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The Erroneous Derivative Examples of Eleventh Grade Students

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Abstract

The derivative is not only an important subject for mathematics but also is an important subject for engineering, physics, economy, chemistry, and statistics. Especially, mathematics depends on strongly preceding learning and the subject of derivative will be used in university education by all students. Therefore, it is one of the most important subjects. This study's purpose is to explore student mistakes and errors in derivative and determine the areas in which students have probable misconceptions. For this purpose, 7 questions were chosen from "the Student Placement Test" (OSS). These questions were transferred into open-ended questions. The results of the study took place at sixth form college are described and discussed. The test administered to 53 students from Balikesir Fatma Emin Kutvar Anatolian High School in the fall-term of 2005-2006. Determining the possible misconceptions should help teachers when they teach this subject. The study findings showed that students could not understand derivative definition that depends on limit, make mistakes in composite functions and trigonometric functions, and establish wrong relations between tangent's slope, and normal's slope. Teachers need to be able to find errors and misconceptions in students' solutions. Teachers also need to be applying meaningful learning strategies such as concept maps, worksheets about derivative (e.g. Appendix B, Appendix C).

Key Words

Errors and Misunderstanding, Misconceptions, Errors and Misunderstanding Towards Derivative

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The derivative of a function represents an infinitesimal change in the function with respect to whatever parameters it may have. The "simple" derivative of a function f with respect to x is denoted either f(x). Students have some misconceptions or errors in derivative. Misconception is defined as erroneous conception, false opinion, or wrong understanding (Big Larousse, 1986). Studies about derivative and ideas related to it (such as tangent lines) have emphasized students' misconceptions and common errors (e.g., Amit & Vinner, 1990; Artique, 1991; Orton, 1983; Ubuz, 1996, 2001; Maurer (1987; Norman & Pritchard, 1994; Krutetski, 1980; Orton, 1983; Donaldson, 1963; Cipra, 1989; Keith et al., 1990). Ubuz (2001, p. 129) showed that students' common misconceptions on derivative were as follows: " (a) derivative at a point gives the function of a derivative, (b) tangent equation is the derivative function, (c) derivative at a point is the tangent equation, and (d) derivative at a point is the value of the tangent equation at that point." Ubuz also found that students seem to think different concepts as the same. He reported that *"(a) the lack of discrimination of concepts which occur in the same context or the confusion of a concept with another concept describing a different feature of the same situation, (b) the inappropriate extension of a specific case to a general case, and (c) the lack of understanding of graphical representation"(p.133)*. Some studies have mainly focused on the constructions of mathematical knowledge in a theoretical perspective rather than students' misconceptions and common errors (Dubinsky & Schwingendorf, 1991) On the other hand, few empirical research were conducted such as Tall (1986a). He revealed that 67% of the experimental students who used Graphic Calculus (Tall, 1986b) chose the right answer with a correct explanation, while only 8% of the control students did. Thus, it is likely that visualization in the graphical context can help students understand the relations between differentiation and integration. Mathematics teaching is directly linked to learning and students' understanding of the concept of derivative is related to their prior knowledge (Kendal, 2001). Kendal (2001) stated that using multiple presentations was important in developing the understanding of the concept of derivative. In the present study the following questions are addressed. What are the errors and misconceptions beyond the difficulties? Is it possible to diminish or eliminate these difficulties with the use of technology or meaningful learning tools? If so, How?

Method

Subject

The sample consists of 53 eleventh grade students in Fatma Emin Kutvar Anatolian High School. Science (Fen) class had 40 students and Turkish and Mathematics (TM) class had 13 students. Both classes are taught by the same mathematics teacher during the 2005-2006 term. 53 students took the test on derivative. The students who took both the test was taken as the sample of the study.

Instruments

The test used for assessing students' learning of derivative consisted of 7 questions some of which had different tasks (altogether 15 tasks), on which students were to work individually to provide written responses. The test was administered after derivative subject had taught at the end of the semester. Each task in the questions was graded by one of the four categories (Abraham *et al.,* 1994): correct (5), partially correct (4), misconception or error (3), incorrect (2) and missing (1). Factor analysis was carried out for the questions in the test. The questions were related to curriculum of derivative.

Treatment

The study was conducted in a mathematics course designed to teach derivative and the basic theorems of differential calculus. After each course, students completed homework exercises about derivative. Teacher of the course was available to answer their questions. At the end of the term, the derivative test was given.

Results and Discussion

The study described eleventh grade students' errors and misconceptions in derivative. The primary goal of the first and fourth questions was to analyze the definition of derivative in the question. These questions were mostly answered correctly. %8 of TM students have errors (table 1a, 1b, 2a, 2b, 3a, 3b, 4a, 4b, 5a, 5b, 6a, 6b, 7a, 7b). These errors were two types: Not knowing the definition of derivative or the type of function. Students memorized the rules not the definition. The second and third questions focused on tangent line and normal line. Students have misconceptions related to square root; the derivative of close function; tangent line, normal line; the derivation of composite functions, and application of rules. Fifth questions focused on the computation of two functions: f(1)+f (1). Students have misconceptions related to the derivation of composite functions. Sixth questions focused on the computation of derivative. Students have misconceptions related to the derivation of composite function; not knowing derivative rules. Similar errors were observed in the last question. To sum, the study found several types of errors and misconceptions in derivative.

The frequent errors in derivative include: not knowing definition of derivative, missing or erroneous square root, not knowing the type of function, the error of formulation, erroneous variable handling, the derivation of composite functions, not knowing tangent line, normal line, the erroneous of the interpretation of the notion 'derivative', composite functions, the mal rule of formulation, erroneous about composite functions, application of wrong derivation rules or wrong application of such rules, misconception about variables, missing domain conditions, slips in computations, and arithmetic or algebraic errors.

Conclusion

Teaching is directly linked to learning and each class developed understanding of the concept of derivative that related to the combined effect of their teacher's privileging characteristics: Calculus content, teaching approach.

The general analysis of students' performance pointed to a misconception or errors in derivative topic. Teachers need to be able to find errors and misconceptions in students' solutions in mathematics topics. Teachers need to be applying meaningful learning strategies such as concept maps or worksheets about derivative (e.g. Appendix B, Appendix C).

Kaynakça/References

Abraham, M. R., Williamson, V. M. (1994). A cross-age student understanding of five chemistry concept. *Journal of Research and Science Teaching*, *31*, 147-165.

Akkuş, O. (2000). *Principles and standarts for school mathematics NCTM.* 11 Aralık 2005 tarihinde mategt.web.ibu.edu.tr/makaleler/OKUL_MATEMATi-Gi.hTM adresinden edinilmiştir.

Altun, M. (Ed). (2002). *İlköğretim ikinci kademede (6, 7 ve 8. sınıflarda) matematik öğretimi* (2. baskı). Bursa: Alfa Yayınları.

Artigue, M.(1991): Analysis. I D. Tall (red.), *Advanced mathematical thinking* (Kapitel 11). Dordrecht: Kluwer Academic Publishers.

Amit, M., & Vinner, S. (1990). Some misconceptions in calculus: Anecdotes or tip of an iceberg. In G. Brooker et al. (Eds.), *Proceedings of 14th International Conference for the Psychology of Mathematics Education 14* (Vol.1, p. 3-10). Oaxtepec, Mexico: CINVESTAV.

Balcı, M. (2003). *Genel matematik* (2. baskı). Ankara: Balcı Yayınları.

BauJaoude, S. B. (1992). The relationship between students' learning strategies and the change in their misunderstanding during a high school chemistry course. *Journal of Research in Science Teaching, 29*, 687-699.

Benk, A. (1986). *Büyük larousse* (13. Cilt) İstanbul: Interpress Basın ve Yayıncılık A.Ş.

Cipra, B., (1989). *Mistakes.* New York: Academic Press.

Doğan, A., Sulak, H. & Cihangir, A. (2002, Eylül). *İlköğretim matematik eğitimi anabilim dalı öğrencilerinin özel fonksiyonlar ile fonksiyonlarda limit, türev ve türev uygulamaları konularındaki yeterlikleri üzerine bir araştırma.* V. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresinde sunulan bildiri, ODTÜ, Ankara.

Driver, R., & Easley, Y. (1978). Pupils and paradigms: A review of literature related to concept development in adolescent science students. *Studies in Science Education, 5*, 61-84.

Donaldson, M., (1963). *A study of children's thinking.* London: Tavistock Publications.

Dubinsky, E., & Schwingendorf, E. K. (1991). Constructing calculus concepts: cooperation in a computer laboratoy. In C. Leinbach (Ed.), *The laboratory approach to teaching calculus.* Mathematical Association of America, Notes and Reports, 20.

Fensham, P. J. Garrard, J., & West, L. W. (1981). The use of cognitive mapping in teaching and learning strategies. *Research in science Education, 11*, 121-129.

Gronlund, N. E., & Linn, R. L. (1990). *Measurement and evaluation in teaching* (6th ed.). New York, London: MacMillan, Collier MacMillan.

Gür, H. & Seyhan S. (2004). İlköğretim 7. ve 8. sınıf öğrencilerinin ondalık konusundaki hataları ve kavram yanılgıları. 11.11.2005 tarihinde http://matder.org.tr adresinden edinilmiştir.

Gür, H. & Barak, B. (2006, Nisan). Çağdaş Yönelimler III *(Yapılandırmacılık ve Eğitime Yansımaları) Sempozyumu.* İzmir Özel Tevfik Fikret Lisesi, İzmir.

Hirst, K. (2002, Jully). Hirst, K. (2002), *Classifying students' mistakes in Calculus.* Paper presented at the 2nd International Conference on the Teaching of Mathematics (at the undergraduate level), University of Crete.

Hirst, K. (2004), Student expectations of studying mathematics at university. In I. Putt, R. Faragher & M.McLean (Eds), *Proceedings of the 27th Annual Conference of the Mathematics Education Research Group of Australasia (MERGA27)* (Vol. 1, pp. 295-302). Townsville, Queensland, Australia.

Kendal, M. (2001). *Teaching and learning introductory differential calculus with a computer algebra system*. Unpublished doctorate dissertsation, The University of Melbourne.

Krutetski, V. A. (1980), *The psychology of mathematical abilities in school children.* Chicago: University of Chicago Pres.

Mayer, R. E. (1987). *Educational psychology: A cognitive approach.* Toronto: Little, Brown and Company.

Maurer, S. B., (1987), New knowledge about errors and new views about learners: What they mean to educators and more educators would like to know. In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 165-188). Hillsdale NJ: Erlbaum.

Melis, E. (2004). Erroneous examples as a source of learning in mathematics erroneous examples. In D. G. Sampson, & P. Isaias (Eds.), *International conference: Cognition and exploratory Learning in the Digital Age* (pp. 311-318). Kinshuk.

Moralı, S., Köroğlu, H. & Çelik, A. (2004). Buca Eğitim Fakültesi matematik öğretmen adaylarının soyut matematik dersine yönelik tutumları ve rastlanan kavram yanılgıları. *Gazi Üniversitesi Gazi Eğitim Fakültesi Dergisi*, *24*(1), 161–175.

Norman, F. A., & Pritchard, M. K. (1994). Cognitive obstacles to the learning of calculus: a krutetskiian perspective. In J. J. Kaput, & E. Dubinsky (Eds.), *Research issues in undergraduate mathematics learning.* Mathematical Association of America.

Orton, A. (1983). Students' understanding of differentiation. *Educational Studies in Mathematics, 5* (15), 235-250.

Osborne, R. J., & Cosgrove, M. M. (1983). Children's conceptions of the change of states of water. *Journal of Research in Science Teaching, 20*, 825-835.

Osborne, R., & Freyberg, P. (1985). *Learning in science: the implication of children's science.* Auckland: Heinemann.

Payne S. J., & Juibb, H. R. (1990). Algebra mal-rules and cognitive account of error. *Cognitive Science, 14*, 445-481.

Senemoğlu, N. (2004). *Gelişim öğrenme ve öğretim kuramdan uygulamaya* (10. baskı). Ankara: Gazi Kitabevi.

Sleeman, D. (1984). An attempt to understand students` understanding of basic algebra. *Cognitive Science, 8*, 367-412.

Sulak, H. & Ardahan, H. (1999). Ondalık kesirlerin öğretimindeki yanılgıların teşhisi ve alınması gereken tedbirler, *Selçuk Üniversitesi Araştırma Vakfı Projesi*, 1996-1997, Proje No: 96/123, Konya.

Tall, D. (1986a). *Building and testing a cognitive approach to the calculus using interactive computer graphics.* Unpublished doctorate dissertation, University of Warwick.

Tall, D. (1986b). *Graphic calculus I, II, III, (3 packs of computer programs, with accompanying texts)*. London: Glentop Publishers.

Treagust, D. F. (1986). Evaluating student's misconceptions by means of diagnostic multiple choice items. *Research in Science Education, 16*, 199-207.

Ubuz, B. (1999). 10. ve 11. sınıf öğrencilerinin temel geometri konularındaki hataları ve kavram yanılgıları. *Hacattepe Üniversitesi Eğitim Fakültesi Dergisi*, 16-17, 95-104

Ubuz, B. (2001). First year engineering students' learning of point of tangency, numerical calculation of gradients, and the approximate value of a function at a point through computers. *Journal of Computers in Mathematics and Science Teaching, 20* (1), 113-137.

Ubuz, B. (2002). Development of calculus concepts through a computer based learning environment. *Proceedings of the 2th International Conference on Teaching of Mathematics* (pp. 1-10), Greece.

Yıldırım, A. & Şimşek, H. (2003). *Sosyal bilimlerde nitel araştırma yöntemleri* (3. baskı). Ankara: Seçkin.

EK-A

1. $f : R \rightarrow R$, $y = f(x) = -4x^2$ fonksiyonu veriliyor. f, fonksiyonunun x =1 noktasındaki türevini, türev tanımından yararlanarak bulunuz.

2. $y < 0$ olmak üzere $x^2 + y^2 = 9$ çemberinin $x = \sqrt{3}$ noktasındaki teğetinin eğimi kaçtır?"

3. Denklemi f(x)=sin(cos5x) olan eğrinin $x = \frac{\pi}{10}$ noktasındaki normalinin eğimi kaçtır?"

4. $f(x)=2x^2 + 3$ olduğuna göre $\lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$ değeri kaçtır?"

5. $f(3x-5) = 2x^2 + x - 1$ olduğuna göre $f'(1)+f(1)$ kaçtır?"

6. y = f (x) fonksiyonu $\frac{1}{x}+\frac{1}{y}=1$ olarak tanımlı olduğuna göre $f'(2)$ değeri kaçtır?"

7. $\frac{d^2}{dx^2}(\sin^2 3x)$ değeri kaçtır?"

EK-B

BİR ARABA KADAR HIZLI KOŞABİLİR MİSİN?

Bil bakalım!

Seoul'de düzenlenen 1998 Olimpiyat Oyunları'nda 100 metre koşusunda, Florence Griffith Joyner, 10 metreyi 0.91 saniyede koşmuştu. Acaba bu hızla saatte 15 mil hızla okuluna giden bir arabayı geçebilir misin?

Laboratuarlardan inşaat alanlarına, mutfaklara, her yerde yapılan ölçümlerin birimleri arasında çevirme yapmak gerekir. Aşçılar, marangozlar, bilim adamları ve mühendislerin hepsi işlerinde kullandıkları ölçümlerin birimlerini birbirine çevirmelilerdir.