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The Investigation of Prospective Mathematics Teachers' Proof Writing Skills and Proof Self- Efficacy

Filiz Tuba Dikkartin Övez [a] *, Emine Özdemir[a]

[a] *Balıkesir University, Education Faculty of Necatibey, Elementary Mathematics Education Department,Balıkesir and 10100, Turkey*

Abstract

The aim of this study is to investigate proof self-efficacy and proof writing skills of prospective mathematics teachers. As a means of data collection, "Proof Self-Efficacy Scale (PSE)" developed by Iannone & Inglis and four geometry theorems selected randomly from Euclidean geometry are used. PSE scale has been adapted to Turkish and this form of the scale has been administered to prospective mathematics teachers for the validity and reliability of analysis. Prospective mathematics teachers' proof writings have been classified according to the Stylianides and Stylianides' scheme. Proof self-efficacy scores are classified according to the measurement of Guttman. Data are analyzed by using one-way Anova and descriptive statics. It is determined that there is a meaningful difference between proof self efficacy levels and proof writing skills.

Keywords: Proof self-efficacy, Proof writing skills, Prospective mathematics teachers

1. Introduction

The primary aim of advanced-level mathematics and geometry courses is to make students acquire proof skills (Weber, 2001). Although proof is at the core of doing and knowing mathematics and geometry, literature shows that prospective mathematics teachers have some difficulties in understanding proof and proof writing (Moore, 1994; Almeida, 2000; Jones, 2000; Stylianides, Stylianides, and Philippou, 2007) and it is observed that the proof skills of prospective mathematics teachers are not at a sufficient level (Jones, 2000; Stylianides, Stylianides, and Philippou, 2004). Prospective teachers' perceptions and experiences regarding proof are effective in the process of gaining proof skills (Almeida, 2000). In this regard, it can be said that prospective teachers' proof skills, proof perceptions, and their self-efficacies they have are associated with one another. Thus, self-efficacy belief is defined as the students' judgment of belief in their capabilities of performing and organizing necessary activities to obtain a certain achievement (Roberts, Henson, Tharp and Moreno, 2001). Conducted studies reveal that there is a positive relation between mathematical performance and mathematical self-efficacy (Sewell and George, 2000). It is considered that this relation plays an important role in developing proof and proof writing skills that have an important place in teaching mathematics. For example Anapa and Şamkar's (2010) findings in their study regarding prospective teachers that do not believe in themselves adequately to write proofs and understand the accuracy of proofs are

* Corresponding Author: Dr.Filiz Tuba DİKKARTIN ÖVEZ. Tel.: +90 266 241 27 62
E-mail address: tdikkartin@balikesir.edu.tr

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capable of supporting this view. In this context, revealing the relation between prospective teachers' self-efficacies for proof and their proof writing skills is considered important. Accordingly, the aim of the study is to examine whether or not there is a significant relation between prospective teachers' proof writing skills and their proof self-efficacy levels.

2. Method

2.1. Research Model and Study Group

A qualified descriptive scan model that aims at revealing the existing state in the research as it should be (Karasar, 2006) was adopted. The study group of the study was formed from 153 prospective elementary mathematics teachers studying at Balıkesir University and selected by means of a simple random sampling method during the 2012-2013 academic years. The data collected from 310 elementary mathematics students in the 3rd grade were utilized for the validity and reliability study of the proof self –efficacy scale that was used as a data collection tool.

2.2. Data Collection Tools

In order to specify proof writing skills in the study the scale consisting of theorems selected from Euclidean geometry was used. These theorems are as follows:

1. *Demonstrate that the distances of a point on the bisector to straight lines are equal.*
2. *Demonstrate that in an isosceles triangle the bisectors of equilaterals are equal to one another.*
3. *Demonstrate that the sum of any two sides of the triangle is greater than the length of the third side.*
4. *Demonstrate that the lengths of tangent lines drawn externally to two externally tangent circles are equal to one another.*

In order to specify students' proof self-efficacies the proof self –efficacy scale developed by Iannone & Inglis (2010) in their study was used after adapting it to Turkish. First, this scale prepared as a five point likert scale was translated to Turkish for the language validity and then its Turkish version was translated by two language experts and two field experts to English again and this way an agreement was reached on the intelligibility of the expressions. Factor analysis was used in order to examine the structure validity of the scale. Prior to the factor analysis, the compliance of the data with the analysis was tested by means of the Keiser- Meyer-Olkin and Barlett-Sphericity test. KMO value of .87 and p value [X^2=1458,733 p<0,001] at the end of the Barlett-Sphericity test were found to be significant. It was observed that as a result of these obtained values the data were in compliance with the factor analysis (Büyüköztürk, 2006). In exploratory factor analysis a varimax rotation method was used and then a principal components analysis was applied to the data. As a result of analyses, it was determined that the items of 1,2,3,4,5,7 were included in the factor of 'proof self- efficacy knowledge and the items of 6,8,9, and 10 were included in the factor of "use of proof self-efficacy knowledge." An item-total correlation analysis was used in order to specify the discrimination power of each item for the persons. When the item-total correlations were examined, it was seen that the correlation values of all items were greater than .30 and that the factor loads of items varied between .43 and .79. According to the results of the principal components factor analysis that was conducted with rotation, it was determined that the items were grouped under two factors and that the adapted scale had a two-factor structure. These factors were named as "proof self- efficacy knowledge" and "use of proof self-efficacy knowledge, respectively." It was observed that the items accounted for 65% of total variance and that they were above .41, which was an acceptable amount (Kline, 1994). Cronbach's alpha internal consistency coefficient was calculated for the reliability of the scale and then found as .813. It was observed that the Pearson Product-Moment Correlation Coefficient was positively related and significant at the level of p<.01. Cronbach's alpha internal consistency coefficients relating to two sub factors were calculated as .90 and .76, respectively. It was determined that the obtained values were reliable together with the sub dimensions of the scale. Results that were similar to the original scale were obtained.

2.3. Analysis of Data

In the study, answers of students provided to four geometry theorems measuring their proof writing skills, were categorized by using Stylianides & Stylianides (2009) diagram. In addition, the categories of C2 and C6 were added to the diagram in line with the answers provided by students. Accordingly, the answers were coded as such C1: A clear, persuasive, and reasonable proof by using the mathematical notations, language, and the definitions (scored full marks), C2: proof conducted by using deficient mathematical language and notations (representations), C3: reasonable attempts at producing a proof – containing a general argument, C4: irrelevant responses to the solution of the problem, C5: empirical arguments, i.e. proofs entirely based on one numerical example, and C6: Blank or writing the provided ones. After both researchers coded the data separately for the reliability of the study, the codes assigned to the answers were compared. In coding, the percentage of compliance was observed to be 100%. Obtained results were analyzed descriptively and the values of % and frequency were calculated. The scores obtained from proof self-efficacy were organized according to Guttman measuring shape. According to the scale evaluation system the scores varying between 10 and 50 were taken from the scale. In comparisons conducted with quantitative measures self-efficacy levels are evaluated as high (40-50), middle (20-39), and low (10-19). While specifying the scores of proof writing skills, a full score in the meaning of 5 scores for the category of C1, 3 scores for the category of C2, 1 score for the category of C3, and 0 score for the categories of C4-C5-C6 were given, considering the scoring system used by Iannone & Inglis (2010) in their study. One-Way Variance Analysis (ANOVA) was used in order to specify whether or not the proof writing skills of prospective elementary mathematics teachers differ according to their proof self-efficacies in line with the second sub problem of the research.

3. Findings and Interpretation

3.1. Finding and interpretation regarding the first sub problem

The findings regarding the question of "how are the proof writing skills of prospective elementary mathematics teachers?" which is the first sub problem of the research, are provided in Table 1.

Table 1: Descriptive findings regarding prospective teachers' proof writing skills

Item No	Categories												
	C1		C2		C3		C4		C5		C6		
	f	%	f	%	f	%	f	%	f	%	f	%	Total
Theorem 1	10	6.54	32	20.9	32	20.9	22	14.4	4	2.61	53	34.74	153
Theorem 2	11	7.19	35	22.9	24	15.7	18	11.8	2	1.31	63	41.1	153
Theorem 3	6	3.92	15	9.8	18	11.8	14	9.15	12	7.84	88	57.49	153
Theorem 4	13	8.5	45	29.4	23	15.03	15	9.8	11	7.09	46	30.81	153

When the obtained data were examined, it was observed that the students were able to provide answers at the rate of 6.54% for the first theorem, at the rate of 7.19% for the second theorem, at the rate of 3.92% for the third theorem, and at the rate of 8.5% for the fourth theorem in the category of C1, in other words, they were able to conduct clear and persuasive proof by accurately using the provided theorems, mathematical notations, language, and the definitions. As was observed in these obtained results, it was determined that 6.5% of students in average were in the category of C1 and that they reached the accurate proof. It was seen that the given proofs were obtained with formal proof methods. It was observed that the students were able to provide answers at the rate of 20.9% for the first theorem, at the rate of 22.9% for the second theorem, at the rate of 9.8% for the third theorem, and 29.4% for the fourth theorem in the category of C2 and in other words, they tried to conduct proofs by using deficient mathematical language and notations. As is seen in these obtained results, it was ascertained that 20.75% of students in average tried to conduct proofs in the category of C2. It was observed that the students providing such answers assumed that they were conducting proofs by demonstrating the provided features over the figure and making

several calculations to conduct the proof procedure. It was determined that they did not support their hypotheses with necessary mathematical language and symbols and that they did not state the hypothesis and judgments for the theorem accurately or used the proof methods deficiently. According to the averages of the answers given, it was considered that 15.85% of the students provided answers in the category of C3 in other words; they put forward a general argument or an idea, which is not a proof. It was specified that 11.28% of the students provided answers in the category of C4 in other words; they provided unrelated answers without understanding the theorem accurately. It was ascertained that 4.71% of the students provided answers in the category of C5 in other words; they tried to verify the result with quantitative values or conducted proof from a special sample. It was considered that 41.11% of the students in average provided answers in the category of C6 in other words; they were not able to make a judgment for the theorem and they provided answers by leaving blank or writing the given data.

3.2. Findings regarding the second sub problem

The question of "is there a significant relation between prospective elementary mathematics teachers' proof writing skills and proof self-efficacies?," which is the second sub problem of the research, was examined. One-Way Variance Analysis was used in order to determine whether or not the proof writing skills of prospective elementary mathematics teachers differ according to their proof self-efficacies or not (Büyüköztürk, 2006). The results of analysis are provided in Table 2.

Table 2: Descriptive Data For Proof Self –Efficacy Levels and the Results of One-Way Variance Analysis (ANOVA)

Proof Self –Efficacy Levels	$\overline{X}$	SS	N
High	8.70	4.32	31
Middle	4.48	2.67	87
Low	3.68	3.34	35
The Source of Variance	Total of Squares	df	F
Between groups	506.581	2	24.45
Within groups	1553.654	150	
Total	2060.235	152	

When the averages in Table 2 were examined, it was observed that they were in favor of prospective teachers with high self-efficacy among the scores of the proof writing skills of prospective teachers with different proof self-efficacy levels. There is a 4.22 and 5.02 score difference, respectively among the proof writing average scores of the prospective teachers with high-middle and high-low level of self-efficacy. According to the results of one-way ANOVA conducted in order to specify whether the difference observed among the averages is significant or not, it was confirmed that prospective teachers' proof writing scores indicated a significant difference according to proof self-efficacy levels they owned [$F(2\text{-}150)=24.45$; $p<.05$]. For the value of F that appeared to be significant in order to determine the source of the difference, the Scheffe Test was used as one of the multiple comparison tests (Büyüköztürk, 2006). According to the results of the Scheffe test, it was concluded that proof writing scores of prospective teachers with high self-efficacy level indicated a significant difference when compared with the proof writing scores of prospective teachers with low and middle level.

4. Conclusion and Recommendations

When the results of the conducted study were examined, it was observed that 80% of the prospective teachers had proof self-efficacy at the low and middle level. Self-efficacy is defined as a self-judgment of an individual regarding the capacity of organizing necessary activities and realizing them successfully to show a particular performance. When considered that the most effective source was the experiences undergone by the individual himself/herself among the self-efficacy sources (Acar, 2007), it is an expected situation for a significant relation between the self-efficacy levels of prospective teachers' failures in writing proofs to appear. This finding demonstrates a resemblance

with the results encountered in literature (Hackett and Betz, 1989). It was observed that prospective teachers had various deficiencies and faults regarding proof answers they provided, understanding the proof, and completion of the proof process. It is considered that this negative situation has an impact on self-efficacy because the proof writing skills of individuals with low and middle level of self-efficacy was also found to be low. The students encountering proof and formal mathematics for the first time at universities and their having no idea about what to do when they are expected to conduct the proof may be indicated as a reason for this. While teaching the axiomatic structure, meaning of the proof and its methods to the students in education faculties, it is considered to be important to make its logic of proof meaningful and to have the logic of proof, which is known to have an important place in mathematics education, comprehended correctly. In case the education to be provided in this direction may also contain informal proofs, their thinking and reasoning correctly and rationally will be ensured. In this context, as a condition that was also observed in the study, in courses whose rationale is based on the sense of proof, it is recommended to make up the students' deficiencies in the past on the subjects such as hypothesis, writing judgments, and listing hypothesis and have them prepared for advanced learning.

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