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On ϕ-quasiconformally symmetric Sasakian manifolds

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ABSTRACT

We study locally and globally ϕ-quasiconformally symmetric Sasakian manifolds. We show that a globally ϕ-quasiconformally symmetric Sasakian manifold is globally ϕ-symmetric. Some observations for a 3-dimensional locally ϕ-symmetric Sasakian manifold are given. We also give an example of a 3-dimensional locally ϕ-quasiconformally symmetric Sasakian manifold.

1. INTRODUCTION

Let (M, g), $n \geqslant 3$, be a Riemannian manifold. The notion of the quasi-conformal curvature tensor was introduced by Yano and Sawaki [10]. According to them a *quasiconformal curvature tensor* is defined by

$$(1.1)\qquad \begin{aligned} C^*(X, Y)Z &= aR(X, Y)Z \\ &\quad + b[S(Y, Z)X - S(X, Z)Y + g(Y, Z)QX - g(X, Z)QY] \\ &\quad - \frac{r}{n}\left[\frac{a}{n-1} + 2b\right][g(Y, Z)X - g(X, Z)Y], \end{aligned}$$

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where a and b are constants, S is the Ricci tensor, Q is the Ricci operator defined by $S(X,Y)=g(QX,Y)$ and r is the scalar curvature of the manifold M^n. If $a=1$ and $b=-\frac{1}{n-2}$, then (1.1) takes the form

$$\begin{aligned}C^*(X,Y)Z &= R(X,Y)Z-\frac{1}{n-2}\\ &\quad\times[S(Y,Z)X-S(X,Z)Y+g(Y,Z)QX-g(X,Z)QY]\\ &\quad+\frac{r}{(n-1)(n-2)}[g(Y,Z)X-g(X,Z)Y]\\ &=C(X,Y)Z,\end{aligned}$$

where C is the conformal curvature tensor [9]. In [3], De and Matsuyama studied a quasiconformally flat Riemannian manifold satisfying a certain condition on the Ricci tensor. From Theorem 5 of [3], it can be proved that a 4-dimensional quasi-conformally flat semi-Riemannian manifold is the Robertson–Walker space–time. Robertson–Walker space–time is the warped product $I\times_f M^*$, where M^* is a space of constant curvature and I is an open interval [6]. From (1.1), we obtain

$$\begin{aligned}(1.2)\qquad (\nabla_W C^*)(X,Y)Z &= a(\nabla_W R)(X,Y)Z\\ &\quad+b[(\nabla_W S)(Y,Z)X-(\nabla_W S)(X,Z)Y\\ &\qquad+g(Y,Z)(\nabla_W Q)(X)-g(X,Z)(\nabla_W Q)(Y)]\\ &\quad-\frac{dr(W)}{n}\left[\frac{a}{n-1}+2b\right][g(Y,Z)X-g(X,Z)Y].\end{aligned}$$

If the condition

$$\nabla R=0$$

holds on M, then M is called *locally symmetric*, where ∇ denotes the Levi–Civita connection on M. It is known that for a locally symmetric Sasakian manifold, the manifold is a space of constant curvature [5]. This fact means that a locally symmetric space condition is too strong for a Sasakian manifold. In [7], Takahashi introduced a weaker condition for a Sasakian manifold that satisfies the condition

$$(1.3)\qquad \phi^2\big((\nabla_X R)(Y,Z,W)\big)=0,$$

where X,Y,Z and W are horizontal vector fields which means that it is horizontal with respect to the connection form η of the local fibering; namely, a horizontal vector is nothing but a vector which is orthogonal to ξ. A Sasakian locally ϕ-symmetric space is an analogous notion of Hermitian symmetric space [7]. In [7], it was shown that a Sasakian manifold is a locally ϕ-symmetric space if and only if each Kaehlerian manifold, which is a base space of a local fibering, is a Hermitian locally symmetric space. Later in [2], Blair, Koufogiorgos and Sharma studied locally ϕ-symmetric contact metric manifolds.

In (1.3), if X, Y, Z and W are not horizontal vectors then we call the manifold *globally ϕ-symmetric.*

In this paper, we define locally ϕ-quasiconformally symmetric and globally ϕ-quasiconformally symmetric contact metric manifolds. A contact metric manifold (M, g) is called *locally ϕ-quasiconformally symmetric* if the condition

$$(1.4) \qquad \phi^2\big((\nabla_X C^*)(Y, Z, W)\big) = 0$$

holds on M, where X, Y, Z and W are horizontal vectors. If X, Y, Z and W are arbitrary vectors then the manifold is called *globally ϕ-quasiconformally symmetric.*

In the present paper, we study locally and globally ϕ-quasiconformally symmetric Sasakian manifolds. Some observations for 3-dimensional locally ϕ-quasiconformally symmetric Sasakian manifolds are also given.

The paper is organized as follows. In Section 2, we give a brief account of Sasakian manifolds. In Section 3, we study globally ϕ-quasiconformally symmetric Sasakian manifolds. We prove that if a Sasakian manifold is globally ϕ-quasiconformally symmetric, then the manifold is an Einstein manifold. We also show that a globally ϕ-quasiconformally symmetric Sasakian manifold is globally ϕ-symmetric. In Section 4, we study 3-dimensional locally ϕ-quasiconformally symmetric Sasakian manifolds. We prove that a 3-dimensional Sasakian manifold is locally ϕ-quasiconformally symmetric if and only if it is locally ϕ-symmetric. We also give an example of a 3-dimensional locally ϕ-quasiconformally symmetric Sasakian manifold.

2. SASAKIAN MANIFOLDS

Let (M^n, g), $n = 2m + 1$, be a contact Riemannian manifold with contact form η, the associated vector field ξ, $(1, 1)$-tensor field ϕ and the associated Riemannian metric g. If ξ is a Killing vector field then M^n is called a K-contact Riemannian manifold [1]. If in such a manifold the relation

$$(2.1) \qquad (\nabla_X \phi)Y = g(X, Y)\xi - \eta(Y)X$$

holds, where ∇ denotes the Levi–Civita connection of g, then M^n is called a *Sasakian manifold.*

Let R, Q, r denote the curvature tensor of type $(1, 3)$, Ricci operator and scalar curvature of M^n, respectively. It is known that in a contact manifold M^n the Riemannian metric may be so chosen that the following relations hold [1,9]:

$$(2.2) \qquad (a) \quad \phi\xi = 0, \qquad (b) \quad \eta(\xi) = 1, \qquad (c) \quad \eta \circ \phi = 0;$$

$$(2.3) \qquad \phi^2 X = -X + \eta(X)\xi,$$

$$(2.4) \qquad g(X, \xi) = \eta(X),$$

$$(2.5) \qquad g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y)$$

for any vector fields X, Y. If M^n is a Sasakian manifold, then besides (2.2)–(2.5), the following relations hold [1,9]:

$$(2.6)\qquad \nabla_X\xi = -\phi X, \qquad (\nabla_X\eta)Y = g(X,\phi Y),$$

$$(2.7)\qquad \Phi(X,Y) = (\nabla_X\eta)Y,$$

$$(2.8)\qquad \Phi(X,Y) = -\Phi(Y,X),$$

$$(2.9)\qquad \Phi(X,\xi) = 0,$$

$$(2.10)\qquad R(X,Y)\xi = \eta(Y)X - \eta(X)Y,$$

$$(2.11)\qquad R(\xi,X)Y = (\nabla_X\phi)Y,$$

$$(2.12)\qquad S(X,\xi) = (n-1)\eta(X).$$

3. GLOBALLY ϕ-QUASICONFORMALLY SYMMETRIC SASAKIAN MANIFOLDS

Definition 1. A Sasakian manifold M is said to be globally ϕ-quasiconformally symmetric if the quasiconformal curvature tensor C^* satisfies

$$(3.1)\qquad \phi^2\big((\nabla_X C^*)(Y,Z,W)\big) = 0,$$

for all vector fields $X, Y, Z \in \chi(M)$.

It is well known that if the Ricci tensor S of the manifold is of the form $S(X,Y) = \lambda g(X,Y)$, where λ is a constant and $X, Y \in \chi(M)$, then the manifold is called an Einstein manifold.

Let us suppose that M is a globally ϕ-quasiconformally symmetric Sasakian manifold. Then by definition

$$\phi^2\big((\nabla_W C^*)(Y,Z,W)\big) = 0.$$

Using (2.3) we have

$$-\big(\nabla_W C^*\big)(X,Y)Z + \eta\big((\nabla_W C^*)(X,Y)Z\big)\xi = 0.$$

From (1.2) it follows that

$$\begin{aligned}
&-ag\big((\nabla_W R)(X,Y)Z,U\big) - bg(X,U)(\nabla_W S)(Y,Z)\\
&\quad + bg(Y,U)(\nabla_W S)(X,Z)\\
&\quad - bg(Y,Z)g\big((\nabla_W Q)X,U\big) + bg(X,Z)g\big((\nabla_W Q)Y,U\big)\\
&\quad + \frac{1}{n}dr(W)\left[\frac{a}{n-1}+2b\right]\big(g(Y,Z)g(X,U) - g(X,Z)g(Y,U)\big)\\
&\quad + a\eta\big((\nabla_W R)(X,Y)Z\big)\eta(U) + b(\nabla_W S)(Y,Z)\eta(U)\eta(X)\\
&\quad - b(\nabla_W S)(X,Z)\eta(U)\eta(Y)\\
&\quad + bg(Y,Z)\eta\big((\nabla_W Q)X\big)\eta(U) - bg(X,Z)\eta\big((\nabla_W Q)Y\big)\eta(U)\\
&\quad - \frac{1}{n}dr(W)\left[\frac{a}{n-1}+2b\right]\big(g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\big)\eta(U) = 0.
\end{aligned}$$

Putting $X = U = e_i$, where $\{e_i\}$, $i = 1, 2, \ldots, n$, is an orthonormal basis of the tangent space at each point of the manifold, and taking summation over i, we get

$$\begin{aligned}
&-(a+nb-2b)(\nabla_W S)(Y,Z)\\
&-\Big\{bg\big((\nabla_W Q)e_i,e_i\big)-\frac{n-1}{n}dr(W)\Big(\frac{a}{n-1}+2b\Big)\\
&-b\eta\big((\nabla_W Q)e_i\big)\eta(e_i)+\frac{1}{n}dr(W)\Big(\frac{a}{n-1}+2b\Big)\Big\}g(Y,Z)\\
&+bg\big((\nabla_W Q)Y,Z\big)\\
&+a\eta\big((\nabla_W R)(e_i,Y)Z\big)\eta(e_i)-b(\nabla_W S)(\xi,Z)\eta(Y)\\
&-b\eta\big((\nabla_W Q)Y\big)\eta(Z)\\
&+\frac{1}{n}dr(W)\Big(\frac{a}{n-1}+2b\Big)\eta(Y)\eta(Z)=0.
\end{aligned}$$

Putting $Z = \xi$, we obtain

$$\begin{aligned}
(3.2)\qquad &-(a+nb-2b)(\nabla_W S)(Y,\xi)\\
&-\eta(Y)\Big\{bdr(W)-\frac{n-1}{n}dr(W)\Big(\frac{a}{n-1}+2b\Big)\\
&-b\eta\big((\nabla_W Q)e_i\big)\eta(e_i)+\frac{1}{n}dr(W)\Big(\frac{a}{n-1}+2b\Big)\Big\}\\
&+a\eta\big((\nabla_W R)(e_i,Y)\xi\big)\eta(e_i)\\
&-b(\nabla_W S)(\xi,\xi)\eta(Y)+\frac{1}{n}dr(W)\Big(\frac{a}{n-1}+2b\Big)\eta(Y)=0.
\end{aligned}$$

Now

$$\begin{aligned}
(3.3)\qquad \eta\big((\nabla_W Q)e_i\big)\eta(e_i)&=g\big((\nabla_W Q)e_i,\xi\big)\eta(e_i)\\
&=\eta\big((\nabla_W Q)\xi\big)=g(Q\phi X,\xi)\\
&=S(\phi X,\xi)=0.
\end{aligned}$$

$$(3.4)\qquad \eta\big((\nabla_W R)(e_i,Y)\xi\big)\eta(e_i)=g\big((\nabla_W R)(e_i,Y)\xi,\xi\big)g(e_i,\xi).$$

$$\begin{aligned}
g\big((\nabla_W R)(e_i,Y)\xi,\xi\big)&=g\big(\nabla_W R(e_i,Y)\xi,\xi\big)-g\big(R(\nabla_W e_i,Y)\xi,\xi\big)\\
&\quad-g\big(R(e_i,\nabla_W Y)\xi,\xi\big)-g\big(R(e_i,Y)\nabla_W\xi,\xi\big).
\end{aligned}$$

Since $\{e_i\}$ is an orthonormal basis $\nabla_X e_i = 0$ and using (2.10) we find

$$\begin{aligned}
g\big(R(e_i,\nabla_W Y)\xi,\xi\big)&=g\big(\eta(e_i)\nabla_W Y-\eta(\nabla_W Y)e_i,\xi\big)\\
&=\eta(e_i)\eta(\nabla_W Y)-\eta(\nabla_W Y)\eta(e_i)\\
&=0.
\end{aligned}$$

As

$$g\big(R(e_i, Y)\xi, \xi\big) + g\big(R(\xi, \xi)Y, e_i\big) = 0,$$

we have

$$g\big(\nabla_W R(e_i, Y)\xi, \xi\big) + g\big(R(e_i, Y)\xi, \nabla_W \xi\big) = 0.$$

Using this we get

$$g\big((\nabla_W R)(e_i, Y)\xi, \xi\big) = 0. \tag{3.5}$$

By the use of (3.3)–(3.5), from (3.2) we obtain

$$(\nabla_W S)(Y, \xi) = \frac{1}{n} dr(W)\eta(Y), \tag{3.6}$$

since $a + (n-2)b \neq 0$. Because if $a + (n-2)b = 0$ then from (1.1), it follows that $C^* = aC$. So we can not take $a + (n-2)b = 0$. Putting $Y = \xi$ in (3.6) we get $dr(W) = 0$. This implies r is constant. So from (3.6), we have

$$(\nabla_W S)(Y, \xi) = 0.$$

Using (2.6), this implies

$$(n-1)g(W, \phi Y) + S(Y, \phi W) = 0.$$

Changing W with ϕW and using (2.3), we obtain

$$S(Y, W) = \lambda g(Y, W),$$

where $\lambda = n - 1$. Hence we can state the following theorem:

Theorem 1. *If a Sasakian manifold is globally ϕ-quasiconformally symmetric, then the manifold is an Einstein manifold.*

Next suppose $S(X, Y) = \lambda g(X, Y)$, i.e. $QX = \lambda X$. Then from (1.1) we have

$$\begin{aligned} C^*(X, Y)Z &= aR(X, Y)Z \\ &\quad + \left[2b\lambda - \frac{r}{n}\left(\frac{a}{n-1} + 2b\right)\right][g(Y, Z)X - g(X, Z)Y], \end{aligned} \tag{3.7}$$

which gives us

$$\big(\nabla_W C^*\big)(X, Y)Z = a(\nabla_W R)(X, Y)Z.$$

Applying ϕ^2 on both sides of the above equation we have

$$\phi^2\big(\nabla_W C^*\big)(X, Y)Z = a\phi^2(\nabla_W R)(X, Y)Z.$$

Hence we can state the following theorem.

Theorem 2. *A globally ϕ-quasiconformally symmetric Sasakian manifold is globally ϕ-symmetric.*

Remark 1. Since a globally ϕ-symmetric Sasakian manifold is always a globally ϕ-quasiconformally symmetric manifold, from Theorem 2, we conclude that on a Sasakian manifold, globally ϕ-symmetry and globally ϕ-quasiconformally symmetry are equivalent.

4. 3-DIMENSIONAL LOCALLY ϕ-QUASICONFORMALLY SYMMETRIC SASAKIAN MANIFOLDS

In a 3-dimensional Riemannian manifold, since $C = 0$ we have

$$R(X,Y)Z = g(Y,Z)QX - g(X,Z)QY + S(Y,Z)X - S(X,Z)Y + \frac{r}{2}[g(X,Z)Y - g(Y,Z)X]. \tag{4.1}$$

Now putting $Z = \xi$ in (4.1) and using (2.12) and (2.10) we get

$$\left(1 - \frac{r}{2}\right)[\eta(Y)X - \eta(X)Y] = \eta(X)QY - \eta(Y)QX. \tag{4.2}$$

Putting $Y = \xi$ in (4.2), we find

$$QX = \left(\frac{r}{2} - 1\right)X + \left(3 - \frac{r}{2}\right)\eta(X)\xi. \tag{4.3}$$

Therefore it follows from (4.3) that

$$S(X,Y) = \left(\frac{r}{2} - 1\right)g(X,Y) + \left(3 - \frac{r}{2}\right)\eta(X)\eta(Y). \tag{4.4}$$

Thus from (4.3) and (4.4), we get

$$\begin{aligned} R(X,Y)Z = {} & \left(\frac{r}{2} - 2\right)[g(Y,Z)X - g(X,Z)Y] \\ & + \left(3 - \frac{r}{2}\right)[g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi \\ & + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y]. \end{aligned} \tag{4.5}$$

Putting (4.3), (4.4) and (4.5) into (1.1) we have

$$\begin{aligned} C^*(X,Y)Z = {} & \left[\frac{(a+b)r}{3} - 2(a+b)\right][g(Y,Z)X - g(X,Z)Y] \\ & + \left(3 - \frac{r}{2}\right)(b+1)[g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi \\ & + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y]. \end{aligned} \tag{4.6}$$

Taking the covariant differentiation to the both sides of the equation (4.6), we have

$$
\begin{aligned}
(4.7) \qquad (\nabla_W C^*)(X,Y)Z &= \frac{dr(W)}{3}[a+b][g(Y,Z)X - g(X,Z)Y] \\
&\quad - \frac{dr(W)}{2}(b+1)[g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi \\
&\qquad + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y] \\
&\quad + \left(3 - \frac{r}{2}\right)(b+1) \\
&\quad \times [g(Y,Z)(\nabla_W \eta)(X)\xi + g(Y,Z)\eta(X)\nabla_W \xi \\
&\qquad - g(X,Z)(\nabla_W \eta)(Y)\xi - g(X,Z)\eta(Y)\nabla_W \xi \\
&\qquad + g(Y,\nabla_W \xi)\eta(Z)X \\
&\qquad + g(Z,\nabla_W \xi)\eta(Y)X - g(X,\nabla_W \xi)\eta(Z)Y \\
&\qquad - g(Z,\nabla_W \xi)\eta(X)Y].
\end{aligned}
$$

Now assume that X, Y and Z are horizontal vector fields. So equation (4.7) becomes

$$
\begin{aligned}
(4.8) \qquad (\nabla_W C^*)(X,Y)Z &= \frac{dr(W)}{3}[a+b][g(Y,Z)X - g(X,Z)Y] \\
&\quad + \left(3 - \frac{r}{2}\right)(b+1) \\
&\quad \times [g(Y,Z)(\nabla_W \eta)(X)\xi - g(X,Z)(\nabla_W \eta)(Y)\xi].
\end{aligned}
$$

Since X, Y and Z are horizontal vector fields, using (2.3) equation (4.8) gives us

$$
(4.9) \qquad \phi^2(\nabla_W C^*)(X,Y)Z = \frac{dr(W)}{3}[a+b]\big(-g(Y,Z)X + g(X,Z)Y\big).
$$

Assume that $\phi^2(\nabla_W C^*)(X,Y)Z = 0$. If $a+b=0$ then putting $a=-b$ into (1.1) we find

$$
C^*(X,Y)Z = aC(X,Y)Z,
$$

where C is the Weyl conformal curvature tensor. But for a 3-dimensional Riemannian manifold since $C=0$, we obtain $C^*=0$. Therefore $a+b\neq 0$. Then the equation (4.9) implies $dr(W)=0$. Hence we conclude the following theorem:

Theorem 3. *A 3-dimensional Sasakian manifold is locally ϕ-quasiconformally symmetric if and only if the scalar curvature r is constant.*

In [8], Watanabe proved this corollary.

Corollary 1. *A 3-dimensional Sasakian manifold is locally ϕ-symmetric if and only if the scalar curvature r is constant.*

Using Corollary 1, we state the following theorem:

Theorem 4. *A 3-dimensional Sasakian manifold is locally ϕ-quasiconformally symmetric if and only if it is locally ϕ-symmetric.*

Example 1. We consider the three-dimensional manifold $M = \{(x, y, z) \in \mathbb{R}^3\}$, where (x, y, z) are the standard coordinates in $\mathbb{R}^3$. The vector fields

$$e_1 = \frac{\partial}{\partial x}, \qquad e_2 = \frac{\partial}{\partial y} - x\frac{\partial}{\partial x} + x\frac{\partial}{\partial z}, \qquad e_3 = \frac{1}{2}\frac{\partial}{\partial z}$$

are linearly independent at each point of M. Let g be the Riemannian metric defined by

$$g(e_1, e_3) = g(e_2, e_3) = g(e_1, e_2) = 0,$$
$$g(e_1, e_1) = g(e_2, e_2) = g(e_3, e_3) = 1.$$

Let η be the 1-form defined by $\eta(Z) = g(Z, e_3)$ for any $Z \in \chi(M)$. Let ϕ be the $(1, 1)$ tensor field defined by $\phi(e_1) = e_2$, $\phi(e_2) = -e_1$, $\phi(e_3) = 0$. Then using the linearity of ϕ and g we have

$$\eta(e_3) = 1, \qquad \phi^2 Z = -Z + \eta(Z)e_3,$$
$$g(\phi Z, \phi W) = g(Z, W) - \eta(Z)\eta(W),$$

for any $Z, W \in \chi(M)$. Thus for $e_3 = \xi$, (ϕ, ξ, η, g) defines an almost contact metric structure on M.

Let ∇ be the Levi–Civita connection with respect to the metric g. Then we have

$$[e_1, e_2] = -e_1 + 2e_3, \qquad [e_1, e_3] = 0, \qquad [e_2, e_3] = 0.$$

The Riemannian connection ∇ of the metric g is given by

$$2g(\nabla_X Y, Z) = Xg(Y, Z) + Yg(Z, X) - Zg(X, Y) - g(X, [Y, Z]) - g(Y, [X, Z]) + g(Z, [X, Y]),$$

which is known as Koszul's formula.

Koszul's formula yields

$$\nabla_{e_1} e_3 = -e_2, \qquad \nabla_{e_1} e_2 = -\frac{1}{2}e_1 + e_3, \qquad \nabla_{e_1} e_1 = e_2,$$
$$\nabla_{e_2} e_3 = e_1, \qquad \nabla_{e_2} e_2 = 0, \qquad \nabla_{e_2} e_1 = \frac{1}{2}e_1 - e_3,$$
$$\nabla_{e_3} e_3 = 0, \qquad \nabla_{e_3} e_2 = e_1, \qquad \nabla_{e_3} e_1 = -e_2.$$

From the above expressions it is easy to see that equations (2.1) and (2.6) hold. Hence the manifold is Sasakian.

With the help of the above results we can verify the following results:

$$R(e_1, e_2)e_1 = \frac{9}{2}e_2, \qquad R(e_2, e_3)e_2 = \frac{1}{2}e_1 - e_3,$$
$$R(e_1, e_2)e_2 = -\frac{13}{4}e_1 + \frac{1}{2}e_3, \qquad R(e_2, e_3)e_3 = e_2,$$
$$R(e_1, e_2)e_3 = 0, \qquad R(e_1, e_3)e_1 = -e_3 - \frac{1}{2}e_1,$$
$$R(e_2, e_3)e_1 = \frac{1}{2}e_2, \qquad R(e_1, e_3)e_2 = \frac{1}{2}e_2,$$
$$R(e_1, e_3)e_3 = e_1$$

and

$$S(e_1, e_1) = -\frac{9}{4}, \qquad S(e_2, e_2) = -\frac{7}{2} \qquad S(e_3, e_3) = 2, \quad r = -\frac{15}{4}.$$

Thus the scalar curvature r is constant. Hence from Corollary 1 and Theorem 4, M is a locally ϕ-quasiconformally symmetric Sasakian manifold.

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