Apparent Power Definitions: A Comparison Study

Murat Erhan Balci[1] and Alexander Eigeles Emanuel[2]

Abstract – *This paper examines the concept of apparent power: It advocates the idea that the apparent power quantifies an ideal situation that represents optimum energy flow conditions not only for the energy supplier, but for the consumers as well. Different apparent power definitions were evaluated based on the simulation of a typical power network that supplies linear and nonlinear loads. The effects of different methods of nonactive power compensation on the power factor, unbalance, harmonic distortion, motor power losses and converters voltage ripple are analyzed. It was concluded that the present apparent power definition needs improvement, a mathematical expression that will favor in equal measure the provider and the user of electric energy.*

***Keywords**: Power Definitions, Power Quality, Harmonics, Unbalance, Standards.*

Nomenclature

t, ω : Time and the angular frequency of sinusoidal voltage and current, respectively.

V, α : The rms value and phase angle of the sinusoidal voltage.

I, β : The rms value and phase angle of the sinusoidal current.

θ : The phase angle difference between sinusoidal voltage and current.

ω_1 : The angular frequency of fundamental harmonic voltage and current.

V_h, α_h : The rms value and phase angle of the hth harmonic voltage.

I_h, β_h : The rms value and phase angle of the hth harmonic current.

θ_h: The phase angle difference between the hth harmonic voltage and current.

V_{mh}, I_{mh} : The rms values of the hth harmonic voltage and current measured at phase m=a,b,c.

θ_{mh}: The phase angle difference between the hth harmonic voltage and current measured at phase m=a,b,c.

V_h^+, V_h^-, V_h^0 : The rms values of the hth harmonic positive-, negative- and zero- sequence components of three-phase voltages.

I_h^+, I_h^-, I_h^0 : The rms values of the hth harmonic positive-, negative- and zero- sequence components of three-phase currents.

α_h^+, α_h^-, α_h^0 : Phase angles of the hth harmonic positive-, negative- and zero- sequence components of three- phase voltages.

v_{an}, v_{bn}, v_{cn} : Instantaneous line-to-neutral point voltages.

V_{an}, V_{bn}, V_{cn} : The rms values of the line-to-neutral point voltages.

V_{ab}, V_{bc}, V_{ca} : The rms values of the line-to-line voltages.

I_a, I_b, I_c, I_n : The rms values of the line currents.

v_{a0}, v_{b0}, v_{c0}, v_{n0} : Instantaneous line-to-virtual neutral point voltages.

V_{a0}, V_{b0}, V_{c0}, V_{n0} : The rms values of the line-to-virtual neutral point voltages.

P_1^+, Q_1^+, S_1^+ : Fundamental harmonic positive- sequence active, reactive and apparent powers, respectively.

PF_1^+ : Power factor calculated with P_1^+ and S_1^+ (P_1^+/S_1^+).

I. Background

The evaluation of an electric load that converts electric energy in thermal, mechanical, chemical, electrical, or other forms of energy, is based on a number of measurements, out of which the most significant are:

- Input rms values of voltages and currents: their frequency spectra and unbalance,
- Powers: active, nonactive and apparent *S* (VA),
- Energy conversion efficiency.

Prototype acceptance tests rely on the measurement of such values. Actual operation of equipment is continuously or periodically monitored by measuring some of the above values. Among these *S* is threefold important:

1. The equipment size is a function of apparent power. Engineering Economics sensitivity studies pivot around variables like \$/kVA or kg/kVA
2. Insulation's thermal aging of equipment is correlated to *S*. It is customary to recover the invested capital in function of the monthly maximum kVA demand [1] measured at the customer's mains
3. The power factor, ratio |*P*|/*S,* is an indicator of how effectively are utilized the conductors that supply the power *P*.

The concept of apparent power *S*, was explained and interpreted in many engineering publications, nevertheless it is still a topic that remained controversial, a subject that still puzzles engineers and scientists [2]-[11].

For single-phase sinusoidal conditions the definition, $S = VI$, is universally accepted, moreover, from the careful scrutiny of this basic case, good knowledge can be gained and extrapolated to the disputed three-phase conditions. To prove this claim a linear load, supplied with the rms voltage and current phasors $\underline{V} = V\angle\alpha$ and $\underline{I} = I\angle\beta$, from the voltage source V_S

Manuscript received July 2011, revised November 2011

through the equivalent line impedance $r + jx$, is considered, Fig.1. The load draws the active power

$$P = VI\cos(\theta), \quad \theta = \alpha - \beta \tag{1}$$

and the apparent power

$$S = VI \tag{2}$$

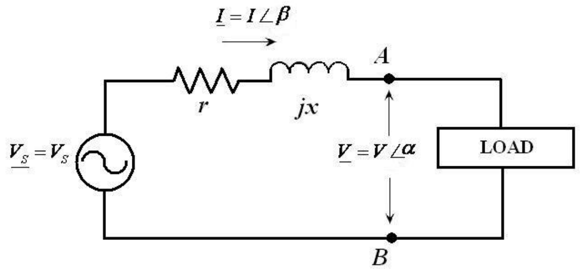

Fig. 1. Single-phase load.

Many researchers [4], [5] have concluded that *S*, the apparent power measured at the terminals A-B, represents the maximum active power that can be accommodated by the supplying equipments (feeder, cable, transformers and generator windings), while the thermal stress on the conductors' insulation remains the same as when the load at the terminals A-B was supplied with the initial phasors voltage $\underline{V}$ and current $\underline{I}$. This interpretation, however, is based on a two constrains:

1. The load topology must be modified to allow unidirectional instantaneous power flow by bringing the phasors $\underline{V}$ and $\underline{I}$ in-phase. The load voltage must remain unchanged, this requirement being tantamount with the condition that the energy conversion process, as produced by the load, remains unchanged, (load output is the same)
2. The supplying line power loss, $\Delta P = rI^2$, remains unchanged; i.e. the rms line current value stays constant, (the thermal stress on conductors' insulation is the same).

The first condition materialized in the use condition realized by means of power factor correction capacitor, or the overexcited synchronous motor that provide a capacitive current that compensates the current component $I\sin(\theta)$. When this is done, the rms line current decreases from I to $I\cos(\theta)$. To maintain the rms current constant an additional hypothetical load, with unity power factor, must be connected in parallel with the original load. In this case, if the voltage source V_S (Fig. 1) remains unchanged, the load voltage increases from

$$V = \left|\underline{V_S} - (r + jx)\underline{I}\right| \tag{3}$$

to

$$V' = \sqrt{V_S^2 - (xI)^2} - rI \tag{4}$$

If no additional load is connected, the line current is reduced to about $I\cos(\theta)$ and the load voltage increases to

$$V'' = \sqrt{V_S^2 - (xI\cos(\theta))^2} - rI\cos(\theta) > V' \tag{5}$$

Evidently, the load voltage must be maintained within a certain tolerance band. Power factor correction at loads supplied by a soft line may lead to excessive load voltage that causes iron core devices saturation, overheating of uncontrolled heaters, interruption of motors, etc. This observation leads to conclusion that the power factor correction is, in theory, a two-step operation: first is the line current adjustment in-phase with the voltage. This is done using shunt capacitors, static filters, switched shunt reactors or active reactors and active filters. This operation should be the consumer responsibility. The second step is the consumer's voltage adjustment within the recommended range. This is the responsibility of the utility that distributes the electric energy.

One learns from these observations that the concept of *S* implies a hypothetical situation – not always possible to materialize – and in spite of the fact that *S* can be measured, *S* is only an indicator of what can be achieved under ideal conditions. This conclusion implies that the ideal condition should cover not only optimum energy transfer to a single isolated load, but should be extended to all the loads supplied by the same feeder. Such a rule has a significant impact on the definition of *S* for nonlinear loads and for three-phase systems.

As above mentioned, the power factor is the immediate consequence of *S* definition and can be interpreted as the ratio between the monitored energy supplied to a load during a certain time and the maximum energy that can be supplied during the same time under the most advantageous conditions for the supplier and the consumer of energy, while maintaining the line losses and the load output histories unchanged during the observation time, $0 \le t \le \tau$.

$$PF = \left[\int_0^\tau P dt\right] / \left[\int_0^\tau S dt\right] \approx \frac{<P>}{<S>} \tag{6}$$

Another *PF* definition is based on the power loss expression:

$$\Delta P = rI^2 = \frac{r}{V^2}S^2 = \frac{r}{V^2}(P^2 + Q^2) \tag{7}$$

thus

$$PF = \frac{P}{S} = \frac{VI\cos(\theta)}{VI} = \sqrt{\frac{r[I\cos(\theta)]^2}{rI^2}} = \sqrt{\frac{\Delta P_C}{\Delta P}} \tag{8}$$

where $\Delta P_C = r\left[I\cos(\theta)\right]^2$ and ΔP are the losses after compensation and losses prior to compensation. This definition has an interesting physical interpretation, Fig.2: According to (7) the utilization of the conductor cross sectional area can be proportionally allocated to P^2 and Q^2, resulting that the utilization of the conductor is P^2/S^2 and not *P/S.*

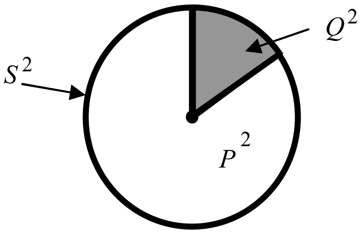

Fig. 2. Geometrical interpretation of *PF*.

In the case of single-phase nonsinusoidal condition, the compensation is implemented by means of tuned, active, or hybrid filters [12]. If the load voltage and current are

$$v = \sqrt{2}\sum_h V_h\, sin(h\omega_1 t + \alpha_h) \tag{9}$$

$$i_L = \sqrt{2}\sum_h I_h\, sin(h\omega_1 t + \beta_h) \tag{10}$$

the compensator current is $i_C=i-i_L$, where i is the line current with a waveform that is a perfect replica of the voltage waveform, i.e.

$$i = \sqrt{2}\, G\sum_h V_h\, sin(h\omega_1 t + \alpha_h) \tag{11}$$

with the conductance $G=P/V^2$; $V^2 = \sum_h V_h^2$ and

$$P = \sum_h V_h I_h cos(\theta_h), \qquad \theta_h = \alpha_h - \beta_h \tag{12}$$

The problem with this approach is that once the line current is modified from i_L to i, the voltage harmonics V_h are changing and the actual conductance does not equal $G=P/V^2$. Moreover, one is entitled to ask the following question: based on the hypothetical nature of S, is not the current compensation to a perfect sinusoidal waveform a better deal? i.e.:

$$i = \sqrt{2}\, G_1 V_1^*\, sin(\omega_1 t + \alpha_1), \ \ G_1 = P/(V_1^*)^2 \tag{13}$$

where V_1^* is the rms value of a fictitious sinusoidal voltage that yields the same load output as the nonsinusoidal voltage v. Such conditions can be achieved if all the loads supplied by the same feeder are compensated to draw sinusoidal currents.

II. Three-Phase System

This section presents brief descriptions of the most common apparent power expressions recommended for three-phase conditions.

II.1. The Vector Apparent Power (Budeanu-Curtis-Silsbee) Definition

This is one of the oldest [13], [14] and probably the most commonly used definition today [15]. The powers are measured individually for each of the three phases a, b and c:

Active powers;

$$P_m = \sum_h V_{mh} I_{mh}\, cos(\theta_{mh}) \tag{14}$$

Reactive powers;

$$Q_m = \sum_h V_{mh} I_{mh}\, sin(\theta_{mh}) \tag{15}$$

Apparent powers;

$$S_m = \sqrt{\left(\sum_h V_{mh}^2\right)\left(\sum_h I_{mh}^2\right)}, \quad m = a,b,c \tag{16}$$

and the calculated Distortion powers;

$$D_m = \sqrt{S_m^2 - P_m^2 - Q_m^2} \tag{17}$$

giving the Vector apparent power;

$$S_V = \sqrt{\left(\sum_{m=a,b,c} P_m\right)^2 + \left(\sum_{m=a,b,c} Q_m\right)^2 + \left(\sum_{m=a,b,c} D_m\right)^2} = \sqrt{P^2 + Q_V^2 + D_V^2} \tag{18}$$

The above expressions of D_m and Q_m were proved imperfect [16], [17], moreover the major flaw of this method consists in the fact that (18) violates the S definition that results from (7) and (8): If for example one assumes a three-wire system with Q_a, $Q_b > 0$, (inductive) and $Q_c<0$ (capacitive), partial cancellation of reactive power is assumed to take place, and if $V_a=V_b=V_c=V$, then

$$\sum_{m=a,b,c} \frac{S_m^2}{V^2} > \frac{S_V^2}{V^2} \tag{19}$$

Since each phase of the load side is treated as an independent load the PF correction is also done for each phase separately. For example if the total nonactive power, including both Q_V and D_V, is compensated, the remaining equivalent load may well be unbalanced, consisting of three resistances that dissipate the powers P_a, P_b and P_c and $S_V = P_a + P_b + P_c$, hence $PF_V = P/S_V = 1$. From (8) results that such an unbalanced system does not have PF=1.

II.2. The DIN Approach: Fryze-Buchholz-Depenbrock Method

The DIN std. 40110 approach relies on collective rms voltage and current defined as follows:

$$V_\Sigma = \sqrt{\frac{1}{4}\left(V_{an}^2 + V_{bn}^2 + V_{cn}^2 + V_{ab}^2 + V_{bc}^2 + V_{ca}^2\right)} \tag{20}$$

$$I_\Sigma = \sqrt{I_a^2 + I_b^2 + I_c^2 + I_n^2} \tag{21}$$

giving the apparent power [9];

$$S_\Sigma = V_\Sigma I_\Sigma = \sqrt{P^2 + Q_{tot\Sigma}^2} \tag{22}$$

The nonactive power has two components:

$$Q_{tot\Sigma} = \sqrt{Q_{tot\Sigma\perp}^2 + Q_{tot\Sigma\|v}^2} \tag{23}$$

where $Q_{tot\Sigma\|v}$ is caused by the load unbalance and $Q_{tot\Sigma\perp}$ is caused by the current component orthogonal with the voltage.

The DIN gives the following expressions for the nonactive power components:

$$Q_{tot\,\Sigma\|v} = V_{\Sigma}\sqrt{\sum_{m=a,b,c,n}\left(G_m - G\right)^2 V_{m0}^2} \qquad (24)$$

$$Q_{tot\,\Sigma\perp} = V_{\Sigma}\sqrt{\sum_{m=a,b,c,n}\left[I_m^2 - G_m^2 V_{m0}^2\right]} \qquad (25)$$

where the conductances $G_m = P_m / V_{m0}^2$, $G = P/V_{\Sigma}^2$ and the voltage V_{m0} is

$$V_{m0} = \sqrt{\sum_h\left(V_h^+\right)^2 + \sum_h\left(V_h^-\right)^2 + \frac{1}{16}\sum_h\left(V_h^0\right)^2},\ m = a,b,c \qquad (26)$$

$$V_{n0} = \sqrt{\frac{9}{16}\sum_h\left(V_h^0\right)^2},\ \ m=n \qquad (27)$$

and V_h^+, V_h^-, V_h^0 are the positive-, negative- and zero-sequence voltage harmonics of order h, (see Appendix).

In this method the compensated load is equivalent to four equal resistances $R_\Sigma = V_\Sigma / I_\Sigma$ connected in star and producing a virtual neutral point 0. Thus, each line current waveforms is a perfect replica of its respective line-to-a virtual neutral voltage. The neutral wire is treated as a fourth phase, consequently after the compensation some imbalance may persist, the neutral current is not nil and the line currents are still distorted.

If the substation voltage is sinusoidal and the DIN compensation is applied to all the loads in the system, then a perfect sinusoidal condition results for all the customers.

II.3. The IEEE Method

According to the IEEE method the compensated system is assumed to have perfect sinusoidal and balanced line currents. It defines the apparent power as $S_e = 3V_e I_e$ using the equivalent rms current and voltage [8], [18], [19];

$$I_e = \sqrt{\frac{1}{3}\left(I_a^2 + I_b^2 + I_c^2 + I_n^2\right)} = \sqrt{I_{e1}^2 + I_{eH}^2} \qquad (28)$$

$$V_e = \sqrt{\frac{1}{18}\left[3\left(V_{an}^2 + V_{bn}^2 + V_{cn}^2\right) + V_{ab}^2 + V_{bc}^2 + V_{ca}^2\right]} = \sqrt{V_{e1}^2 + V_{eH}^2} \qquad (29)$$

where I_{e1} and I_{eH} are fundamental and nonfundamental harmonic components of I_e, V_{e1} and V_{eH} are fundamental and nonfundamental harmonic components of V_e.

The resolution of the effective apparent power is as follows:

$$S_e^2 = P^2 + N^2 = S_{e1}^2 + S_{eN}^2 \qquad (30)$$

where

N is the nonactive power (var),

$$S_{e1} = 3V_{e1} I_{e1} \qquad (31)$$

is the fundamental effective apparent power (VA),

$$S_{eN}^2 = S_e^2 - S_{e1}^2 = D_{eI}^2 + D_{eV}^2 + S_{eH}^2 \qquad (32)$$

is the nonfundamental effective apparent power (VA),

$$D_{eI} = 3V_{e1} I_{eH} \qquad (33)$$

is the current distortion power (var),

$$D_{eV} = 3V_{eH} I_{e1} \qquad (34)$$

is the voltage distortion power (var),

$$S_{eH} = 3V_{eH} I_{eH} = \sqrt{P_H^2 + D_{eH}^2} \qquad (35)$$

is the harmonic apparent power (VA), where P_H is the total harmonic active power (W) and D_{eH} is the harmonic distortion power (var).

This approach emphasizes S_{e1}, the fundamental component whom, in turn, can be resolved in the positive-, negative- and zero-sequence apparent powers. For moderately unbalanced and distorted loads the IEEE and the DIN methods give almost identical results.

II.4. Arithmetic and Geometric Apparent Powers

These seemingly convenient definitions,

$$S_A = \sum_{m=a,b,c} S_m \qquad (36)$$

for the arithmetic [15], and

$$S_G = \sqrt{3\sum_{m=a,b,c} S_m^2} \qquad (37)$$

for the geometric, yield the nonactive powers $Q_A = \sqrt{S_A^2 - P^2}$ and $Q_G = \sqrt{S_G^2 - P^2}$ that totally lack physical or practical meaning.

The IEEE and the DIN methods treat the three or four wire systems as one entity, one energy flow path. In the case of sinusoidal waveforms and perfectly symmetrical conditions all the above methods give the same apparent power, $S = 3V_{\ell n} I_\ell = \sqrt{3} V_{\ell\ell} I_\ell$, (where $V_{\ell n}$ and $V_{\ell\ell}$ are the line-to-neutral and the line-to-line voltages, I_ℓ is the line current.)

III. Application

The system studied is presented in Fig. 3. An ideal 1000 V, 60 Hz, adjustable three-phase source, supplies a radial feeder with the loads clustered in two groups: On the source side, Fig.3(a), are loads meant to cause voltage distortion as well as unbalanced conditions. On the remaining side, Fig. 3(b), are four busses supplying a set of balanced nonlinear loads and unbalanced linear loads, representative of modern equipment:

1. An induction motor rated 440 V, 249.92A, 200HP (148.60 kW), 60 Hz, 1775 rev/min, PF=0.82, and efficiency $\eta = 96.0\%$ (bus MP2),
2. A 200kW, six-pulse rectifier, meant to supply a 2000V dc to an adjustable speed drive simulated by means of a variable resistance (bus MP3),

3. A 56kW, six-pulse controlled rectifier, meant to supply 28.0 A and 2000 V equivalent voltage source (bus MP4),
4. A linear but unbalanced load.

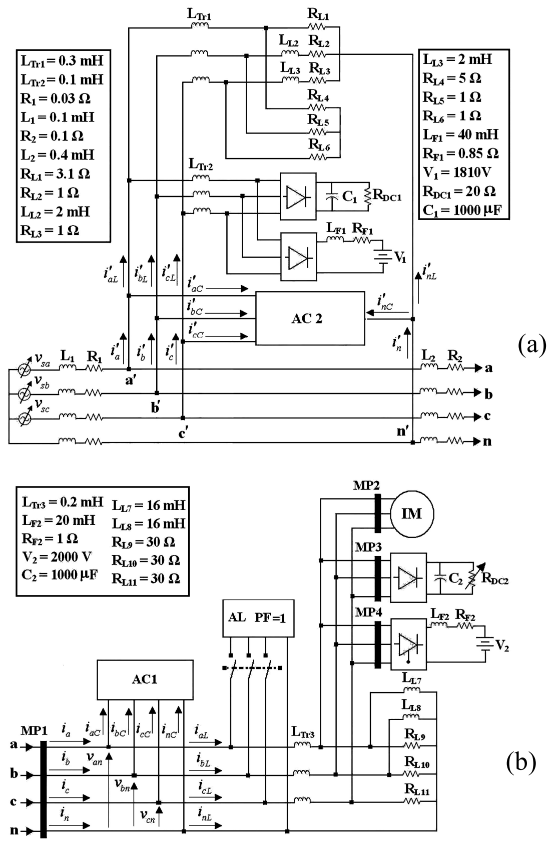

Fig. 3. The simulated system : (a) source side and (b) load side.

These four loads are supplied by a transformer with an equivalent short-circuit inductance L_{Tr3} = 0.2 mH/phase. The system is equipped with two active compensators, AC1 at the terminals *a, b, c, n,* (Fig. 3b) and AC2 at the terminals *a', b', c', n'* (Fig. 3a). The compensators are assumed ideal, i.e. able to adjust the compensator currents spectra to any desired sequence.

The results are divided in five groups:

1. **BC** (before the compensation): The apparent powers and their components are computed for the system as it is. This is the most important information since it represents the actual measurement and allows for comparing different definitions.
2. **AC1** (after the compensator AC1 activation): The compensator performs differently for different definitions of *S.* Thus, one can observe and compare the actual effects of a certain *S* definition.
3. **AL** (additional load is connected): To keep the line rms current unchanged a unity power factor load is connected at the terminals *a, b, c, n*. This is a pure hypothetical step.
4. **AC2** (after the compensator AC2 activation): This hypothetical step assumes that all the loads are compensated. This step may precede step 3, or may be implemented at the same time as step 3.
5. **AV** (adjusted voltage): The supply voltages are adjusted to maintain the MP1 bus voltages equal to their original values, i.e. same V_e for the IEEE, same V_Σ for DIN and same line-to-neutral rms voltages (V_{an}, V_{bn}, V_{cn}) for the Vector approach.

During all these steps the output power of each load was maintained nearly constant. Measurements were taken at the four busses labeled MP1, MP2, MP3 and MP4 (Fig.3b). All the reported voltages and currents are normalized rms values. The base values (unless differently stated) are: $V_e = 891.38$ *V*, $I_e = 261.13$ *A*, $S_e = 698.32$ kVA all calculated according to IEEE Std. 1459-2010. The voltage rms values and spectra measured at MP1 are summarized in Table I. The total harmonic voltage distortions are around 6%. The voltage unbalance is characterized by the ratios $V_1^-/V_1^+ = 0.0098$ and $V_1^0/V_1^+ = 0.0527$. The presence of 3rd and 9th harmonics among the line-to-line voltages indicates a significant non zero-sequence component among triplen harmonics.

Table II describes the line currents. Since the rectifiers and the motor are three-wire loads, that command about 84% of the total power, the neutral current is due only to the unbalanced linear load and has a small, 1.96%, total harmonic distortion. The fundamental harmonic negative- and zero-sequence currents measured at MP1 have significant values: $I_1^-/I_1^+ = 0.2354$ and $I_1^0/I_1^+ = 0.1901$.

Table III displays the total harmonic voltage and current distortions as well as unbalance indices measured at the MP1 bus after compensations. After the compensation AC1, the Vector and the DIN definitions of *S* yield line currents with 2.3% distortion and more than 2% negative-sequence current. The IEEE approach holds the promise for perfectly sinusoidal and balanced currents, but the voltage distortion is about the same as the values predicted from DIN and Vector. If all five steps are implemented, all three methods yield perfect sinusoidal waveforms, but the Vector method yields at MP1 nearly 5% zero-sequence voltage and 4% zero-sequence current. This result points to the deficiency of S_V definition. (The perfect sinusoidal and balanced situations are printed on a gray background).

Tables IV, V and VI summarize the apparent powers and their components' values. For BC case evidently the active powers are identical but the nonactive powers differ: $N = \sqrt{S_e^2 - P^2} = 72.71\%$, $Q_{tot\Sigma} = 72.65\%$ and $\sqrt{Q_V^2 + D_V^2} = 59.33\%$. As a result, one observes that $S_V < S_\Sigma < S_e$; nevertheless, the difference between S_Σ and S_e is only 0.04%, while the S_V is 9.26% lower than S_e. After all five steps are performed the difference

TABLE I
BC: NORMALIZED VOLTAGES MEASURED AT POINT MP1.

	V_{an}	V_{bn}	V_{cn}	V_{ab}	V_{bc}	V_{ca}
RMS	1.0433	0.9460	1.0102	1.7440	1.7150	1.7332
h=1	1.0417∠-0.76°	0.9442∠-123.16°	1.0081∠114.35°	1.7408∠26.48°	1.7117∠-93.37°	1.7301∠147.39°
h=3	0.0085∠100.95°	0.0054∠-42.62°	0.0043∠-104.52°	0.0137∠120.59°	0.0051∠5.84°	0.0124∠-81.55°
h=5	0.0486∠-8.54°	0.0477∠118.30°	0.0537∠-123.49°	0.0862∠-34.87°	0.0871∠85.38°	0.0863∠-154.17°
h=7	0.0211∠108.32°	0.0227∠-18.34°	0.0219∠-137.74°	0.0392∠136.04°	0.0386∠11.31°	0.0361∠-105.36°
h=9	0.0029∠-130.05°	0.0014∠37.02°	0.0016∠70.56°	0.0044∠-134.38°	0.0009∠-42.93°	0.0044∠57.17°
h=11	0.0158∠116.84°	0.0161∠-116.96°	0.0206∠-1.57°	0.0285∠89.60°	0.0312∠-153.65°	0.0314∠-27.84°
THD (%)	5.64	6.24	6.47	5.59	5.72	5.65

TABLE II
BC: NORMALIZED CURRENTS MEASURED AT POINT MP1.

	I_a	I_b	I_c	I_n
RMS	1.0846	1.0182	0.7210	0.5165
h=1	1.0775∠-49.72°	1.0105∠-171.73°	0.7096∠101.06°	0.5164∠31.64°
h=3	0.0272∠-141.07°	0.0303∠71.90°	0.0175∠-43.49°	0.0010∠160.90°
h=5	0.1041∠98.39°	0.1035∠-141.48°	0.1096∠-23.13°	0.0064∠135.27°
h=7	0.0538∠-148.42°	0.0558∠90.13°	0.0558∠-31.84°	0.0022∠128.97°
h=9	0.0056∠-51.18°	0.0043∠163.67°	0.0033∠81.12°	0.0001∠-43.58°
h=11	0.0162∠-122.39°	0.0179∠-12.19°	0.0201∠114.33°	0.0010∠-125.02°
THD (%)	11.45	12.36	18.00	1.96

TABLE III
THE UNBALANCE OF VOLTAGE AND CURRENT AND DISTORTION INDICES AT MP1.

		THD (%)						V_1^- / V_1^+	V_1^0 / V_1^+	I_1^- / I_1^+	I_1^0 / I_1^+
		V_{an}	V_{bn}	V_{cn}	I_a	I_b	I_c				
VECTOR	AC1	2.18	2.32	2.55	2.18	2.32	2.55	0.0220	0.0303	0.0381	0.0461
	AL	2.10	2.25	2.42	2.10	2.25	2.42	0.0219	0.0292	0.0322	0.0390
	AC2	0.00	0.00	0.00	0.00	0.00	0.00	0.0143	0.0436	0.0178	0.0314
	AV	0.00	0.00	0.00	0.00	0.00	0.00	0.0090	0.0493	0.0018	0.0411
DIN	AC1	2.25	2.28	2.54	2.28	2.35	2.40	0.0228	0.0331	0.0228	0.0082
	AL	2.16	2.20	2.42	2.18	2.27	2.29	0.0227	0.0330	0.0227	0.0082
	AC2	0.00	0.00	0.00	0.00	0.00	0.00	0.0000	0.0000	0.0000	0.0000
	AV	0.00	0.00	0.00	0.00	0.00	0.00	0.0000	0.0000	0.0000	0.0000
IEEE	AC1	2.42	2.50	2.78	0.00	0.00	0.00	0.0234	0.0339	0.0000	0.0000
	AL	2.44	2.49	2.77	0.00	0.00	0.00	0.0236	0.0342	0.0000	0.0000
	AC2	0.00	0.00	0.00	0.00	0.00	0.00	0.0000	0.0000	0.0000	0.0000
	AV	0.00	0.00	0.00	0.00	0.00	0.00	0.0000	0.0000	0.0000	0.0000

between S_V and S_e reduces to 0.20%. On the other hand, the minor differences about 0.04% between the DIN and the IEEE results can be traced to the definitions of V_Σ and V_e. These voltages respond differently to the presence of zero-sequence voltage [10].

In addition to above mentioned results, once the AC1 is activated the nonactive powers of S_Σ and S_V become nil. Not so for the IEEE method. Since the voltage at MP1 is still nonsinusoidal (see Table III), the voltage distortion power D_{eV}=1.68%. Once AC2 is activated D_{eV}=0.

It was found that following AC1 the active powers increase. The reason for this result is tied to the incremental change in voltage and to wave form distortion reduction. Rectifier loads are sensitive to peak input voltage as well as to negative-sequence voltage. A slight increase in peak voltage may lead to a drastic increase in the dc current.

Table VII summarizes the values of apparent powers and power factors for the Arithmetic and Geometric definitions in BC case and other four cases of the Vector approach, which threats each phase individually.

Manuscript received July 2011, revised November 2011

TABLE IV
VECTOR APPARENT POWER QUANTITIES MEASURED AT MP1.

	S_V	P	Q_V	D_V	$PF_V = P/S_V$	$PF_{IEEE} = P/S_e$
BC	90.74	68.65	56.86	16.96	0.7566	0.6865
AC1	70.57	70.57	0.00	0.00	1.0000	0.9967
AL	101.35	101.35	0.00	0.00	1.0000	0.9977
AC2	102.88	102.88	0.00	0.00	1.0000	0.9988
AV	99.80	99.80	0.00	0.00	1.0000	0.9979

TABLE V
DIN QUANTITIES MEASURED AT MP1.

	S_Σ	P	$Q_{tot\Sigma}$	$Q_{tot\Sigma\|v}$	$Q_{tot\Sigma\perp}$	$PF_\Sigma = P/S_\Sigma$	PF_{IEEE}
BC	99.96	68.65	72.65	30.34	66.02	0.6868	0.6865
AC1	70.62	70.62	0.00	0.00	0.00	1.0000	0.9998
AL	101.59	101.59	0.00	0.00	0.00	1.0000	0.9998
AC2	102.95	102.95	0.00	0.00	0.00	1.0000	1.0000
AV	99.96	99.96	0.00	0.00	0.00	1.0000	1.0000

TABLE VI
IEEE QUANTITIES MEASURED AT MP1.

	APPARENT POWERS					ACTIVE POWERS			NONACTIVE POWERS					
	S_e	S_{e1}	S_1^+	S_{eN}	S_{eH}	P	P_1^+	P_H	Q_1^+	D_{eV}	D_{eI}	D_{eH}	PF_{IEEE}	PF_1^+
BC	100.00	99.02	90.33	13.96	0.76	68.65	69.57	-0.16	57.61	6.14	12.52	0.74	0.6865	0.7701
AC1	70.90	70.88	70.84	1.68	0.00	70.84	70.84	0.00	0.00	1.68	0.00	0.00	0.9991	1.0000
AL	101.59	101.55	101.50	2.85	0.00	101.50	101.50	0.00	0.00	2.85	0.00	0.00	0.9991	1.0000
AC2	102.95	102.95	102.95	0.00	0.00	102.95	102.95	0.00	0.00	0.00	0.00	0.00	1.0000	1.0000
AV	100.00	100.00	100.00	0.00	0.00	100.00	100.00	0.00	0.00	0.00	0.00	0.00	1.0000	1.0000

TABLE VII
ARITHMETIC AND GEOMETRIC APPARENT POWERS AND POWER FACTORS
MEASURED AT MP1

	S_A	$PF_A = P/S_A$	S_G	$PF_G = P/S_G$
BC	94.11	0.7295	95.55	0.7185
AC1	70.57	1.0000	70.90	0.9953
AL	101.35	1.0000	101.73	0.9963
AC2	102.88	1.0000	103.02	0.9986
AV	99.80	1.0000	100.03	0.9977

TABLE VIII
THE THDS AND V_1^- / V_1^+ MEASURED AT MP2-MP3-MP4.

		THD (%)			V_1^- / V_1^+
		V_{an}	V_{bn}	V_{cn}	
	BC	7.32	8.15	8.18	0.0069
VECTOR	AC1	5.22	5.93	5.00	0.0194
	AV	4.35	4.58	4.00	0.0078
DIN	AC1	4.91	5.41	4.67	0.0195
	AV	4.46	4.21	4.24	0.0041
IEEE	AC1	5.09	5.98	5.51	0.0200
	AV	4.62	4.42	4.40	0.0041

It is seen in BC case that for the apparent power and power factor values both definitions are much closer to the DIN and IEEE definitions than Vector definition. In addition to that, Arithmetic approach gives unity power factor like Vector approach in AV case.

On the secondary side, at the point of common coupling, this includes MP2, MP3 and MP4 (Table VIII), the voltage distortions are higher than at MP1. Compensation at MP1 helps reduce voltage distortions from about 8% to less than 5%. For DIN and IEEE the voltage unbalance improves from 0.69% to 0.41%, but for the Vector method the unbalance increases to 0.78%. This condition, acceptable in this case, may escalate to situations unfavorable for induction motors and rectifiers.

Manuscript received July 2011, revised November 2011

If $V^{-}/V^{+} \geq 0.01$, induction motors need be derated. If negative-sequence or voltage distortion is present rectifiers will inject atypical harmonics.

The voltage measurements at the induction motor terminals are presented in Table VIII. The mechanical load was maintained nearly constant, the load torque $T \approx 720$ Nm. The main results are the normalized total motor power losses, $\Delta P / \Delta P_{Rated}$ (Table IX). After all compensations are performed, the normalized losses are reduced from 1.0358 pu to about 0.988 pu, i.e. by more than 4.5% when the IEEE and the DIN methods are used, followed by 3.3% for the Vector approach.

TABLE IX
THE ACTIVE AND APPARENT POWERS, POWER FACTOR AND NORMALIZED POWER LOSSES OF INDUCTION MACHINE CONNECTED TO MP2.

		S	P	PF	$\Delta P / \Delta P_{Rated}$
VECTOR	**BC**	24.10	19.81	0.8220	1.0358
DIN		24.20	19.81	0.8186	
IEEE		24.20	19.81	0.8186	
VECTOR	**AC1**	25.38	21.08	0.8306	1.1685
	AL	24.77	20.59	0.8312	1.1401
	AC2	25.26	21.12	0.8361	1.0778
	AV	23.78	19.88	0.8360	1.0014
DIN	**AC1**	26.24	21.07	0.8030	1.1778
	AL	25.61	20.58	0.8036	1.1489
	AC2	25.26	21.09	0.8349	1.0483
	AV	23.80	19.89	0.8357	0.9881
IEEE	**AC1**	26.29	21.07	0.8014	1.1857
	AL	25.69	20.57	0.8007	1.1607
	AC2	25.26	21.09	0.8349	1.0483
	AV	23.82	19.90	0.8347	0.9889

The rectifiers' performances are described in Tables X and XI. The V_{dc} is normalized to the rated voltage (2000 V). The quantity of main concern is the voltage ripple $\Delta V / V_{dc}$, across the capacitance C_2 (Table X), and the current ripple $\Delta I / I_{dc}$ through L_{F2} (Table XI). For the capacitor filter rectifier the voltage ripple increases from 4.75% to more than 10% for AC1, and more than 6% after the implementation of all five steps. The explanation is as follows: The ideal input voltage that leads to minimum ripple is a square wave voltage. For the same rms value a perfect sinusoidal voltage will cause more ripple than a trapezoidal voltage. Voltage oscillograms recorded at MP3 revealed nearly trapezoidal waveforms. After compensation the voltage crest factor increases, hence the ripple will also increase. This observation is reflected in Table XI too, where in spite of the fact that the power is maintained constant by adjusting the firing angle, the current ripple increases from 16.52% to 28.2% for DIN and IEEE methods and to 32.08% for the Vector approach. Since the dc current supplied by the L-filtered rectifier is $I_{dc} = (V_{dc} - 2000) / R_{F2}$ a slight increase in V_{dc} leads to a large increase in the output power. On a

TABLE X
THE ACTIVE AND APPARENT POWERS, POWER FACTOR, DC OUTPUT VOLTAGE AND DC OUTPUT VOLTAGE RIPPLE OF SIX-PULSE RECTIFIER CONNECTED TO MP3.

		S	P	PF	V_{dc} (%)	$\Delta V/V_{dc}$ (%)
VECTOR	**BC**	34.48	29.46	0.8544	101.31	4.75
DIN		34.48	29.46	0.8544		
IEEE		34.48	29.46	0.8544		
VECTOR	**AC1**	43.08	29.46	0.6838	106.61	10.33
	AL	42.88		0.6870	105.36	10.64
	AC2	42.54		0.6925	108.00	8.61
	AV	41.80		0.7048	104.81	7.85
DIN	**AC1**	43.19	29.46	0.6821	106.61	10.55
	AL	42.97		0.6856	105.36	10.71
	AC2	42.12		0.6994	107.96	6.50
	AV	41.57		0.7087	104.81	6.77
IEEE	**AC1**	43.05	29.46	0.6843	106.60	10.69
	AL	42.62		0.6912	105.40	10.71
	AC2	42.12		0.6994	107.96	6.50
	AV	41.59		0.7083	104.86	6.65

controlled rectifier this situation is corrected by adjusting the firing angles. For an uncontrolled rectifier supplying an inverter (typically used for adjustable speed drives) the inverter is controlled to supply the load power, however, following compensation the input dc voltage may be excessively high.

TABLE XI
THE ACTIVE AND APPARENT POWERS, POWER FACTOR AND DC OUTPUT CURRENT RIPPLE OF SIX-PULSE RECTIFIER CONNECTED TO MP4.

		S	P	PF	$\Delta I/I_{dc}$ (%)
VECTOR	**BC**	8.68	8.19	0.9435	16.52
DIN		8.68	8.19	0.9435	
IEEE		8.68	8.19	0.9435	
VECTOR	**AC1**	9.03	8.19	0.9070	55.70
	AL	8.89		0.9213	47.96
	AC2	9.04		0.9060	51.75
	AV	8.71		0.9403	32.08
DIN	**AC1**	9.04	8.19	0.9060	57.35
	AL	8.90		0.9202	47.53
	AC2	9.04		0.9060	48.51
	AV	8.72		0.9392	28.20
IEEE	**AC1**	9.04	8.19	0.9060	58.02
	AL	8.89		0.9213	48.14
	AC2	9.04		0.9060	48.51
	AV	8.71		0.9403	28.29

IV. Conclusion

It was learned from this study that the DIN and the IEEE approaches give very close results before and after the compensation, nevertheless some basic differences between these approaches are obvious: The IEEE requires a compensation that yields sinusoidal positive-sequence currents, but the voltage may remain slightly distorted. The compensation based on DIN leads to line currents that are still slightly distorted and unbalanced. If all the nonlinear loads in the system are compensated the

Manuscript received July 2011, revised November 2011

result will be, for both DIN and IEEE approaches, an ideal situation, a power system with positive-sequence currents and voltages at fundamental frequency only.

The Vector apparent power gives different results than the DIN and the IEEE. Even if all the nonlinear loads are compensated, the zero- and negative- sequence components will continue to be present and may cause undesirable effects. The authors recommend that the vector definition should be avoided.

The provider of electric energy must be responsible for the voltage quality and the consumer should be accountable for the current distortion and load balancing. This practical approach should be reflected in the definition of $S = 3V_{\ell n} I_\ell$: The apparent power definition should cover the optimal conditions for the transmission and distribution of energy as well as for the conversion of energy at the loads; it must be based on provisions for the most economical conditions for both user and supplier of electricity. The effective rms current squared, I_e^2, must quantify the line losses and the ageing rate of equipment. (There are conditions when skin and proximity effects affect the line losses. In such cases the current I_e must be corrected).The voltage V_e should take the value that leads to the best overall operating conditions of the monitored loads. In the past, when the incandescent light was the dominant load, the interpretation of load voltage [4], [5], [7] was based on the thermoelectric effect. In spite of the proliferation of motors and rectifiers, the modern S expressions (given in IEEE std. 1459 and DIN std. 40110) are still based on the thermoelectric effect. The measurements obtained at rectifiers and motors input terminals, after different stages of power factor correction, indicate that the actual apparent power definitions can be challenged and eventually improved. The actual definitions do not represent the best possible conditions from the customer perspective. The voltage V must be a voltage that satisfies the conditions for optimal energy flow as well as good energy conversion efficiency at loads. It is in the spirit of power systems design to expect such a three-phase voltage to be sinusoidal and symmetrical, i.e. positive-sequence only [20]. The amplitude value of such voltage is a matter of debate especially when the monitored point of common coupling serves a variety of loads. One obvious possibility is to opt for the value that minimizes power loss in the system, while operating each load within the constraint of an acceptable voltage range. Such conditions are hard to implement at the present time unless the considered system consists of a few customers. Nevertheless, as the acceptance of active filters will increase, the apparent power fictitiousness (especially for three-phase systems) will be replaced by the actual implementation of ideally compensated loads, with perfectly sinusoidal line currents and voltages.

Appendix

The instantaneous line-to-virtual neutral point voltage is

$$v_{m0} = v_{mn} - \frac{1}{4}\left(v_{an} + v_{bn} + v_{cn}\right); \quad m = a,b,c\ ,n \qquad \text{(A.1)}$$

The line-to-neutral voltages v_{an} , v_{bn} and v_{cn} can be expressed as;

$$\begin{aligned} v_{mn} &= \sum_h \sqrt{2}V_h^+ sin\left(h\omega_1 t + \alpha_h^+ - k\frac{2\pi}{3}\right) + \\ &+ \sum_h \sqrt{2}V_h^- sin\left(h\omega_1 t + \alpha_h^- + k\frac{2\pi}{3}\right) + \sum_h \sqrt{2}V_h^0 sin\left(h\omega_1 t + \alpha_h^0\right) \end{aligned} \qquad \text{(A.2)}$$

where k is equal to 0, 1 and 2, for $m=a,\ b,\ c$, respectively. Thus:

$$v_{an} + v_{bn} + v_{cn} = 3\sum_h \sqrt{2}V_h^0 sin\left(h\omega_1 t + \alpha_h^0\right) \qquad \text{(A.3)}$$

Substituting (A.2) and (A.3) in (A.1) for $m=a,\ b,\ c$, results:

$$\begin{aligned} v_{m0} &= \sum_h \sqrt{2}V_h^+ sin\left(h\omega_1 t + \alpha_h^+ - k\frac{2\pi}{3}\right) + \\ &+ \sum_h \sqrt{2}V_h^- sin\left(h\omega_1 t + \alpha_h^- + k\frac{2\pi}{3}\right) + \frac{1}{4}\sum_h \sqrt{2}V_h^0 sin\left(h\omega_1 t + \alpha_h^0\right) \end{aligned} \qquad \text{(A.4)}$$

and for $m=n$

$$\begin{aligned} v_{n0} &= v_{nn} - \frac{3}{4}\sum_h \sqrt{2}V_h^0 sin\left(h\omega_1 t + \alpha_h^0\right) \\ &= -\frac{3}{4}\sum_h \sqrt{2}V_h^0 sin\left(h\omega_1 t + \alpha_h^0\right) \end{aligned} \qquad \text{(A.5)}$$

The positive-, negative- and zero- sequence components are orthogonal to each other. As a result; the rms value of line-to-virtual neutral point voltage, V_{mo} , can be expressed as given in (26) and (27).

Acknowledgements

During this work, Dr. Murat Erhan Balci is financially supported by The Scientific and Technological Research Council of Turkey (TUBITAK).

References

[1] *Handbook for Electricity Metering* (Edison Electric Institute, EEI Publication No. 06-84-56).

[2] P. M. Lincoln, Polyphase Power Factor, *AIEE Trans., vol. 39 n. 2,* July 1920, p.p. 1477-1479.

[3] C. L. Fortescue, Polyphase Power Representation by Means of Symmetrical Coordinates, *AIEE Trans., vol. 39 n.2*, July 1920, p.p. 1481-1484.

[4] Special AIEE Joint Committee, Power Factor in Polyphase Circuits, *AIEE Trans., vol. 39*, 1920, p.p. 1449-1520.

[5] C. I. Budeanu, *Reactive and Fictive Powers* (*National Romanian Institute for the Management and Utilization of Energy Sources* No.2, 1927).

[6] A. E. Knowlton, Reactive Power Concepts in Need of Clarification, *AIEE Trans., vol. 52 n. 3*, September 1933, p.p. 744-747.

[7] M. Depenbrock, The FBD Method, A Generally Applicable Tool for Analyzing Power Relations, *IEEE Trans. on Power Systems*, *vol. 8 n.2*, May 1993, p.p. 381-387.

[8] IEEE Std. 1459 – 2010, *IEEE Standard Definitions for the Measurement of Electric Power Quantities under Nonsinusoidal, Balanced, or Unbalanced Conditions* (IEEE standards, 2010).
[9] DIN std. 40110, *Part 1: Single-Phase Circuits, Part 2: Polyphase Circuits* (DIN standards, Part 1: 1994 and Part 2: 2002) (in German).
[10] J. L. Willems, J. Ghijselen and A. E. Emanuel, The Apparent Power Concept and the IEEE Standard 1459-2000, *IEEE Trans. on Power Delivery*, *vol. 20 n. 2*, April 2005, p.p. 876-886.
[11] A. E. Emanuel, Reflections on the Effective Voltage Concept, *L'Energia Electrica (Research), vol. 81*, 2004, p.p. 30-36.
[12] H. Akagi, E. H. Watanabe and M. Aredes, *Instantaneous Power Theory and Applications to Power Conditioning* (IEEE Press, John Wiley & Sons, 2007).
[13] H. L. Curtis and F. B. Silsbee, Definitions of Power and Related Quantities, *AIEE Trans.*, *vol. 54 n. 4*, April 1935, p.p. 394-404.
[14] *American Standard Definitions of Electrical Terms* (American Institute of Electrical Engineers, 1941).
[15] IEEE Standard 100, *The Authoritative Dictionary of IEEE Standards Terms* (IEEE Press, 2000).
[16] V. Lyon, (Discussion to [13]), *AIEE Trans., vol. 54 n. 10*, 1935, p.p. 1121.
[17] L. S. Czarnecki, What is Wrong with Budeanu Concept of Reactive and Distortion Power and Why It Should be Abandoned, *IEEE Trans. on Instrum*entation *and Measurement*, *vol. IM36 n. 3*, September 1987, p.p. 834-837.
[18] A. E. Emanuel, On the Assessment of Harmonic Pollution, *IEEE Trans. on Power Deliv*ery, *vol. 10 n. 3*, July 1995, p.p. 1693-1698.
[19] A. E. Emanuel, *Power Definitions and the Physical Mechanism of Power Flow,* (IEEE Press, John Wiley & Sons, 2010).
[20] A. J. Berrisford, Should a utility meter harmonics?, Proceedings of the IEE 7th Intnl. Conf. on Metering Apparatus and Tariffs for Electricity Supply, 1992, Glascow, U.K.

Authors' information

[1]Balikesir University, Department of Electrical and Electronics Engineering, Balikesir, Turkey.
[2]Worcester Polytechnic Institute, Department of Electrical Engineering, Worcester, MA, USA.

Murat Erhan Balci
was born in Istanbul, Turkey. In 2001, 2004 and 2009, respectively, he received the B.Sc. degree from Kocaeli University, M.Sc. and D.Sc. degrees from Gebze Institute of Technology, Turkey. Since 2009, he has been with the Electrical and Electronics Engineering Department of Balikesir University, Turkey as Assistant Professor. During 2008 he was a visiting scholar at Worcester Polytechnic Institute.

Alexander Eigeles Emanuel
was born in Bucharest, Rumenia. In 1963, 65 and 69, respectively, he earned B.Sc., M.Sc. and D.Sc. degrees, all from the Technion, Israel Institute of Technology. In 1969 he started working for High Voltage Power Engineering, designing shunt reactances and SF6 insulated cables. In 1974 he joined Worcester Polytechnic Institute where he teaches and conducts research.