ENERGYCON 2014 • May 13-16, 2014 • Dubrovnik, Croatia

On the No-load Loss of Power Transformers under Voltages with Sub-harmonics

E. Arslan[1]
BEST Transformers Co.[1]
Balikesir, TURKEY
earslan@besttransformer.com[1]

S. Sakar[2]
Gediz University[2]
Izmir, TURKEY
selcuk.sakar@gediz.edu.tr[2]

Murat E. Balci[3]
Balikesir University[3]
Balikesir, TURKEY
mbalci@balikesir.edu.tr[3]

Abstract—**Power transformers are traditionally designed for their utilization in sinusoidal voltage and current conditions. However, the non-linear loads are largely proliferated in the modern power systems. Thus, pcc (point of common coupling) voltages and line currents usually have harmonically distorted or non-sinusoidal waveshapes. This study focuses on the parametrical analysis of transformer no-load loss under the excitation voltage with several sub-harmonic contents. For this aim, a computationally efficient technique is developed by combining both the harmonic-domain model of transformer windings and Finite Element Method (FEM) based modeling of transformer core. The obtained simulation results figured out that effect of sub-harmonic voltages on the transformer core loss is negligible. However, it is also seen that small amount of sub-harmonic voltages may highly contribute the winding loss under the no-load condition, and they should carefully be handled for the derating of power transformers.**

Index Terms—**Transformers, no-load loss, sub-harmonics, harmonic distortion, FEM analysis.**

I. INTRODUCTION

Transformers are one of the most important equipment for transmission and distribution of electric energy in ac power networks. Transformer losses can be separated into no-load loss and load loss [1]. In the electrical engineering literature, measurement and calculation of both losses are well known for sinusoidal voltage and linear load (sinusoidal current) cases. No-load loss of a transformer is sum of the active powers consumed by its core and energized winding when it is not supplying a load. The no-load loss is practically regarded as core loss due to the fact that winding loss is notably small in the no-load case under sinusoidal excitation voltage at rated frequency. In addition, since a power transformer's winding loss is considerably larger than its core loss for the rated loading case, the winding loss is generally called as the load loss.

In modern power systems, the large proliferation of nonlinear loads has leaded to the high level of voltage and current harmonic distortion [2]. Thus, great interests have been focused on the effect of harmonics on the power transformers [3]-[7]. The main conclusion of these studies is that non-sinusoidal currents cause excessive winding losses, which result in overheating and useful life reduction of the transformers. As a result, in [8]-[13], an efficient technique, widely called as derating, is employed to prevent the excessive winding losses related with the current harmonics. Derating can basically be interpreted as the intentional reduction in loading capability of a transformer, which is dedicated to supply a non-linear load [9], [11]. Derating ratio (or maximum loading capability) of transformers is generally determined via (i) FHL method [8], (ii) K factor method [9], (iii) measurement of current harmonic losses [10] and (iv) computation of current harmonic losses with the electric and magnetic circuit models of transformers [11] and FEM based analysis [12], [13].

On the other hand, in the literature, ref. [14]-[20] analyzed the no-load loss of transformers under harmonically contaminated voltages. It was concluded in [14] and [15] that both magnitude and phase angle of voltage harmonics affect the no-load loss of transformers and lower-order harmonics have a greater effect on the no-load loss than higher-order harmonics. It was figured out in [16]-[18] that the no-load loss of transformers may notably increase with voltage total harmonic distortion. However, [19] and [20] clearly interpreted that under the supply voltage total harmonic distortion levels below 5%, effect of the harmonic voltages on the core loss can be negligible, and load current harmonics passing through the transformer do not significantly affect the core loss of the transformer. In addition, the effect of voltage sub-harmonics, which have the frequencies considerably lower than fundamental frequency (50 or 60 Hz), on the single-phase and three-phase distribution transformers are analyzed in quite a few studies [21], [22]. Both studies clearly confirmed that very low frequency sub-harmonic voltages, even with seemingly insignificant magnitudes, can lead to magnetic core saturation and the dramatic increase in rms value and harmonic distortion of the excitation current. According to these results, it can be mentioned that winding and core losses of transformers should particularly be investigated for the no-load condition under the excitation voltages with sub-harmonics.

In this paper, a method is developed to compute no-load loss of transformers utilized under voltages with sub-harmonics. By means of the developed method, the

---

This work is supported by Turkish Republic Ministry of Science, Industry and Technology and BEST Transformers Co. under the project number of 01008.STZ.2011-2.

parametrical analysis results on the excitation currents, winding loss and core loss of power transformers in the no-load case under the supply voltages with sub-harmonics are presented in this study. Unlike the modeling methodology used in [21], [22], the developed method considers the frequency dependency of transformer's winding resistance. This is important for the precise determination of the transformer no-load loss under the sub-harmonic voltage conditions. On the other hand, in FEM analysis, for exact modelling of the frequency dependency of the winding resistance, a detailed geometry of the windings should be introduced to the FEM analysis software. This leads to slow simulations and large memory requirements. To avoid these difficulties, in the developed method, FEM analysis is only employed for the determination of the excitation current and core loss. In addition, under the excitation current, the winding loss is separately calculated by using the transformer's harmonic domain equivalent electric circuit.

II. DEVELOPED METHOD

The developed method is based on two parts as the transformer equivalent electrical circuit, which is employed to calculate the transformer winding loss, and FEM model, which is used to find the magnetization current and core loss of transformer.

A. *Harmonic-domain equivalent electric circuit of a transformer*

In the developed method, the winding loss of transformer is calculated by means of the harmonic domain electric circuit model illustrated below (Figure 1). In the illustration, for k=a, b, c phases, V_{pk}^h, V_{sk}^h, I_{pk}^h, I_{sk}^h and I_{mk}^h are the h^{th} harmonic phase-to-neutral voltages and phase currents of the primary/secondary sides and the h^{th} harmonic magnetization currents. These voltages and currents are phasor quantities referred to per-unit (pu) values. In addition, according to [8], [9], h^{th} harmonic primary/secondary winding resistances (in pu) can be expressed as;

$$R_p^h = R_{dcp} + h^2 R_{ecp},\quad R_s^h = R_{dcs} + h^2 R_{ecs} \quad \text{for h} \geq 0 \qquad (1)$$

where R_{dcp} and R_{dcs} are dc parts of the winding resistances, R_{ecp} and R_{ecs} are the equivalent resistances corresponding to the winding eddy-current loss.

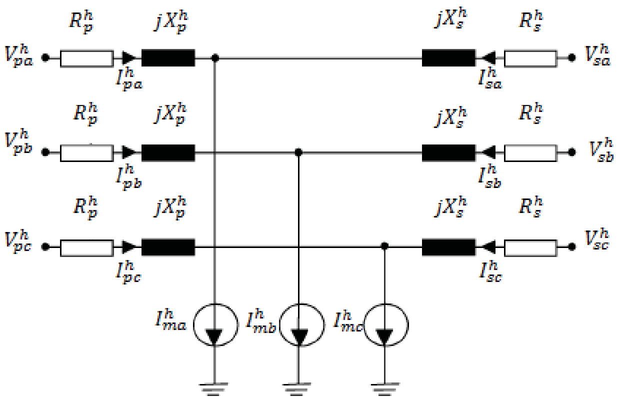

Figure 1: Harmonic domain electric circuit model of a power transformer.

The last parameters of the model are the primary/secondary winding reactances (in pu):

$$X_p^h = hX_p^1,\quad X_s^h = hX_s^1 \quad \text{for h} \geq 0 \qquad (2)$$

According to the above detailed model, when the primary side is energized under no-load condition ($I_{sa}^h = I_{sb}^h = I_{sc}^h = 0$), winding loss of a transformer can be calculated as:

$$\Delta P_w = \sum_{h\geq 0}\left[\left(\left|I_{pa}^h\right|^2+\left|I_{pb}^h\right|^2+\left|I_{pc}^h\right|^2\right)R_p^h\right] \qquad (3)$$

B. *FEM Based Core Model of a transormer*

The FEM is a numerical method for attaining approximate solutions to boundary-value problems in applied physics, and it is widely used in electrical engineering for the solution of electromagnetic problems [23]. Maxwell Ansys FEM analysis software [24] can be employed to determine h^{th} harmonic magnetization currents, which are included in aforementioned harmonic-domain equivalent electric circuit, and core loss due to voltage sub-harmonics. The results of the FEM software is primarily based on solution of Maxwell equations for each mesh of the transformer geometry.

In this paper, 12.5 MVA, 11 kV/ 31.5 kV, delta/star connected three-phase power transformer, of which losses are measured as 8.92 kW and 76.38 kW under no-load and short-circuit tests, is considered for the exemplary analysis. For the no-load loss analysis of the transformers, it is enough to have a FEM model, which consists of a magnetic core and an energized winding on each core leg. Accordingly, a two-dimensional (2D) cross-section of the considered three-phase transformer is first drawn in the FEM analysis software. Figure 2 shows a completely meshed 2D model of the transformer. Certainly, three-dimensional (3D) FEM analysis leads to much more sensitive results, but it takes a longer computation time. Due to this fact, 2D FEM model is widely preferred in the literature [12], [13], [17].

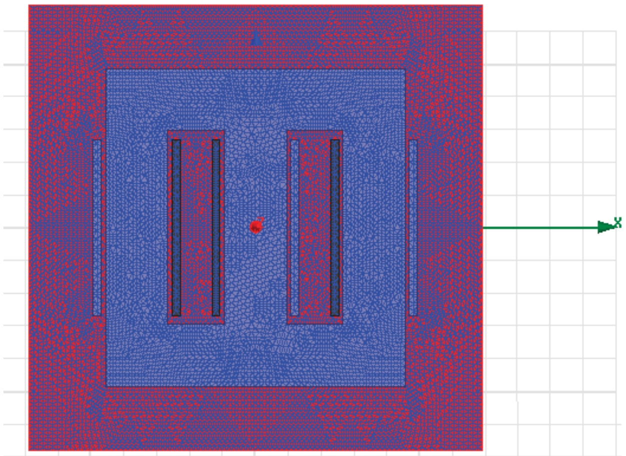

Figure 2. 2D cross section area of the studied three-phase power transformer.

And then, the 2D model depth, the properties, such as magnetic flux density-magnetic field strength (B-H) and loss-magnetic flux density (P-B) curves, of the core material, number of turns, resistances and excitation voltages of three windings should be introduced to the software. Here, note that there is no need to insert the leakage inductance value of three windings into the software since it calculates the winding leakage inductances, which is a function of the number of winding turns, current density in the windings and the leakage flux. The B-H and P-B curves of the studied transformer's core material (M-5 oriented steel, 0.3 mm thick) are given in Figure 3 and Figure 4, respectively.

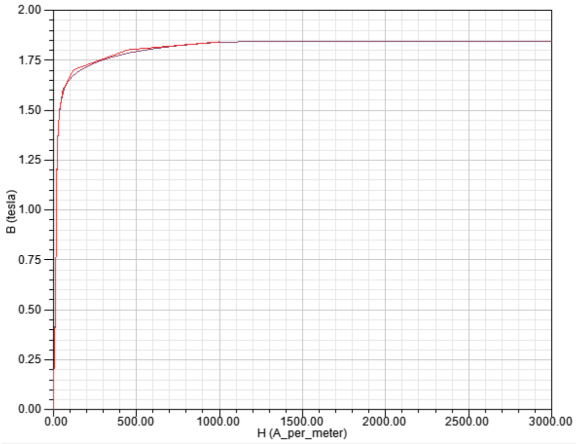

Figure 3. B-H curve of the core material.

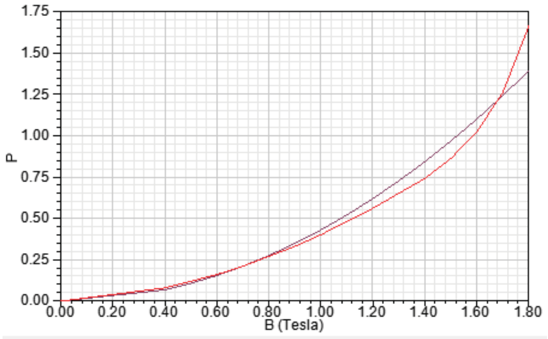

Figure 4. P-B curve of the core material.

Finally, the Transient mode simulation of the FEM analysis software should be allowed to run for several periods until the simulated instantaneous three-phase excitation currents reach the steady state condition. Under the rated sinusoidal and balanced three-phase voltages, the instantaneous excitation currents and the instantaneous core loss for the simulation time (10 second) are plotted in Figure A 1 and Figure A 2 (see appendix). The core loss can be calculated by averaging the instantaneous core loss (p_c):

$$\Delta P_C = \frac{1}{T}\int_{\tau}^{\tau+T} p_c dt \tag{4}$$

where τ and T are the starting time and period of averaging operation.

For the balanced and sinusoidal excitation, the core loss is calculated as 8.42 kW (see Table A 1 and Figure A 2 given in appendix). The calculated core loss value is close to the measured loss value (8.92 kW), which is obtained via the no-load test of the transformer. It should also be pointed out from the FEM simulation that maximum flux density of the studied transformer core under sinusoidal excitation is around 1.8T (see Table A 1 and Figure A 3 in appendix).

Consequently, sum of the ΔP_w and ΔP_C values, which are found from (3) and (4), gives the no-load loss (ΔP_{nl}) of the studied transformer.

III. EXAMPLARY CASES RELATED WITH VOLTAGE SUB-HARMONICS

In this section, the excitation currents, winding loss and core loss of the modelled transformer are analyzed for the no-load case under the primary side phase-to- neutral rated sinusoidal voltages superimposed with a sub-harmonic:

$$v_{pa} = \sqrt{2}V_S \cos\left(2\pi f_S t\right) + \sqrt{2}V_R \cos\left(100\pi t\right) \tag{5}$$

$$v_{pb} = \sqrt{2}V_s \cos\left(2\pi f_s t + \frac{2\pi}{3}\right) + \sqrt{2}V_R \cos\left(100\pi t + \frac{2\pi}{3}\right) \tag{6}$$

$$v_{pc} = \sqrt{2}V_s \cos\left(2\pi f_s t - \frac{2\pi}{3}\right) + \sqrt{2}V_R \cos\left(100\pi t - \frac{2\pi}{3}\right) \tag{7}$$

In eq. (5)-(7), V_S and f_S denote the rms value and frequency of the sub-harmonic voltage, and V_R is the rated value of the phase-to-neutral voltage of the studied transformer.

Accordingly, two types of the parametrical analysis are presented to show the relation between transformer no-load loss and voltage sub-harmonics.

A. *Case I : variation of V_S*

In the first analysis case (Case I), V_S/V_R is varied from 0.25% to 1% where f_s is kept constant as 1Hz. For V_S/V_R of 1%, Figure 5 shows that maximum flux density of the core is around 2.4 T. Since the observed maximum flux density value (2.4 T) is notably larger than the rated maximum flux density (1.8T), the transformer's core is saturated for V_S/V_R of 1%. Accordingly, it draws dramatically large excitation (or magnetization) currents, which have highly distorted waveshapes (see Figure 6). The average value of the three-phase rms excitation currents, calculated as 693 A, is almost equal to the studied transformer's rated current. These results are consistent with the findings reported in [21], [22].

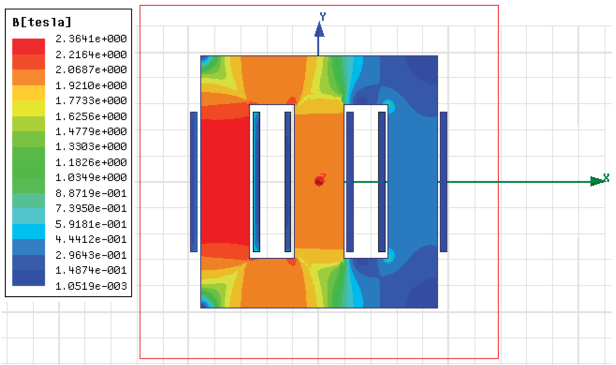

Figure 5. Flux distribution on the transformer core observed for $V_S/V_R = 1\%$.

One can observe from the results plotted in Figure 7 that average value (I_S) of the sub-harmonic rms currents measured at each phase, average value (I_1) of the fundamental harmonic rms currents measured at each phase and average value (I_H) of total rms of all harmonic currents, which except fundamental and sub-harmonic ones, measured at each phase are proportional to V_S/V_R ratio. In addition, this figure also indicates that the winding losses based on current harmonics has significant amount as compared with the winding losses based on fundamental and sub-harmonic current harmonics due to the fact that I_H values are very close to I_1 and I_s values.

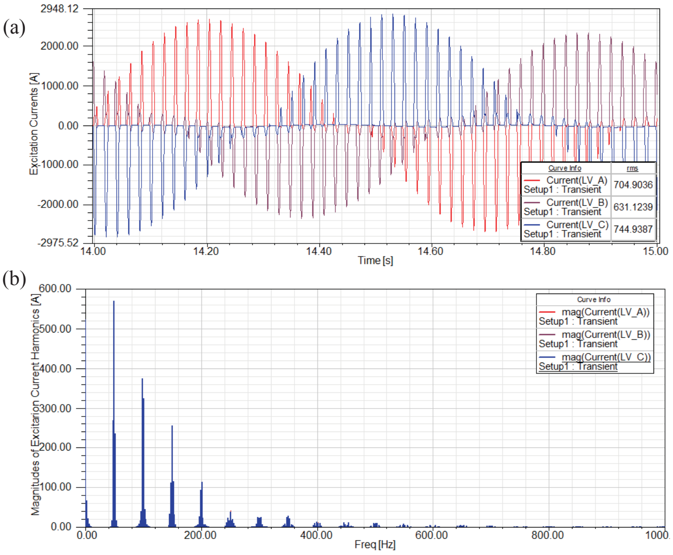

Figure 6. (a) Waveshapes and (b) harmonic spectrums of three-phase excitations currents observed for $V_S/V_R = 1\%$.

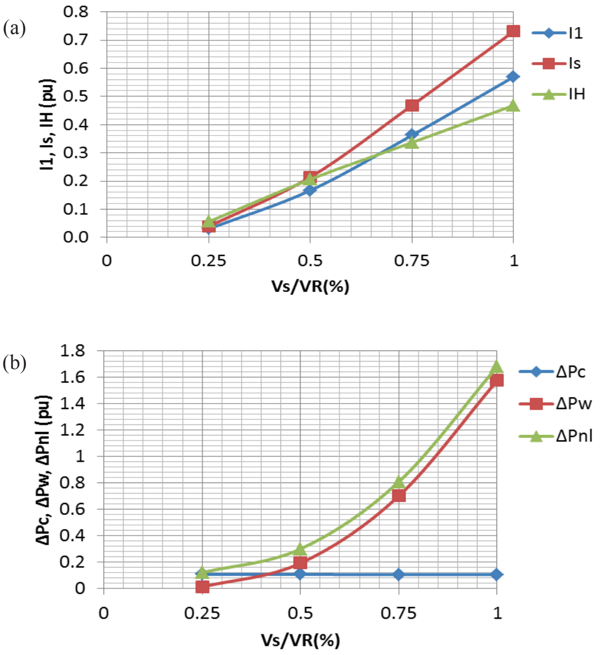

Figure 7. (a) I_1, I_S and I_H values and (b) ΔP_C, ΔP_w and ΔP_{nl} values observed for Case I.

On the other hand, with the increment of V_S/V_R from 0.25% to 1%, the winding loss (ΔP_w) varies from 0.013 pu to 1.575 pu and the core loss (ΔP_C) is almost constant (0.1 pu). Thus, for the same V_S/V_R interval, the no-load loss (ΔP_{nl}) of the transformer increases from 0.113 pu to 1.675 pu. Here, it should be note that for per-unitization, base power and base current values are selected as the rated load loss of the transformer under sinusoidal excitation and the rated current of the transformer primary side, respectively.

B. Case II : variation of f_S

In the second analysis case, f_S is varied from 1 Hz to 5 Hz where V_S/V_R is kept constant as 1%. The obtained results are given in Figure 8. It can be pointed out from these results that with the increment of f_S from 1 Hz to 5 Hz, ΔP_w varies from 1.575 pu to 0.005 pu and ΔP_C is almost the same (0.1 pu) for the considered sub-harmonic frequencies. Therefore, for the same f_S interval, ΔP_{nl} decreases from 1.675 pu to 0.105 pu. In addition, I_S, I_1 and I_H are inversely proportional to f_S.

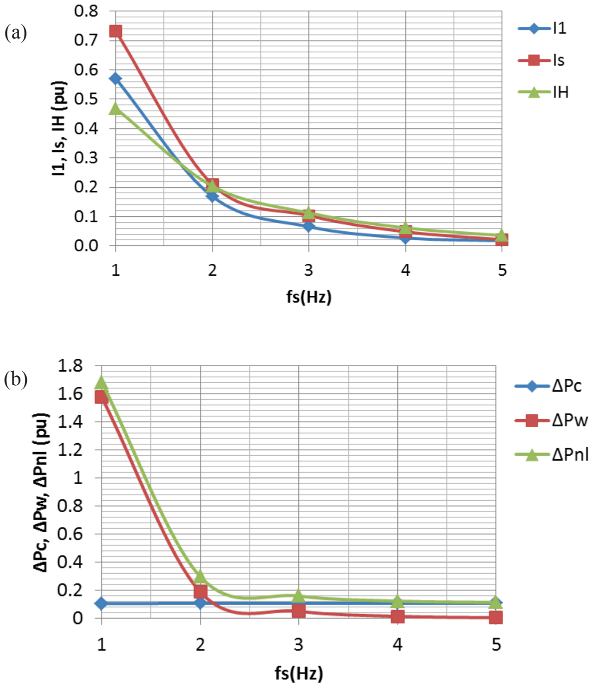

Figure 8. (a) I_1, I_S and I_H values and (b) ΔP_c, ΔP_w and ΔP_{nl} values observed for Case II.

IV. CONCLUSION

In this study, a computationally efficient method is developed to calculate no load loss of transformers under excitation voltages with sub-harmonics. The method is based on two main parts as the harmonic domain equivalent electric circuit and time domain FEM analysis.

It can clearly be concluded from the obtained numerical results that voltage sub-harmonics, even with seemingly insignificant magnitudes, may highly contribute to maximum flux density and excitation current of the transformer core. Thus, for the same voltage conditions, it is revealed that the winding losses may attain excessive levels. On the other hand,

978-1-4799-2449-3/14/$31.00 ©2014 IEEE

effect of sub-harmonic voltages on the transformer core loss is negligible.

This case practically means that voltage sub-harmonics should carefully be considered for the calculation of the transformer no-load losses and derating of power transformers.

V. REFERENCES

[1] A. E. Fitzgerald, C. Kingsley, S. D. Umans, "Electric Machinery", 6th Edition, McGraw Hill Co., NY, 2003.

[2] G. K. Singh, "Power system harmonics research: A survey", Eur. Trans. Electr. Power, vol. 19, no. 2, pp.151 -172, 2007.

[3] M. D. Hwang, M. W. Grady, H. W. Sanders, "Distribution transformer winding losses due to nonsinusoidal currents", IEEE Trans. on Power Deliv., vol. PWRD-12, no. 1, pp. 140-146, 1987.

[4] J. Alan, C. Forrest, "Harmonic load losses in HVDC converter transformers", IEEE Trans. on Power Del., vol. 6, no. 1, pp.153 -157, 1991.

[5] D. Henderson, P. J. Rose, "Harmonics: The effects on power quality and transformers," IEEE Trans. on Industrial Appl., vol. 30, no. 3, pp. 528-532, 1994.

[6] J. Driesen, T. V. Craenenbroeck, B. Brouwers, K. Hameyer, and R. Belmans, "Practical method to determine additional load losses due to harmonic currents in transformers with wire and foil windings", Proc. IEEE Power Engineering Society Winter Meeting, vol. 3, pp.2306 -2311, 2000.

[7] A. Damnjanovic, G. Feruson, "The measurement and evaluation of distribution transformer losses under nonlinear loading", Proc. Power Eng. Soc. General Meeting, pp.1416 - 1419, 2004.

[8] D. Yildirim, E. F. Fuchs, "Measured transformer derating and comparison with harmonic loss factor approach", IEEE Tran. on Power Del., vol. 15, no.1, pp.186 -191, 2000.

[9] E. F. Fuchs, D. Yildirim, W. M. Grady, "Measurement of eddy-current loss coefficient PEC-R, derating of single-phase transformers, and comparison with K-factor approach", IEEE Trans. on Power Del., vol. 15, no. 1, pp.148 -154, 2000.

[10] E. F. Fuchs , D. Lin, J. Martynaitis, "Measurement of three-phase transformer derating and reactive power demand under nonlinear loading conditions", IEEE Trans. on Power Del., vol. 21, no. 2, pp.665 -672, 2006.

[11] M. A. S. Masoum, P. S. Moses, A. S. Masoum, "Derating of asymmetric three-phase transformers serving unbalanced nonlinear loads", IEEE Trans. on Power Del., vol. 23, no. 4, pp.2033 -2041, 2008.

[12] M. B. B. Sharifian, J. Faiz, S. A. Fakheri, A. Zraatparvar, "Derating of distribution transformers for non-sinusoidal load currents using finite element method", Proc. 10th IEEE Int. Conf. Electronics, Circuits and Systems, vol. 2, pp.754 -757, 2003.

[13] M. B. B. Sharifian, J. Faiz, "Derating of a distribution transformer for non-linear loads", Eur. Trans. Elect. Power, vol. 16, no. 2, pp.189 -203, 2006.

[14] T. Stensland, E. F. Fuchs, W. M. Grady, M. T. Diyle, "Modeling of magnetizing and core-loss currents in single-phase transformers with voltage harmonics for use in power flow", IEEE Trans. on Power Del., vol. 12, no. 2, pp.768 -774, 1997.

[15] A. Gaurdeau, P. Picher, L. Bolduc, A. Coutu, "No load losses in transformer under overexcitation/inrush current conditions: Tests and a new model", IEEE Trans. on Power Del., vol. 17, no. 4, pp.1009 -1017, 2002.

[16] B. Suechoey, S. Tadsuan, C. Thammarat, M. Leelajindakrairerk, "Estimation of core loss of transformer under non-sinusoidal voltage supply", IEEE Int. Conf. on Power System Technology (POWERCON) 2004, vol. 1, pp. 511-516, 2004.

[17] T. D. Kefalas, A. G. Kladas, "Harmonic impact on distribution transformer no-load loss", IEEE Trans. on Ind. Electron., vol. 57, no. 1, pp.193 -200, 2010.

[18] M. Yazdani-Asrami, M. Mirzaie, A. A. S. Akmal, "No-load loss calculation of distribution transformers supplied by nonsinusoidal voltage using three-dimensional finite element analysis", Energy, vol. 50, no. 1, pp. 205-219, 2013.

[19] M. Shareghi, B. T. Phung, M. S. Naderi, T. R. Blackburn, E. Ambikairajah, "Effects of current and voltage harmonics on distribution transformer losses", International Conference on Condition Monitoring and Diagnosis (CMD) 2012, pp.633-636, 2012.

[20] T. L. G. Soh, D. M. Said, N. Ahmad, K. M. Nor, F. Salim, "Experimental study on the impact of harmonics on transformer", IEEE 7th International Power Engineering and Optimization Conference (PEOCO), pp.686-690, 2013.

[21] R. Langella, A. Testa, A. E. Emanuel, "On the effects of subsynchronous interharmonic voltages on power transformers: single phase units", IEEE Trans. on Power Deliv., vol. 23, no. 4, pp. 2480-2487, 2008.

[22] R. Langella, A. Testa, A. E. Emanuel, "On the effects of subsynchronous interharmonic voltages on power transformers: three phase units", IEEE Trans. on Power Deliv., vol. 23, no. 4, pp. 2461-2471, 2008.

[23] J. Jin, "The Finite Element Method in Electromagnetics", 2nd edition, John Wiley & Sons, Inc., 2002.

[24] The User's Guide of ANSYS/ Maxwell Release 15 Electromagnetics Simulation Software, 2010.

VI. APPENDIX

The results plotted in Figure A1- A3 are summarized in Table A 1.

Table A 1: Summary of the results plotted in Figure A1-A3.

Average value of the calculated no-load phase currents at primary side	1.30 A
Calculated core loss	8.42 kW
Maximum flux density under sinusoidal excitation	1.8 T

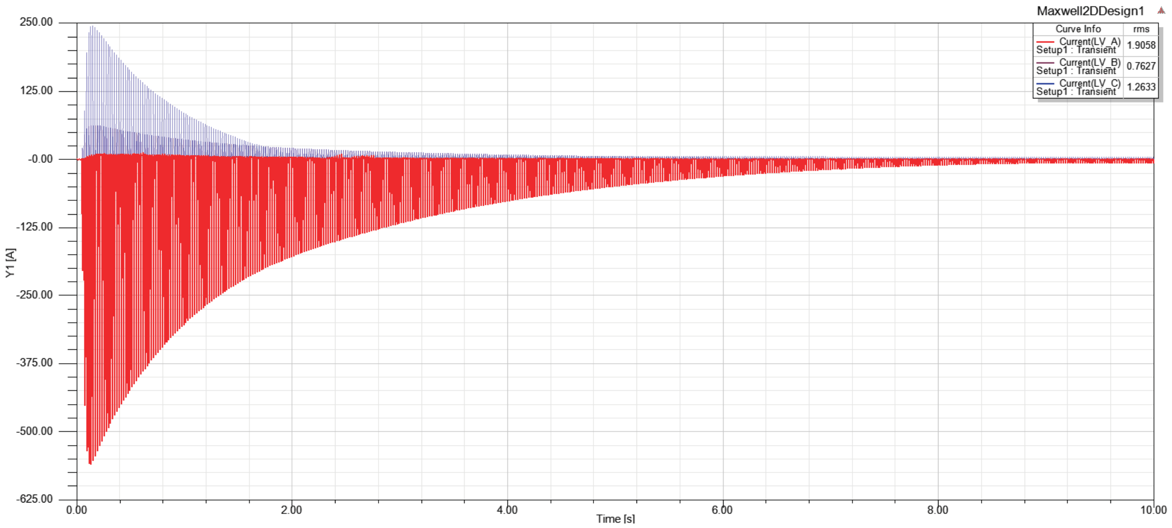

Figure A 1. Three-phase excitation currents versus time under sinusoidal and balanced voltage conditions.

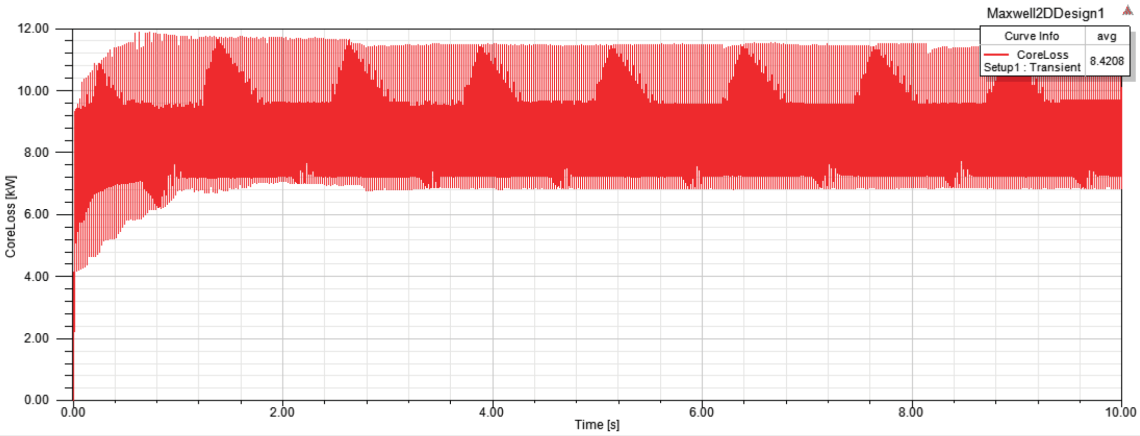

Figure A 2. Core loss versus time under sinusoidal and balanced voltage conditions.

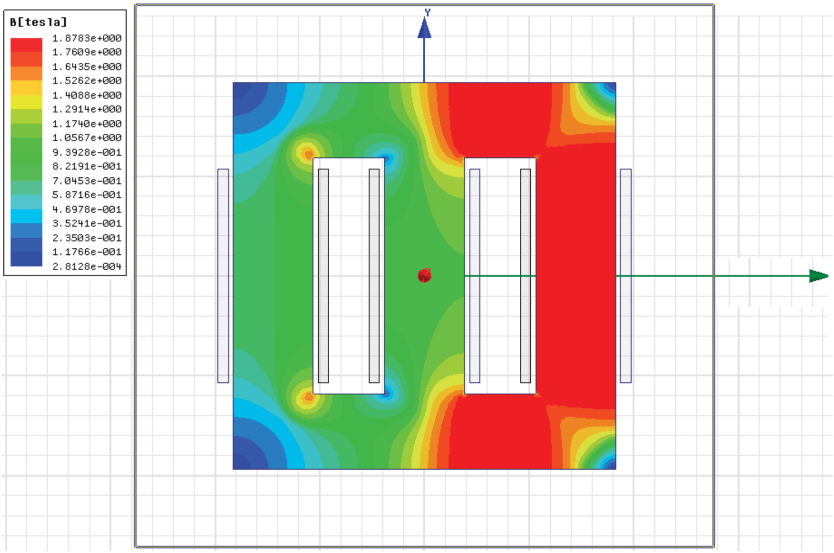

Figure A 3. Flux distribution on the transformer core under sinusoidal and balanced voltage conditions.