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On some special classes of Kenmotsu manifolds

SUNGPYO HONG[1], CİHAN ÖZGÜR[2] AND MUKUT MANI TRIPATHI[3]

[1]*Department of Mathematics, Pohang University of Science and Technology, San 31 Hyoja Dong, Nam-Gu, Pohang 790-784, Republic of Korea, e-mail: sungpyo@postech.ac.kr*

[2]*Department of Mathematics, Faculty of Arts and Sciences, Balikesir University, Campus of Cagis, 10145, Cagis, Balikesir, Turkey, email: cozgur@balikesir.edu.tr*

[3]*Department of Mathematics and Astronomy, Lucknow University, Lucknow 226 007, India, e-mail: mmtripathi66@yahoo.com*

ABSTRACT

We investigate the classes of Kenmotsu manifolds which satisfy the condition of being η-Einstein, having η-parallel Ricci tensor, $R(\xi, X) \cdot Z = 0$, $R(\xi, X) \cdot R = 0$, $Z(\xi, X) \cdot Z = 0$, $Z(\xi, X) \cdot R = 0$, $Z(\xi, X) \cdot S = 0$ or being Ricci-pseudosymmetric, where R, Z and S denote the curvature tensor, the concircular curvature tensor and the Ricci tensor, respectively. We also prove that a transformation in a Kenmotsu manifold under certain conditions is an isometry.

Keywords: Kenmotsu manifold; η-parallel Ricci tensor; concircular curvature tensor; η-Einstein manifold and Ricci-pseudosymmetric manifold.

AMS Subject Classification: 53C25.

INTRODUCTION

A transformation in an n-dimensional Riemannian manifold M, which transforms every geodesic circle of M into a geodesic circle, is called a *concircular transformation* (Yano 1940, Kuhnel 1988). A concircular transformation is always a conformal transformation (Kuhnel 1988). Thus, the geometry of concircular transformations, that is the concircular geometry, is a generalization of inversive geometry in the sense that the change of metric is more general than that induced by a circle preserving diffeomorphism (see also Blair 2000). An interesting invariant of a concircular transformation is the *concircular curvature tensor* Z as defined by Yano (1940) and Yano & Kon (1984):

$$Z(X, Y)U = R(X, Y)U - \frac{r}{n(n-1)}\{g(Y, U)X - g(X, U)Y\}$$

for all $X, Y, U \in TM$, where R is the curvature tensor and r is the scalar

curvature. Riemannian manifolds with vanishing concircular curvature tensor are of constant curvature. Thus, the concircular curvature tensor is a measure of the failure of a Riemannian manifold to be of constant curvature. In other words, it represents deviation of the manifold from being of constant curvature.

On the other hand, Tanno (1969) classified $(2n+1)$-dimensional connected almost contact metric manifolds M with almost contact metric structure (φ, ξ, η, g), whose automorphism groups have the maximum dimension $(n+1)^2$. For such a manifold, the sectional curvature of plane sections containing ξ is a constant, say c. If $c > 0$, M is a homogeneous Sasakian manifold of constant φ-sectional curvature. If $c = 0$, M is the global Riemannian product of a line or a circle with a Kähler manifold of constant holomorphic sectional curvature. If $c < 0$, M is a warped product space $\mathrm{R}\times_f \mathrm{C}^n$. In 1972, K. Kenmotsu abstracted the differential geometric properties of the third case. In particular, the almost contact metric structure in this case satisfies the equation

$$(\nabla_X \varphi) Y = g(\varphi X, Y)\xi - \eta(Y)\varphi X, \qquad X, Y \in TM,$$

where ∇ is the Levi-Civita connection of the Riemannian metric, and an almost contact metric manifold satisfying the above equation is called a *Kenmotsu manifold* (Kenmotsu 1972). It is known that certain Legendre curves in a Kenmotsu manifold are circles (Tripathi 2000). Recently, Kirichenko (2001) obtained Kenmotsu structures from cosymplectic structures (that is, $\nabla\varphi = 0$ (Blair 2002)) by the canonical concircular transformations (Yano 1940). Moreover, the concircular curvature tensor of contact metric manifolds are studied recently (Tripathi & Kim 2004; Blair, Kim & Tripathi 2005). These circumstances motivate us for further study of a Kenmotsu manifold, which is another important class of almost contact metric manifolds apart from Sasakian manifold. The paper is organized as follows. In section 2, some properties of the concircular curvature tensor on a Riemannian manifold are shown. In particular, it is proved that a Riemannian manifold satisfying $Z \cdot S = 0$ is a Ricci-pseudosymmetric manifold. Section 3 contains a brief account of almost contact metric manifolds and Kenmotsu manifolds. In section 4, among other results it is proved that a 3-dimensional Kenmotsu manifold with η-parallel Ricci tensor is an Einstein manifold with constant scalar curvature -6 and constant curvature -1 and is locally φ-symmetric. In section 5, the conditions $R(\xi, X) \cdot Z = 0$, $R(\xi, X) \cdot R = 0$, $Z(\xi, X) \cdot Z = 0$ and $Z(\xi, X) \cdot R = 0$ on a Kenmotsu manifold are studied and several statements are proved to be equivalent. In section 6, Ricci-pseudosymmetric Kenmotsu manifold and the condition $Z(\xi, X) \cdot S = 0$ on a Kenmotsu manifold are studied. In the last section, a transformation in a Kenmotsu manifold under certain conditions is proved to be an isometry.

SOME PROPERTIES OF THE CONCIRCULAR CURVATURE TENSOR

It is well known that every $(1,1)$ tensor field A on a differentiable manifold determines a derivation $A\cdot$ of the tensor algebra on the manifold, commuting with contractions. For example, the $(1,1)$ tensor field $R(X,Y)$ induces the derivation $R(X,Y)\cdot$, thus associating with a (r,s) tensor field T, the $(r,s+2)$ tensor $R\cdot T$ is defined by

$$\begin{aligned}(R\cdot T)(X_1,X_2,\ldots,X_s;X,Y) &= (R(X,Y)\cdot T)(X_1,X_2,\ldots,X_s)\\ &= -\,T(R(X,Y)X_1,X_2,\ldots,X_s)\\ &\quad -\cdots-\;T(X_1,X_2,\ldots,R(X,Y)X_s).\end{aligned}$$

Now, we begin with the following:

Proposition 1. Let (M,g) be an n-dimensional Riemannian manifold. Then $R\cdot R = R\cdot Z$.

Proof. Denoting

$$R_0(X,Y)U = g(Y,U)X - g(X,U)Y, \qquad X,Y,U\in TM, \tag{1}$$

the concircular curvature tensor can be represented as

$$Z = R - \frac{r}{n(n-1)}R_0, \tag{2}$$

which implies that

$$R\cdot Z = R\cdot R - \frac{r}{n(n-1)}R\cdot R_0.$$

By a straightforward calculation it follows that $R\cdot R_0 = 0$.

Similarly, it is easy to prove the following

Proposition 2. Let (M,g) be an n-dimensional Riemannian manifold. Then

$$Z\cdot Z = Z\cdot R.$$

An n-dimensional Riemannian manifold (M,g) is called *Ricci-pseudosymmetric* (Deszcz 1989) if the tensors $R\cdot S$ and $Q(g,S)$ are linearly dependent, where

$$Q(g,S)(U,V;X,Y) = -\,S(R_0(X,Y)U,V) \;-\; S(U,R_0(X,Y)V) \tag{3}$$

for vector fields U, V,X, Y on M. The condition of Ricci-pseudosymmetry is equivalent to

$$(R(U,V)\cdot S)(X,Y) = L_S Q(g,S)(U,V;X,Y) \tag{4}$$

holding on the set

$$U_S = \left\{x \in M : S \neq \frac{r}{n}g \quad at \quad x\right\},$$

where L_S is some function on U_S. If $R\cdot S = 0$ then M is said to be *Ricci-semisymmetric*. Every Ricci-semisymmetric manifold is Ricci-pseudosymmetric but the converse is not true.

Now we prove the following

Proposition 3. Let (M,g) be an n-dimensional Riemannian manifold satisfying the condition $Z\cdot S = 0$. Then M is a Ricci-pseudosymmetric manifold such that $L_S = \frac{r}{n(n-1)}$.

Proof. From (2) we have

$$Z\cdot S = R\cdot S - \frac{r}{n(n-1)}R_0\cdot S.$$

In view of (1) and (3) we get $R_0\cdot S = Q(g,S)$, which implies that

$$Z\cdot S = R\cdot S - \frac{r}{n(n-1)}Q(g,S).$$

Since the condition $Z\cdot S = 0$ holds on M, we obtain

$$R\cdot S = \frac{r}{n(n-1)}Q(g,S),$$

which completes the proof.

KENMOTSU MANIFOLDS

Let M be a $(2n+1)$-dimensional almost contact metric manifold (Blair 2002) equipped with an almost contact metric structure (φ,ξ,η,g) consisting of a $(1,1)$ tensor field φ, a vector field ξ, a 1-form η and a compatible Riemannian metric g satisfying

$$\varphi^2 = -I + \eta\otimes\xi, \qquad \eta(\xi) = 1, \qquad \varphi\xi = 0, \qquad \eta\circ\varphi = 0, \tag{5}$$

$$g(X,Y) = g(\varphi X,\varphi Y) + \eta(X)\eta(Y), \tag{6}$$

$$g(X,\varphi Y) = -g(\varphi X,Y), \qquad g(X,\xi) = \eta(X) \tag{7}$$

for all $X, Y \in TM$. An almost contact metric manifold M is called a *Kenmotsu manifold* if it satisfies (Kenmotsu 1972)

$$(\nabla_X \varphi) Y = g(\varphi X, Y)\xi - \eta(Y)\varphi X, \qquad X, Y \in TM, \tag{8}$$

where ∇ is Levi-Civita connection of the Riemannian metric. From the above equation it follows that

$$\nabla_X \xi = X - \eta(X)\xi, \tag{9}$$

$$(\nabla_X \eta) Y = g(X, Y) - \eta(X)\eta(Y). \tag{10}$$

Moreover, the curvature tensor R, the Ricci tensor S, and the Ricci operator Q satisfy (Kenmotsu 1972)

$$R(X, Y)\xi = \eta(X) Y - \eta(Y)X, \tag{11}$$

$$S(X, \xi) = -2n\eta(X), \tag{12}$$

$$Q\xi = -2n\xi \tag{13}$$

Equation (11) is equivalent to

$$R(\xi, X) Y = \eta(Y)X - g(X, Y)\xi, \tag{14}$$

which implies that

$$R(\xi, X)\xi = X - \eta(X)\xi. \tag{15}$$

From (11) and (14), we have

$$\eta(R(X, Y)\xi) = 0, \tag{16}$$

$$\eta(R(\xi, X) Y) = \eta(X)\eta(Y) - g(X, Y). \tag{17}$$

A plane section Π in T_pM of an almost contact metric manifold M is called a *φ-section* if $\Pi \perp \xi$ and $\varphi(\Pi) = \Pi$. If at each point $p \in M$ the sectional curvature $K(\Pi)$ does not depend on the choice of the φ-section Π of T_pM, then M is of *pointwise constant φ-sectional curvature*.

A Kenmotsu manifold of pointwise constant φ-sectional curvature is called a *Kenmotsu space form*. Interestingly, using the notion of exterior recurrent forms (Datta 1982) on manifolds, Pitiș (1988) proved that there exist no connected Kenmotsu space forms or connected conformally flat manifolds of dimension ≥ 5.

A Kenmotsu manifold is normal (that is, the Nijenhuis tensor of φ equals $-2d\eta \otimes \xi$) but not Sasakian. Moreover, it is also not compact since from equation (9) we get $\mathrm{div}\xi = 2n$. Kenmotsu (1972) showed (i) that locally a Kenmotsu manifold is a warped product $I \times_f N$ of an interval I and a Kähler manifold N with warping function $f(t) = se^t$, where s is a nonzero constant; and (ii) that a Kenmotsu manifold of constant φ-sectional curvature is a space of constant curvature -1, and so it is locally hyperbolic space. Examples of Kenmotsu manifolds of strictly pointwise constant φ-sectional curvature are not known so far and, according to D. Blair (in a private conversation), one doubts that there are any since the warped product structure of a Kenmotsu manifold involves a Kähler structure. Thus, one has to be careful for further study of Kenmotsu space forms with strictly pointwise constant φ-sectional curvature.

$\eta-$EINSTEIN KENMOTSU MANIFOLDS

Let M be a $(2n+1)$-dimensional almost contact metric manifold. Then M is said to be η-*Einstein* (Blair 2002) if the Ricci operator Q satisfies

$$Q = aI + b\eta \otimes \xi, \tag{18}$$

where a and b are smooth functions on the manifold. In particular, if $b = 0$, then M is an Einstein manifold. If a $(2n+1)$-dimensional Kenmotsu manifold is an η-Einstein manifold, then $a + b = -2n$. The Ricci tensor S of an almost contact metric manifold M is said to be η-parallel (Yano & Kon 1984) if

$$(\nabla_U S)(\varphi X, \varphi Y) = 0, \qquad X, Y, U \in TM.$$

We prove the following:

Theorem 1. Let M be a $(2n+1)$-dimensional η-Einstein Kenmotsu manifold. Then the following statements are equivalent:

(i) Ricci tensor of M is η-parallel.

(ii) M is of constant scalar curvature $-2n(2n+1)$.

(iii) M is an Einstein manifold with $S(X, Y) = -2ng(X, Y)$.

Proof. It is known that (Kenmotsu 1972) if a $(2n+1)$-dimensional Kenmotsu manifold is an η-Einstein manifold, then $a + b = -2n$. From (15), we also have $r = (2n+1)a + b$. Thus, we have

$$S(X, Y) = \left(\frac{r}{2n} + 1\right)g(X, Y) - \left(\frac{r}{2n} + (2n+1)\right)\eta(X)\eta(Y). \tag{19}$$

The statements (ii) and (iii) are equivalent from (19). By (19), we obtain

$$(\nabla_U S)(X, Y) = \frac{1}{2n} dr(U) g(\varphi X, \varphi Y) - \left(\frac{r}{2n} + (2n+1)\right)\{g(\varphi U, \varphi X)\eta(Y) + g(\varphi U, \varphi Y)\eta(X)\}.$$

From the above equation, we have

$$(\nabla_U S)(\varphi X, \varphi Y) = \frac{1}{2n} dr(U) g(\varphi X, \varphi Y). \tag{20}$$

By (20), statement (ii) implies statement (i). If Ricci tensor S is η-parallel, then from (20) it follows that the scalar curvature r is a constant. It is known that if a Kenmotsu manifold is an η-Einstein manifold such that one of a and b is a constant, then M is Einstein (Corollary 9, Kenmotsu 1972). Thus, we conclude that a $(2n+1)$-dimensional η-Einstein Kenmotsu manifold with η-parallel Ricci tensor is an Einstein manifold. Therefore, from (19) we get $r = -2n(2n+1)$. This completes the proof.

A 3-dimensional Kenmotsu manifold with η-parallel Ricci tensor is of constant scalar curvature (De & Pathak 2004). However, we prove the following:

Theorem 2. A 3-dimensional Kenmotsu manifold with η-parallel Ricci tensor is an Einstein manifold with constant scalar curvature -6 and constant curvature -1.

Proof. Since in a 3-dimensional Riemannian manifold the Weyl conformal curvature tensor vanishes, therefore it is known that

$$\begin{aligned} R(X, Y)U = {} & g(Y, U)QX - g(X, U)QY + S(Y, U)X - S(X, U)Y \\ & - \tfrac{r}{2}\{g(Y, U)X - g(X, U)Y\}, \end{aligned} \tag{21}$$

where r is the scalar curvature. Now, let M be a 3-dimensional Kenmotsu manifold. From (12) and (21), we have

$$R(X, Y)\xi = \eta(Y)QX - \eta(X)QY + \left(\frac{r+4}{2}\right)\{\eta(X)Y - \eta(Y)X\}. \tag{22}$$

In view of (11) and (22), we obtain

$$\eta(Y)\left(QX - \left(\frac{r+2}{2}\right)X\right) = \eta(X)\left(QY - \left(\frac{r+2}{2}\right)Y\right).$$

Putting $Y = \xi$ in the above equation, we get

$$2QX = (r+2)X - (r+6)\eta(X)\xi, \tag{23}$$

which is equivalent to

$$2S(X, Y) = (r + 2)g(X, Y) - (r + 6)\eta(X)\eta(Y). \tag{24}$$

Initially from (23), we see that M is an η-Einstein manifold. Hence, in view of Theorem 1, we see that M is an Einstein manifold with constant scalar curvature -6. Therefore, from (23) and (24) we get

$$QX = -2X, \qquad S(X, Y) = -2g(X, Y). \tag{25}$$

Using (25) in (21), we obtain

$$R(X, Y)U = g(X, U)Y - g(Y, U)X. \tag{26}$$

Thus M is of constant curvature -1.

A 3-dimensional Kenmotsu manifold is locally φ-symmetric if and only if the scalar curvature is a constant (De & Pathak 2004). Thus, in view of Theorem 2, we have the following:

Corollary 1. A 3-dimensional Kenmotsu manifold with η-parallel Ricci tensor is locally $\varphi-$symmetric.

KENMOTSU MANIFOLDS SATISFYING $\mathbf{R(\xi, X)\bullet Z = 0}$ AND $\mathbf{Z(\xi, X)\bullet Z = 0}$

A necessary and sufficient condition that a Riemannian manifold be reducible to a Euclidean space by a suitable concircular transformation is that its concircular curvature tensor vanishes. A Riemannian manifold with vanishing concircular curvature tensor is said to be *concircularly flat*. A Riemannian manifold is concircularly flat if and only if it is of constant curvature.

Let M be a $(2n + 1)$-dimensional almost contact metric manifold equipped with an almost contact metric structure (φ, ξ, η, g). Then the concircular curvature tensor Z in M becomes

$$Z(X, Y)U = R(X, Y)U - \frac{r}{2n(2n + 1)}\{g(Y, U)X - g(X, U)Y\}. \tag{27}$$

From (27), (11) and (14), we have

$$Z(X, Y)\xi = \left(1 + \frac{r}{2n(2n + 1)}\right)\{\eta(X)Y - \eta(Y)X\}, \tag{28}$$

$$Z(\xi, X)Y = \left(1 + \frac{r}{2n(2n + 1)}\right)\{\eta(Y)X - g(X, Y)\xi\}. \tag{29}$$

Consequently, we have

$$Z(\xi, X)\xi = \left(1 + \frac{r}{2n(2n+1)}\right)(X - \eta(X)\xi). \tag{30}$$

Also, in view of (27) and (11) we have

$$\eta(Z(X, Y)U) = \left(1 + \frac{r}{2n(2n+1)}\right)\{\eta(Y)g(X, U) - \eta(X)g(Y, U)\}, \tag{31}$$

which implies that

$$\eta(Z(X, Y)\xi) = 0, \tag{32}$$

$$\eta(Z(\xi, X)Y) = \left(1 + \frac{r}{2n(2n+1)}\right)\{\eta(X)\eta(Y) - g(X, Y)\}. \tag{33}$$

Now, we prove the following theorem.

Theorem 3. On a $(2n+1)$-dimensional Kenmotsu manifold M the following statements are equivalent:

(i) M is canonically concircular to $\mathrm{C}^n \times \mathrm{R}$.

(ii) M is of constant curvature -1.

(iii) M is concircularly flat.

(iv) M satisfies $R(\xi, X) \cdot Z = 0$.

(v) M satisfies $R(\xi, X) \cdot R = 0$.

Proof. It is known that a $(2n+1)$-dimensional Kenmotsu manifold is of constant curvature -1 if and only if it is canonically concircular to $\mathrm{C}^n \times \mathrm{R}$, therefore statements (i) and (ii) are equivalent (Kirichenko 2001). In view of the fact that a Riemannian manifold is concircularly flat if and only if it is of constant curvature, statement (ii) implies statement (iii). Obviously, statement (iv) is implied by statement (iii). From Proposition 1, statements (iv) and (v) are equivalent. A Kenmotsu manifold, which satisfies $R(\xi, X) \cdot R = 0$, is of constant curvature -1 (Kenmotsu 1972); thus statement (v) implies statement (ii). A straightforward calculation also shows that statement (ii) implies statement (v).

Replacing $R(\xi, X)$ by $Z(\xi, X)$ in the conditions $R(\xi, X) \cdot Z = 0$ and $R(\xi, X) \cdot R = 0$, we have the following:

Theorem 4. Let M be a $(2n+1)$-dimensional Kenmotsu manifold. Then the

following statements are equivalent:

(i) M is either of constant scalar curvature $-2n(2n+1)$ or of constant curvature -1.

(ii) M satisfies $Z(\xi, X) \cdot Z = 0$.

(iii) M satisfies $Z(\xi, X) \cdot R = 0$.

Proof. If M has scalar curvature $r = -2n(2n+1)$ then from (29) it follows that $Z(\xi, X) = 0$; and if M is of constant curvature, then $Z = 0$. Thus, statements (ii) and (iii) follow from statement (i). From Proposition 2, statements (ii) and (iii) are equivalent. Next, assuming statement (iii) we have

$$0 = [Z(\xi, U), R(X, Y)]\xi - R(Z(\xi, U)X, Y)\xi - R(X, Z(\xi, U)Y)\xi.$$

In view of (29), (16) and (30) the above relation gives

$$\begin{aligned} 0 = -\left(1 + \tfrac{r}{2n(2n+1)}\right)\{&g(U, R(X, Y)\xi)\xi + R(X, Y)U - \eta(U)R(X, Y)\xi \\ &+ \eta(X)R(U, Y)\xi - g(U, X)R(\xi, Y)\xi + \eta(Y)R(X, U)\xi - g(U, Y)R(X, \xi)\xi\}, \end{aligned}$$

which in view of (11) yields

$$0 = \left(1 + \frac{r}{2n(2n+1)}\right)\{R(X, Y)U + g(Y, U)X - g(X, U)Y\},$$

which gives us either $r = -2n(2n+1)$ or

$$R(X, Y)U + g(Y, U)X - g(X, U) = 0.$$

This implies that M is of constant curvature -1 showing statement (i).

RICCI-PSEUDOSYMMETRIC KENMOTSU MANIFOLDS

It is proved that a Kenmotsu manifold satisfying $R(X, Y) \cdot S = 0$ is an Einstein manifold (Binh *et al.* 2002), while Umnova (2002) has shown that any Einsteinian Kenmotsu manifold has constant sectional curvature -1. In this section we prove the following:

Theorem 5. Let M be a $(2n+1)$-dimensional Kenmotsu manifold. If M is Ricci-pseudosymmetric, then either M is an Einstein manifold with scalar curvature $r = -2n(2n+1)$ (in which case M is Ricci-semisymmetric) or $L_S = -1$ holds on M.

Proof. Let M be a $(2n+1)$-dimensional Ricci-pseudosymmetric Kenmotsu manifold. So the condition

$$(R(U,X)\cdot S)(Y,V) = L_S Q(g,S)(Y,V;U,X) \tag{34}$$

holds on M. Putting $U=\xi$ in (34) we have

$$(R(\xi,X)\cdot S)(Y,V) = L_S Q(g,S)(Y,V;\xi,X). \tag{35}$$

Now we calculate the left hand side of (35). Then

$$(R(\xi,X)\cdot S)(Y,V) = -S(R(\xi,X)Y,V) - S(Y,R(\xi,X)V), \tag{36}$$

which gives

$$(R(\xi,X)\cdot S)(Y,V) = -2ng(X,Y)\eta(V) - \eta(Y)S(X,V) - 2ng(X,V)\eta(Y) - \eta(V)S(X,Y). \tag{37}$$

On the other hand by the use of (3) we can write

$$Q(g,S)(Y,V;\xi,X) = 2ng(X,Y)\eta(V) + \eta(Y)S(X,V) + 2ng(X,V)\eta(Y) + \eta(V)S(X,Y). \tag{38}$$

Suppose that M is Ricci-semisymmetric, then it is trivially Ricci-pseudosymmetric. Therefore we can write $(R(\xi,X)\cdot S)(Y,V)=0$. So from (37) we get

$$2ng(X,Y)\eta(V) + \eta(Y)S(X,V) + 2ng(X,V)\eta(Y) + \eta(V)S(X,Y) = 0. \tag{39}$$

Putting $Y=\xi$ into (39) we obtain

$$S(X,V) = -2ng(X,V),$$

which implies that M is an Einstein manifold with the scalar curvature $r=-2n(2n+1)$.

Now suppose that M is a Ricci-pseudosymmetric manifold, which is not Ricci-semisymmetric. Then in view of (37) and (38) we obtain

$$(1+L_S)\{2ng(X,Y)\eta(V) + \eta(Y)S(X,V) + 2ng(X,V)\eta(Y) + \eta(V)S(X,Y)\} = 0.$$

Since M is not Ricci-semisymmetric, we get $L_S=-1$.

In view of Theorem 5 and Proposition 3 we obtain the following corollary:

Corollary 2. Let M be a $(2n+1)$-dimensional Kenmotsu manifold. If the concircular curvature tensor Z satisfies $Z(\xi,X)\cdot S=0$, then either the scalar curvature r of M satisfies $r=-2n(2n+1)$ or M is Einstein with $r=-2n(2n+1)$.

A TRANSFORMATION IN A KENMOTSU MANIFOLD

We prove the following:

Theorem 6. Let M be a $(2n+1)$-dimensional manifold carrying a Kenmotsu structure (φ, ξ, η, g). If a transformation f transforms the structure (φ, ξ, η, g) into another Kenmotsu structure $(\varphi', \xi', \eta', g')$, which leaves the concircular curvature tensor and the Ricci tensor invariant, such that $\eta(\xi') \neq 0$, then f is an isometry.

Proof. From the assumption $Z = Z'$ and (31) we get

$$\left(1 + \frac{r}{2n(2n+1)}\right)\{\eta(Y)g(X, U) - \eta(X)g(Y, U)\} = \eta(Z'(X, Y)U). \tag{40}$$

Putting $X = \xi'$ in (40) and using (29) we get

$$\eta(Y)\{g(\xi', U) - \eta'(U)\} = \eta(\xi')\{g(Y, U) - g'(Y, U)\}. \tag{41}$$

which implies that

$$\eta(Y)\{g(\xi', U) - \eta'(U)\} = \eta(U)\{g(\xi', Y) - \eta'(Y)\}. \tag{42}$$

Putting $Y = \xi$ in (42), we have

$$g(\xi', U) - \eta'(U) = \eta(U)\{g(\xi', \xi) - \eta'(\xi)\}. \tag{43}$$

From the assumption $S = S'$, in view of (12), we find

$$\eta(\xi') = -\frac{1}{2n}S(\xi, \xi') = -\frac{1}{2n}S'(\xi, \xi') = \eta'(\xi). \tag{44}$$

Using (44) in (43), we get

$$g(\xi', U) - \eta'(U) = 0. \tag{45}$$

From (41) and (45) we obtain

$$\eta(\xi')\{g(Y, U) - g'(Y, U)\} = 0,$$

which implies that $g = g'$, provided $\eta(\xi') \neq 0$.

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حول بعض الأصناف الخاصة لمنطويات كينموتسو

سنجبيو هونغ وجيهان اوزغر ومكوت ماني ترباثي

خلاصة

ندرس في هذا البحث أصناف خاصة لمطويات كينموتسو، ونثبت كذلك أن التحولات لمطوية كينموتسو وتحت ظروف معينة تعطى أبعاد متساوية.