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The attitudes of the elementary school mathematics teacher candidates towards proof in the mathematical modelling process

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Abstract

The need of using and understanding mathematics is gaining importance in daily life and this importance is growing gradually. The most important goal of mathematics education is to obtain logical answers in response to questions ‘why?’ and ‘how?’; in other words, the goal is to ensure the development of reasoning. It is observed in this study that whether there are any changes in the attitudes of the candidates towards proof and proofs based on mathematical modeling. This study consists of two parts which are to test pre-post attitudes. 60 students who are in their third year of BA education attended to this search. They are enrolled in Balikesir University Necatibey Education Faculty Elementary Mathematics Education in 2010-2011 Spring Term. Intentional sampling is used in selecting the sample. ‘Attitude Scale to Proof’, developed by Uzel and Ozdemir (2009)- five point Likert Scale- is used to explore the attitudes of the candidates. This scale Cronbach Alpha coefficient is found .897. The item-total correlations were between .50 - .82. The data is evaluated by SPSS 17.0 Programme to determine percentages, frequencies and t-test for correlated survey. Finally, 10 teachers are interviewed in order to examine the attitudes of teachers in greater depth. As a result of this study, teachers' attitude scores using traditional methods is very low towards the proofs and mathematical modeling show higher scores in the attitude towards the proofs. In addition it is revealed that the candidates think the proof with modeling more understanding, easy and effective than the traditional method. It is recommended to integrate the proof process and the modeling process. However, it may be suggested that different proofs in different areas of mathematics should be evaluated at different levels in mathematical modeling process.

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1. Introduction

One of the most important objectives of the mathematics education is to obtain logical answers in response to the questions “why?” and “how?”, in other words, the goal is to ensure the development of

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reasoning. Mathematics is a science of proof and it differentiates from other disciplines with this characteristics. As proof in mathematics courses is so important, it is a highly emphasized subject area in especially higher education. Although mathematical proof is so important and emphasized in the undergraduate study, the students studying higher mathematics in the university have quite difficulty in proving (Weber, 2004; Stylianides & Stylianides, 2008; Alcock, 2004; Weber, 2001).

Lakatos considers mathematical information as an individual activity and he says that this activity, in which he can imply the observation of the physical world, encompasses the abstract thinking and logical inference activities as well. If the mathematics is a human activity, then it is not possible to seek perfection in its nature. In other words, mathematical information is not absolute true and perfect in its final state that is closed to proofs and questioning some concepts; that information is always open for discussion and improvement. The spiral structure of the proof explains this situation more conveniently (Baki, 2006). Proof has a spiral cycle revolving around the selection of some assumptions, examination and interpretation of these assumptions and a result inference and again production of a new assumption based on this result. Lakatos made out the following result from this point of view: ‘Mathematics is not the discovery of an absolute truth waiting at somewhere before us, but it is the human effort and activity. Mathematics is the construction of the mathematicians. And it is only feasible through discussion, sharing and agreement (Lakatos, 1976).

A proof delivered in school should show not only the accuracy of the assumption, but also the reason why such assumption is considered as accurate. The purpose of such proofs is to illuminate the correlated parts found in the assumptions in an extensive mathematical structure. While it is produced such proofs in classroom environment, it can be said that use of the structures which the students are more familiar with rather than formal structure ensure more efficient, permanent and meaningful learning.

Raman studied the views of the university students and lecturers towards the proof and identified three different ways of thinking according to people’s evaluation of proving and proof. These are heuristic thought, procedural thought and key thought. Heuristic thought is based on informal understanding. It can be built on experimental data or shown with figures. It can be meaningful, but does not lead to formal proof. The heuristic thought gives the feeling that something is true, but it does not convince. However procedural thought is used in proving and it is based upon the formal arrangements leading to logic and formal proof without establishing connection with informal understanding. Procedural thought introduces that something is true, gives the feeling of conviction, but not feeling of understanding. Key thought is the heuristic thought that can be transformed into formal proof together with the appropriate logical validity. It gives the feeling of both understanding and conviction. The heuristic thought is special, whereas the procedural thought is general and the key thought establishes connection in-between them (Raman, 2003). When constructing a proof, it is basically required two special conditions. That’s to say, finding correct assumptions and designing deductive chains with reference to this assumption.

For the realization of the proof, it is expected that the assumption is obtained from a reliable source. To this end, if the assumption obtained is selected from the lives of the students, the students may experience a learning situation as if they constructed the proof themselves for the first time. So they can themselves structure the validations and conceptual verifications included in the proof and follow the processes observed by a mathematician. In formal mathematics, the first of these activities is considered to be already performed. While it is provided proof education, it is paid attention to the development of only second part and the development of the first part is disregarded totally. If the ability of proving is expected to be developed in the individual in the real life, this can only happen if the individual knows how to establish the correct assumptions, which is called as “modeling” in applied science and mathematics.

The modeling consists of a spiral cycle revolving around problem posing, model development, model interpretation, result generation... It was previously mentioned the existence of such a cycle for proving. Proving is far away from a deductive chain that is concluded and limited with a result. So proving and modeling are two inseparable terms. Consequently these two structures should be correlated with each other and used in education effectively (Ron & Dreyfus, 2004). If the proof education is provided by

presenting a problem case that may trigger the Key Thought and following mathematical modeling processes, it can be more efficient, permanent and meaningful. The objective of this study is to observe that whether there are any changes in the attitudes of the mathematics teacher candidates towards proof and proofs based on mathematical modeling.

The data are collected during the study keeping in mind the following question: ‘How do the proofs produced taking into account the mathematical modeling process affect the attitudes and opinions of the teacher candidates to construct proof?’

2. Method

2.1. Study Design

A pre-attitude post-attitude single group trial design model is used in this study. As the study is conducted in the manner of an intervention in a functioning from a real life and the detailed examination of the effects of this intervention on the existing functioning, it also featured as ‘action research’ (Yıldırım & Simsek, 2005).

2.1.1. Target population and Sampling

The target population of the study consists of the third grade students studying in the department of primary school mathematics teaching in the faculty of education. The sampling of the study is made up of the third grade students studying in the department of primary school mathematics teaching in Necatibey Faculty of Education, Balıkesir University in 2010-2011 Spring Term.

2.1.1.1. Data Collection Tools

Before application, it was made pre-attitude application in order to understand the attitudes of the students towards proof via ‘Attitude Scale to Proof’ developed by Üzel ve Özdemir (2009). The mathematical modeling activities were carried out by the researcher in individual level primarily and then in the manner of groups studies. Upon the completion of the activities performed following the mathematical modeling and mathematical modeling process, it was proved three theorems through mathematical modeling. In the end of the study conducted, it was performed post-attitude application in order to understand if there is any change in the attitudes of the teacher candidates. Lastly it was made interviews with fourteen students designated by the researcher according to the performances of the teacher candidates in their activity sheets.

2.1.1.1.1. Attitude Scale to Proof

In order to measure the attitudes of the Primary School Mathematics Teacher candidates towards proof, it was used ‘Attitude Scale to Proof’ developed by Üzel and Özdemir (2009). It is seen that the scale made up of 21 items consists of a single factor. The Cronbach Alpha coefficient of this scale is found .897, which shows that the scale is reliable. The item-total correlations vary between .50 - .82. With reference to $p < ,001$, the scale was found to be valid.

2.1.1.1.1.1. Data Analysis Methods

The scales applied to the teacher candidates were transferred to the computer environment and it was made statistical analyses using t-test in order to determine the percentage-frequency arithmetic averages of attitude scores and correlated measurements with SPSS 11.0 Package Program. The data obtained through the structured interviews with the teacher candidates, which is another data collection tool applied, is analyzed descriptively.

3. Findings and Conclusions

In this section, it is given findings obtained.

Table 1. Findings With Respect to the Primary Sub-problem

	I Agree Totally		I Agree		I’m Neutral		I don’t agree		I don’t agree definitely		X
	f	%	f	%	f	%	f	%	f	%	
Pre-Process	5	8,33	6	10	10	16,66	14	23,33	25	41,66	2,11

When the table is examined, it is seen that 41.66% of the candidates has marked the ‘I agree exactly’ option, while 8.33% of them has marked the ‘I agree totally’ option. The arithmetical average of the candidates’ attitude scores towards proof is found to be 2.11. And this figure corresponds to the ‘I don’t agree’ option.

Table 2. Findings With Respect to the Secondary Sub-problem

	I Agree Totally		I Agree		I'm Neutral		I don't agree		I don't agree definitely		X
	f	%	f	%	f	%	f	%	f	%	
Post-process	47	78,33	8	13,33	4	6,66	47	78,33	8	13,33	4,39

When the table is examined, it is seen that 0% of the candidates has marked the 'I don't agree definitely' option, while 78.33% of them has marked the 'I agree totally' option. The arithmetical average of the candidates' attitude scores towards proof is found to be 4.39. And this figure corresponds to the 'I agree totally' option.

Table 3. Findings With Respect to the Third Sub-problem

	N	X	df	t	ss	p
Pre-Process	60	2,11	59	51,36		
Post-process	60	4,39	59	234,64	,364	,000

When it is looked in the table belonging to t-test for the correlated measurements (because $p=,000<,005$), it is seen that there is a significant difference in the pre-process and post-process attitudes of teacher candidates towards proof. When we look at the arithmetical averages in order to understand that which groups this difference serves for, it is revealed that this difference serves for the post-process group. In this situation, it can be said that proving through mathematical modeling has a positive effect on the attitudes of the teacher candidates towards proof.

Findings With Respect to the Fourth Sub-problem

It was made interviews with fourteen teacher candidates in order to explore the fourth sub-problem of the study. It was made descriptive analysis to analyze the interviews. To this end, it was determined main themes by the researcher for each question and coding was performed. These are specified below:

- 1) Primary Question: Mathematical structures in the mind-Equation, function, graphic, mathematical thinking abilities
- 2) Second Question :The procedure of mathematical processes-Obtaining new models
- 3) Third Question :Connecting life and mathematics-Tangibility -Easy learning-Trouble in application
- 4) Fourth Question:Meaningful learning-Persuasiveness Facilitating learning-Associating with reality

4. Result and suggestions

4.1. Conclusions With Respect to the Primary Sub-problem

It is found out that the attitudes of the teacher candidates towards proof were highly low before proving through mathematical modeling. Besides in the light of the interviews made with the teacher candidates, it is seen that they have difficulty in understanding the proofs produced with the traditional method, have problem to correlate such proofs with each other and cannot give meaning to these theorems.

4.2. Conclusions With Respect to the Secondary Sub-problem

It is observed rising in the attitudes of the teacher candidates towards proof after proving activities based on the mathematical modeling. When this case is studied in depth with the interviews made, it is observed that teacher candidates understand the proofs and establish correlation more easily by the help of modeling. While teacher candidates were validating a theorem, they said that they felt themselves like a mathematician who validates that theorem for the first time. It is also observed teacher candidates stated in the interviews that the proofs based on the mathematical modeling enable them to understand what these proofs correspond in the real life, though any situation in the life can be validated with mathematics. It is seen that they said the proofs constructed with mathematical modeling work to establish a strong connection between mathematics and life.

4.3. Conclusions With Respect to the Third Sub-problem

When the pre and post-process attitudes of the teacher candidates towards proof, it is seen that their post-process attitude scores are higher. The reason of it is the opinion that when the theorems are being

proved, it will be more meaningful, permanent and effective to ensure people to start from the informal information and reach to the formal information.

4.4. Conclusions With Respect to the Fourth Sub-problem

It is seen that all teacher candidates know what ‘Mathematical Model’ and ‘Mathematical Modeling’ mean and explain these terms easily. It is determined that all of teacher candidates think mathematical modeling needs to be used in mathematics teaching. It is found out that all teacher candidates find the proofs constructed with mathematical modeling more meaningful, permanent and easier to remember. It is also observed that some of the candidates tell they can by this means correlate the mathematics and real life.

4.5. Suggestions

In the conclusion of the study, it is determined that the attitudes of the teacher candidates towards the proofs produced in the way as thought in the universities are low. The reason of it may be that the assumptions established for the theorems given are not understandable for the people. So for proving theorems, the use of assumptions fitting to the general level is suggested. While the theorems are proved, it is observed that the deductive chains realized by the teacher giving course are only copied and stored in the volatile memory by the students without understanding and they cannot associate them with daily life. Instead of it, it should be ensured that teacher candidates themselves establish these chains making sense. Therefore they can understand what these theorems correspond in the real life. Moreover their proof and judgment abilities in the real life may develop by this means.

The study conducted remained limited with the third grade students studying primary school mathematics teaching. The study is required to be developed starting from the high school level, where the proof is started to be taught and the differences in the attitudes should be examined. In the study, the proofs of three theorems are studied. In the different branches of mathematics, it can be studied with different proofs. It can be suggested that the study may be validated with mathematical modeling method using visual models. It can be also suggested that the attitudes of the students studying at lower grades towards the proof constructed with mathematical modeling are examined.

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