, Levene test was utilized to control the variance homogeneity of groups which were compared. Furthermore Box's M statistic was utilized for the control of covariance equality assumption in MANOVA test. Equalities of variance and covariance of the groups were found as a result. In order to see whether prospective teachers' attitudes towards proving and general aspect of proof changed or not considering with their perception types were analyzed. When quantitative data were studied, it was seen that factor 1 mean scores for Type A, Type B, Type C and Type D were in order of 3.87, 2.99, 3.05, 2.83 and factor 2 mean scores were in order of 3.63, 3.59, 3.09, 2,89. Covariances related to mean scores of factor 1 and factor 2 were found homogenous. ($F(9\text{-}2597{,}988)= 1.74$, $p>.05$). For the suitability of data variance analyze Levene test was made and it was seen that variances were equal and assumptions were ensured. ($F_1(3\text{-}63)=.459$ and $F_2(3\text{-}63)=.407$; $p>.05$). Whether mean differences of factor 1 and factor 2 determined among proof perceptions were meaningful or not was studied with variance analyze and mean differences were found significantly Wilks Lambda (Λ)=.58, $F(6\text{-}124)=6.47$; $p<.01$). This finding showed that the scores obtained from linear component aroused from factor 1 and factor 2 scores changed considering with perception types. The difference determined considering with perception types, $F(3\text{-}63)=6.15$; $p<.01$ for the mean

score of factor 1 and F(3-63)=4.86; p<.01) for the mean score of factor 2 was meaningful. Tukey-HDS test was carried out to see the meaningfulness arises from which perception type. According to the obtained data, the difference between the mean scores of factor1 of A and B typed prospective teachers was found as .88 and the difference between the mean scores of Factor1of A and D typed prospective teachers was found as 1.04. It was seen that the scores of attitudes towards the proving of B and D typed prospective teachers were lower than A typed ones. The difference between mean scores of Factor2 of B and D typed prospective teacher was found as .70. It was seen that the scores of attitudes towards proof of D typed prospective teachers were lower than B typed ones. These all differences were found meaningful in terms of p values (p<.05). As a result, the scores of attitudes towards proof and proving of A typed prospective teachers were higher than the others.

4. Conclusion and Discussion

In this study the following results are obtained;

1. It was seen that %12 of prospective teachers were A typed, %55 of them were B typed, %9 of them were C typed and % 24 of them were D typed. A typed prospective teachers found formal proofs necessary and refuses informal proof. In A type he proofs based on inductive and deductive were found convincing, proofs based on informal ways were found complicated so informal proofs were not understood. B typed prospective teachers found formal proofs necessary but informal proofs were seen as a tool until adapting to formal proofs. It was determined that B typed prospective teachers expressed that proof actually appeared similar to generalization. When results were evaluated, most of the prospective teachers stated that proof was a conception resulting from previously proven results and axioms or an inference which validated a theorem for all values. However, in Lee's study, the aim of proof was not only clean a trueness of a proposition from all suspicions but also showed the dependency of affirmations each other (Lee, 1999). The reason of developing this consideration could have been arisen from the difference the way the mathematician constructed mathematics and the way he teaches mathematics. Risen from the way mathematics is taught, affected the students' approaches related with proof and caused to care the last process of proof, this inductive evaluation approach was carried out at schools (Almedia, 2000). C typed prospective teachers preferred informal proof and found formal proof difficult because it includes technical difficulties. They mostly relied on their intuitions and experiences. D typed prospective teachers accepted formal proof's necessity but they saw this as a symbolic manipulation. In the study of Moore, proof was a procedure for these typed prospective teachers (Moore, 1994). Some previous studies supported this finding. It was concluded that proof perceptions affected proof choices of prospective teachers. Moreover this typed prospective teachers stated that they memorized proofs. This obtained finding showed parallelism with the findings of the studies of Moore (1994), Baker (1996) and Senk(1985). These studies showed that the prospective teachers' studying methods for exams based on imitative reasoning could be described as a type of reasoning built on copying proof such as recalling the proof by looking at a textbook or course notes or through remembering a proof algorithm, memorizing proofs by rewriting them for many times before exams. They stated that they had lack of self confidence in constructing a proof that they had never seen before.

2. The proving process of the prospective teachers was investigated in view of proof perception types. When the proving process of theorems was analyzed, it was determined that A typed prospective teachers preferred formal ways while proving; B typed prospective teachers utilized informal proof ways to reach formal proof. Some of B typed prospective teachers preferred informal proof ways unlike A typed ones; it was seen that they used mathematical language and symbols truer than the teachers in other perception types. It was indicated that C typed prospective teachers mostly preferred informal proof ways when it was compared with formal proof ways. It was stated that this typed prospective teachers generally used shapes and diagrams in the process of proving and they preferred generalizing instead of being overwhelm with mathematical language and symbols. It was shown that D typed prospective teachers preferred informal and formal proof ways but they could not reach conclusion because of misusing the mathematical language and symbols. Instead of applying different ways, constructing the proofs on a prepared method, they tried to prove the proofs. Moreover, it was stated that the prospective teachers generally had difficulties in proving. It was seen that %49 of the prospective teachers answered the first proof incorrect or left it unanswered, %36 of them answered the second proof wrong or left it unanswered, %34 of them answer the third

proof incorrect or left it unanswered. In some previous studies, it was presented that prospective teachers had troubles in proving processes. Knuth (2002) stated in their study that prospective teachers had difficulties in steps of "understanding", "appreciating and constructing the proof", "following the steps of reasoning" during the processes of constructing the proof. Similarly, Selden & Selden (2003) found that mid-level undergraduates of mathematics had little ability to construct proofs and validate them and preferred using routine ways. Many researchers like Palm (2002) and Lithner (2003) indicated that students had a tendency of using algorithmic and routine solving process. Thus, it was clear that students had various and numerous difficulties related to proof. They could be summarized as the followings: lack of content knowledge (Mingus and Grassl 1999), inability to apply content knowledge (Moore 1994; Weber, 2001), focus on procedures versus context (Baker 1996; Moore 1994; Senk 1985), lack of strategic knowledge, desire to use inductive reasoning (Almeida 2001; Coe and Ruthven 1994) and a lack of understanding of the universal validity of a proven result (Almeida 2001). As a result, it was determined that the proof perception types of the prospective teachers had an effect on their proving processes. This finding matched up with the results of some studies made by Almedia(2000,2001).

3. From the point of the attitudes towards proving, there was a significance difference between A type and B type and between A type and D type. A typed prospective teachers got higher attitude scores towards proof and proving than the others. D typed prospective teachers stated that they had lack of self confidence in constructing a proof that they had never seen before (Bergqvist, 2007). As a result, A typed prospective teachers have more positive attitudes towards proof and proving and also memorizing the proof itself instead of reasoning may cause the differences. There was a significance difference between attitudes towards proof of B type and D type. Both of them accept the necessity of formal proof. if D type does not understand, proof could be disaffected but B type uses informal ones till adapting the formal. Thus, D type could develop more negative attitudes towards proof than B type.

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