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On some submanifolds of Kenmotsu manifolds

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ABSTRACT

In this paper, we study submanifolds of Kenmotsu manifolds. We prove that if the second fundamental form of a submanifold of a Kenmotsu manifold is recurrent, 2-recurrent or generalized 2-recurrent then the submanifold is totally geodesic. Furthermore, we show that a submanifold of a Kenmotsu manifold with parallel third fundamental form is again totally geodesic. We also consider quasi-umbilical hypersurfaces of Kenmotsu space forms. We show that these type hypersurfaces are generalized quasi-Einstein.

1. Introduction

Let (M_1, g_1) and (M_2, g_2) be two Riemannian manifolds and f a positive differentiable function on M_1. The warped product of M_1 and M_2 is the Riemannian manifold

$$M_1 \times_f M_2 = (M_1 \times M_2, g),$$

where $g = g_1 + f^2 g_2$ [3].

It is well known that the notion of warped products plays some important role in differential geometry as well as in Physics. For instance, the best relativistic model of the Schwarzschild space-time that describes the out space around a massive star or a black hole is given as a warped product (see, for instance, [1,6,14]).

In [18], S. Tanno classified $(2n+1)$-dimensional almost contact metric manifolds M with almost contact metric structure (φ, ξ, η, g), whose automorphism group possess the maximum dimension $(n+1)^2$. For such a manifold, the sectional curvature of plane sections containing ξ is a constant, say c. (1) If $c > 0, M$ is a homogeneous Sasakian manifold of constant φ sectional curvature. (2) If $c = 0, M$ is global Riemannian product of a line or a circle with a Kaehler manifold of constant holomorphic sectional curvature. (3) If $c < 0, M$ is a warped product space $\mathbb{R} \times_f \mathbb{C}^n$.

Kenmotsu [10] characterized the differential geometric properties of manifold of class (3); the structure so obtained is now known as *Kenmotsu structure*. A Kenmotsu structure is not Sasakian.

In this study, we consider submanifolds of Kenmotsu manifolds whose second fundamental forms are recurrent, 2-recurrent or generalized 2-recurrent. We also consider quasi-umbilical hypersurfaces of Kenmotsu space forms.

The paper is organized as follows: In Section 2, we give a brief information about recurrent manifolds, submanifolds and quasi-umbilical hypersurfaces. In Section 3, some definitions and notions about Kenmotsu manifolds and their submanifolds are given. In Section 4, we consider submanifolds of Kenmotsu manifolds whose second fundamental forms are recurrent, 2-recurrent or generalized 2-recurrent. We show that these type submanifolds are totally geodesic. We also prove that a submanifold of a Kenmotsu manifold with parallel third fundamental form is again totally geodesic. In the final section, we consider quasi-umbilical hypersurfaces of Kenmotsu space forms. We show that these type hypersurfaces are generalized quasi-Einstein hypersurfaces.

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2. Immersions of recurrent type

We denote by $\nabla^p T$ the covariant differential of the pth order, $p \geqslant 1$, of a $(0,k)$-tensor field $T, k \geqslant 1$, defined on a Riemannian manifold (M,g) with the Levi–Civita connection ∇. According to [17], the tensor T is said to be *recurrent*, respectively, 2- *recurrent*, if the following condition holds on M

$$(\nabla T)(X_1,\ldots,X_k;X)T(Y_1,\ldots,Y_k) = (\nabla T)(Y_1,\ldots,Y_k;X)T(X_1,\ldots,X_k), \tag{2.1}$$

respectively

$$(\nabla^2 T)(X_1,\ldots,X_k;X,Y)T(Y_1,\ldots,Y_k) = (\nabla^2 T)(Y_1,\ldots,Y_k;X,Y)T(X_1,\ldots,X_k),$$

where $X,Y,X_1,Y_1,\ldots,X_k,Y_k \in TM$. From (2.1) it follows that at a point $x \in M$ if the tensor T is non-zero then there exists a unique 1-form ϕ, respectively, a $(0,2)$-tensor ψ, defined on a neighborhood U of x, such that

$$\nabla T = T \otimes \phi, \quad \phi = d(\log \|T\|), \tag{2.2}$$

respectively

$$\nabla^2 T = T \otimes \psi, \tag{2.3}$$

holds on U, where $\|T\|$ denotes the norm of T, $\|T\|^2 = g(T,T)$. The tensor T is said to be *generalized* 2-*recurrent* if

$$\begin{aligned}
&((\nabla^2 T)(X_1,\ldots,X_k;X,Y) - (\nabla T \otimes \phi)(X_1,\ldots,X_k;X,Y))T(Y_1,\ldots,Y_k) \\
&\quad = ((\nabla^2 T)(Y_1,\ldots,Y_k;X,Y) - (\nabla T \otimes \phi)(Y_1,\ldots,Y_k;X,Y))T(X_1,\ldots,X_k)
\end{aligned}$$

holds on M, where ϕ is a 1-form on M. From this it follows that at a point $x \in M$ if the tensor T is non-zero then there exists a unique $(0,2)$-tensor ψ, defined on a neighborhood U of x, such that

$$\nabla^2 T = \nabla T \otimes \phi + T \otimes \psi, \tag{2.4}$$

holds on U.

Let $f : (M,g) \to (\widetilde{M},\tilde{g})$ be an isometric immersion from an n-dimensional Riemannian manifold (M,g) into $(n+d)$ -dimensional Riemannian manifold $(\widetilde{M},\tilde{g}), n \geqslant 2, d \geqslant 1$. The Gauss and Weingarten formulas are given by

$$\tilde{\nabla}_X Y = \nabla_X Y + \sigma(X,Y), \tag{2.5}$$

$$\tilde{\nabla}_X N = -A_N X + \nabla_X^{\perp} N, \tag{2.6}$$

for all vector fields X,Y tangent to M and normal vector field N on M, where ∇ is the Riemannian connection on M determined by the induced metric g and $\nabla^{\perp}$ is the normal connection on $T^{\perp}M$ of M.

The Gauss equation is given by

$$\widetilde{R}(X,Y,Z,W) = R(X,Y,Z,W) - g(\sigma(X,W),\sigma(Y,Z)) + g(\sigma(Y,W),\sigma(X,Z)), \tag{2.7}$$

where Z,W are vector fields tangent to M. The first and second covariant derivative of the second fundamental form σ are given by

$$(\bar{\nabla}_X \sigma)(Y,Z) = \nabla_X^{\perp}\sigma(Y,Z) - \sigma(\nabla_X Y,Z) - \sigma(Y,\nabla_X Z) \tag{2.8}$$

and

$$(\bar{\nabla}^2\sigma)(Z,W,X,Y) = (\bar{\nabla}_X\bar{\nabla}_Y\sigma)(Z,W) = \nabla_X^{\perp}((\bar{\nabla}_Y\sigma)(Z,W)) - (\bar{\nabla}_Y\sigma)(\nabla_X Z,W) - (\bar{\nabla}_X\sigma)(Z,\nabla_Y W) - (\bar{\nabla}_{\nabla_X Y}\sigma)(Z,W), \tag{2.9}$$

respectively, where $\bar{\nabla}$ is called the *van der Waerden–Bortolotti connection* of M [5].

An n-dimensional hypersurface $M, n \geqslant 3$, in a Riemannian manifold $\widetilde{M}$ is said to be *quasi-umbilical* [9] at a point $x \in M$ if at the point x its second fundamental tensor H satisfies the equality

$$H = ag + b\omega \otimes \omega, \tag{2.10}$$

where ω is a 1-form and a and b are some functions on M. If $a = 0$ (respectively, $b = 0$ or $a = b = 0$) holds at x then it is called *cylindrical* (respectively, *umbilical* or *geodesic*) at x. If (2.10) is fulfilled at every point of M then it is called a *quasi-umbilical hypersurface*.

3. Kenmotsu manifolds and their submanifolds

Let $\widetilde{M}$ be a $(2n+1)$-dimensional almost contact metric manifold with structure (φ,ξ,η,g) where φ is a tensor field of type $(1,1)$, ξ is a vector field, η is a 1-form and g is the Riemannian metric on $\widetilde{M}$ satisfying

$$\varphi^2 = -I + \eta \otimes \xi, \quad \varphi\xi = 0, \quad \eta(\xi) = 1, \quad \eta \circ \varphi = 0,$$
$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y),$$
$$\eta(X) = g(X, \xi), \quad g(\varphi X, Y) + g(X, \varphi Y) = 0,$$

for all vector fields X, Y on $\widetilde{M}$ [2]. An almost contact metric manifold $\widetilde{M}$ is said to be a *Kenmotsu manifold* [10] if the relation

$$(\tilde{\nabla}_X \varphi)Y = g(\varphi X, Y)\xi - \eta(Y)\varphi X, \tag{3.1}$$

holds on $\widetilde{M}$, where $\tilde{\nabla}$ is the Levi–Civita connection of g. From the above equation, for a Kenmotsu manifold we also have

$$\tilde{\nabla}_X \xi = X - \eta(X)\xi. \tag{3.2}$$

A Kenmotsu manifold is normal (that is, the Nijenhuis tensor of φ equals $-2d\eta \otimes \xi$) but not Sasakian. Moreover, it is also not compact since from the Eq. (3.2) we get $\mathrm{div}\xi = 2n$. In [10], Kenmotsu showed (1) that locally a Kenmotsu manifold is a warped product $I \times_f N$ of an interval I and a Kaehler manifold N with warping function $f(t) = se^t$, where s is a non-zero constant; and (2) that a Kenmotsu manifold of constant φ-sectional curvature is a space of constant curvature -1, and so it is locally hyperbolic space.

In case of Kenmotsu manifold, $\widetilde{M}$ has constant φ-holomorphic sectional curvature c if and only if

$$\begin{aligned}\widetilde{R}(X, Y)Z = {} & \frac{(c-3)}{4}[g(Y, Z)X - g(X, Z)Y] + \frac{(c+1)}{4}[\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X \\ & + \eta(Y)g(X, Z)\xi - \eta(X)g(Y, Z)\xi + g(X, \varphi Z)\varphi Y - g(Y, \varphi Z)\varphi X + 2g(X, \varphi Y)\varphi Z],\end{aligned} \tag{3.3}$$

where Z is a vector field in $\widetilde{M}$. In this case, we call $\widetilde{M}$ a Kenmotsu space form c. In [16], Pitis proved that there exist no connected Kenmotsu space forms or connected conformally flat manifolds of dimension $\geqslant 5$.

Now assume that M is a submanifold of a Kenmotsu manifold $\widetilde{M}$ such that ξ is tangent to M. So from the Gauss formula

$$\tilde{\nabla}_X \xi = \nabla_X \xi + \sigma(X, \xi),$$

which implies from (3.2) that

$$\nabla_X \xi = X - \eta(X)\xi, \quad \text{and} \quad \sigma(X, \xi) = 0. \tag{3.4}$$

for each vector field X tangent to M (see [11]). It is also easy to see that for a submanifold M of a Kenmotsu manifold $\widetilde{M}$

$$g(\varphi X, \xi) = 0 \tag{3.5}$$

and

$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y). \tag{3.6}$$

4. Recurrent submanifolds of Kenmotsu manifolds

In [11], Kobayashi showed that a submanifold M of a Kenmotsu manifold $\widetilde{M}$ has parallel second fundamental form if and only if M is totally geodesic. As a generalization of this result we state the following theorem:

Theorem 1. *Let M be a submanifold of a Kenmotsu manifold $\widetilde{M}$ tangent to ξ. Then σ is recurrent if and only if M is totally geodesic.*

Proof. Since σ is recurrent, from (2.2) we get

$$(\bar{\nabla}_X \sigma)(Y, Z) = \phi(X)\sigma(Y, Z),$$

where ϕ is a 1-form on M. Then in view of (2.8), the above equation can be written as

$$\nabla^{\perp}_X \sigma(Y, Z) - \sigma(\nabla_X Y, Z) - \sigma(Y, \nabla_X Z) = \phi(X)\sigma(Y, Z). \tag{4.1}$$

Taking $Z = \xi$ in (4.1) we have

$$\nabla^{\perp}_X \sigma(Y, \xi) - \sigma(\nabla_X Y, \xi) - \sigma(Y, \nabla_X \xi) = \phi(X)\sigma(Y, \xi). \tag{4.2}$$

Making use of relation (3.4) in (4.2) we obtain

$$\sigma(X, Y) = 0,$$

which implies that M is totally geodesic. The converse statement is trivial. This completes the proof of the theorem. □

Theorem 2. *Let M be a submanifold of a Kenmotsu manifold $\widetilde{M}$ tangent to ξ. Then M has parallel third fundamental form if and only if it is totally geodesic.*

Proof. Suppose that M has parallel third fundamental form. Then we can write

$$(\bar{\nabla}_X \bar{\nabla}_Y \sigma)(Z, W) = 0.$$

Replacing W with ξ in the above equation and using (2.9) we have

$$\nabla^{\perp}_X((\bar{\nabla}_Y \sigma)(Z,\xi)) - (\bar{\nabla}_Y \sigma)(\nabla_X Z, \xi) - (\bar{\nabla}_X \sigma)(Z, \nabla_Y \xi) - (\bar{\nabla}_{\nabla_X Y} \sigma)(Z, \xi) = 0. \tag{4.3}$$

Taking account of (2.8) in (4.3) and using (3.4), we obtain

$$-2\nabla^{\perp}_X \sigma(Y, Z) + 2\sigma(\nabla_X Z, Y) + 2\sigma(Z, \nabla_X Y) - \eta(Y)\sigma(X, Z) = 0. \tag{4.4}$$

Putting $Y = \xi$ in (4.4), in view of (3.4), we get

$$\sigma(X, Z) = 0,$$

which shows that M is totally geodesic. The converse statement is trivial. Hence, the proof of the theorem is completed. □

Corollary 1. *Let M be a submanifold of a Kenmotsu manifold $\widetilde{M}$ tangent to ξ. Then σ is 2-recurrent if and only if M is totally geodesic.*

Proof. Since σ is 2-recurrent, from (2.3), we have

$$(\bar{\nabla}_X \bar{\nabla}_Y \sigma)(Z, W) = \sigma(Z, W)\phi(X, Y). \tag{4.5}$$

Taking $W = \xi$ in (4.5) and using the proof of Theorem 2 we get

$$\sigma(X, Z) = 0,$$

which shows that M is totally geodesic. The converse statement is trivial. This completes the proof of the corollary. □

Theorem 3. *Let M be a submanifold of a Kenmotsu manifold $\widetilde{M}$ tangent to ξ. Then σ is generalized 2-recurrent if and only if M is totally geodesic.*

Proof. Since σ is generalized 2-recurrent, from (2.4), we can write

$$(\bar{\nabla}_X \bar{\nabla}_Y \sigma)(Z, W) = \psi(X, Y)\sigma(Z, W) + \phi(X)(\bar{\nabla}_Y \sigma)(Z, W), \tag{4.6}$$

where ψ and ϕ are 2-form and 1-form, respectively. Taking $W = \xi$ in (4.6) and taking account of the Eq. (3.4) we get

$$(\bar{\nabla}_X \bar{\nabla}_Y \sigma)(Z, \xi) = \phi(X)(\bar{\nabla}_Y \sigma)(Z, \xi).$$

Then making use of (2.9) and (2.8) in above equation, in view of (3.4), we have

$$-2\nabla^{\perp}_X \sigma(Y, Z) + 2\sigma(\nabla_X Z, Y) + 2\sigma(Z, \nabla_X Y) - \eta(Y)\sigma(X, Z) = -\phi(X)\sigma(Y, Z).$$

Putting $Y = \xi$ in the above equation and using (3.4) we obtain $\sigma(X, Z) = 0$, which means that M is totally geodesic. The converse statement is trivial. Thus our theorem is proved. □

5. Quasi-umbilical hypersurfaces of Kenmotsu manifolds

A Riemannian manifold $(M^n, g), (n > 2)$, is said to be an *Einstein manifold* if its Ricci tensor S satisfies the condition $S = \frac{r}{n}g$, where r denotes the scalar curvature of M. The notion of a quasi-Einstein manifold was introduced by Chaki and Maity in [4]. A non-flat Riemannian manifold $(M^n, g), (n > 2)$, is defined to be a *quasi-Einstein manifold* if the condition

$$S(X, Y) = ag(X, Y) + bA(X)A(Y) \tag{5.1}$$

is fulfilled on M, where a, b scalars of which $b \neq 0$ and A is non-zero 1-form such that

$$g(X, U) = A(X) \tag{5.2}$$

for all vector fields X; U being a unit vector field which is called the generator of the manifold. If $b = 0$ then the manifold reduces to an Einstein manifold. In [8], it was shown that a quasi-umbilical hypersurface in a semi-Riemannian space form is a quasi-Einstein hypersurface. Quasi-Einstein manifolds arose during the study of exact solutions of the Einstein field equations. There are many studies about Einstein field equations (for instance, see [12,13]).

A non-flat Riemannian manifold is called a *generalized quasi-Einstein manifold* [7] if its Ricci tensor S satisfies the condition

$$S(X, Y) = ag(X, Y) + bA(X)A(Y) + cB(X)B(Y), \tag{5.3}$$

where a, b, c are certain non-zero scalars and A, B are two non-zero 1-forms. The unit vector fields U and V corresponding to the 1-forms A and B are defined by

$$g(X,U) = A(X), \quad g(X,V) = B(X), \tag{5.4}$$

respectively, and the vector fields U and V are orthogonal. i.e. $g(U,V) = 0$. The vector fields U and V are called the generators of the manifold. If $c = 0$, then the manifold reduces to a quasi-Einstein manifold.

Now we state the following theorem:

Theorem 4. *Let M be a quasi-umbilical hypersurface of a Kenmotsu space form $\widetilde{M}^{2n+1}(c)$. Then M is a generalized quasi-Einstein hypersurface.*

Proof. Since $\widetilde{M}(c)$ is a Kenmotsu space form, from (3.3), we can write

$$\begin{aligned}\widetilde{R}(X,Y,Z,W) = \frac{(c-3)}{4}[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)] + \frac{(c+1)}{4}[&\eta(X)\eta(Z)g(Y,W) - \eta(Y)\eta(Z)g(X,W) \\ &+ \eta(Y)\eta(W)g(X,Z) - \eta(X)\eta(W)g(Y,Z) + g(X,\varphi Z)g(\varphi Y,W) - g(Y,\varphi Z)g(\varphi X,W) \\ &+ 2g(X,\varphi Y)g(\varphi Z,W)]\end{aligned} \tag{5.5}$$

for all vector fields X, Y, Z, W tangent to M. Let N be the unit normal vector field of M in $\widetilde{M}(c)$. So using $\sigma(X,Z) = H(X,Z)N$ in (2.7) we get

$$\widetilde{R}(X,Y,Z,W) = R(X,Y,Z,W) - H(Y,Z)H(X,W) + H(X,Z)H(Y,W). \tag{5.6}$$

From (2.10), for a quasi-umbilical hypersurface, we know that

$$H(X,Z) = ag(X,Z) + b\omega(X)\omega(Z). \tag{5.7}$$

Putting (5.7) in (5.6) we get

$$\begin{aligned}\widetilde{R}(X,Y,Z,W) = R(X,Y,Z,W) - [&(ag(Y,Z) + b\omega(Y)\omega(Z))(ag(X,W) \\ &+ b\omega(X)\omega(W))] + [(ag(X,Z) + b\omega(X)\omega(Z))(ag(Y,W) + b\omega(Y)\omega(W))].\end{aligned}$$

Then by the use of (5.5) we find

$$\begin{aligned}&\frac{(c-3)}{4}[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)] + \frac{(c+1)}{4}[\eta(X)\eta(Z)g(Y,W) \\ &- \eta(Y)\eta(Z)g(X,W) + \eta(Y)\eta(W)g(X,Z) - \eta(X)\eta(W)g(Y,Z) \\ &+ g(X,\varphi Z)g(\varphi Y,W) - g(Y,\varphi Z)g(\varphi X,W) + 2g(X,\varphi Y)g(\varphi Z,W)] \\ = &R(X,Y,Z,W) + a^2[g(X,Z)g(Y,W) - g(Y,Z)g(X,W)] + ab[g(X,Z)\omega(Y)\omega(W) + g(Y,W)\omega(X)\omega(Z) \\ &- g(Y,Z)\omega(X)\omega(W) - g(X,W)\omega(Y)\omega(Z)].\end{aligned}$$

Contracting above equation over X and W and using (3.5), (3.6) we obtain

$$\begin{aligned}S(Y,Z) = &\left[\frac{(c-3)}{4}(2n-1) + \frac{(c+1)}{2} + a^2(2n-1) + ab\right]g(Y,Z) \\ &- \frac{(c+1)}{4}(2n+1)\eta(Y)\eta(Z) + ab(2n-2)\omega(Y)\omega(Z).\end{aligned}$$

Hence by (5.3), M is a generalized quasi-Einstein hypersurface. Thus, the proof of the theorem is completed. □

6. Conclusions

Study of warped products plays some important role in Differential Geometry as well as in Physics. S. Tanno classified $(2n+1)$-dimensional almost contact metric manifolds M with almost contact metric structure (φ, ξ, η, g), whose automorphism group possess the maximum dimension $(n+1)^2$. For such a manifold, if the sectional curvature of plane sections containing ξ is a constant < 0, then M is a warped product space $\mathbb{R}\times_f \mathbb{C}^n$. Kenmotsu characterized the differential geometric properties of manifold of this class which is known as Kenmotsu structure. In this paper, we study submanifolds of Kenmotsu manifolds whose second fundamental forms are recurrent, 2-recurrent and generalized 2-recurrent. It was proved that these type submanifolds are totally geodesic. The study of space-times admitting fluid viscosity and electromagnetic fields require some generalizations of Einstein manifolds and is under process (see [15]). In the final section, it is shown that a quasi-umbilical hypersurface of a Kenmotsu manifold is generalized quasi-Einstein.

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