Journal of Mathematical Inequalities
Volume 4, Number 2 (2010), 285–299

TRIGONOMETRIC APPROXIMATION IN GENERALIZED LEBESGUE SPACES $L^{p(x)}$

ALI GUVEN AND DANIYAL M. ISRAFILOV

Abstract. The approximation properties of Nörlund (N_n) and Riesz (R_n) means of trigonometric Fourier series are investigated in generalized Lebesgue spaces $L^{p(x)}$. The deviations $\|f-N_n(f)\|_{p(x)}$ and $\|f-R_n(f)\|_{p(x)}$ are estimated by $n^{-\alpha}$ for $f\in Lip(\alpha,p(x))$ $(0<\alpha\leqslant 1)$.

Mathematics subject classification (2010): 41A25, 42A10, 46E30.
Keywords and phrases: Generalized Lebesgue space, Lipschitz class, modulus of continuity, Nörlund mean, Riesz mean.

REFERENCES

[1] P. CHANDRA, *Approximation by Nörlund operators*, Mat. Vestnik 38 (1986), 263–269.
[2] P. CHANDRA, *Functions of classes L_p and $Lip(\alpha,p)$ and their Riesz means*, Riv. Mat. Univ. Parma (4) 12 (1986), 275–282.
[3] P. CHANDRA, *A note on degree of approximation by Nörlund and Riesz operators*, Mat. Vestnik 42 (1990), 9–10.
[4] P. CHANDRA, *Trigonometric approximation of functions in L_p-norm*, J. Math. Anal. Appl. 275 (2002), 13–26.
[5] D. CRUZ-URIBE, A. FIORENZA, C. J. NEUGEBAUER, *The maximal function on variable L^p spaces*, Ann. Acad. Sci. Fenn. Math. **28** (2003), 223–238, and 29 (2004), 247–249.
[6] R. A. DEVORE, G. G. LORENTZ, *Constructive Approximation*, Springer-Verlag (1993).
[7] L. DIENING, M. RUZICKA, *Calderon-Zygmund operators on generalized Lebesgue spaces $L^{p(x)}$ and problems related to fluid dynamics*, J. Reine Angew. Math. 563 (2003), 197–220.
[8] L. DIENING, *Maximal function on generalized Lebesgue spaces $L^{p(x)}$*, Math. Inequal. Appl. 7 (2004), 245–253.
[9] D. E. EDMUNDS, J. LANG, A. NEKVINDA, *On $L^{p(x)}$ norms*, Proc. R. Soc. Lond. A 455 (1999), 219–225.
[10] X. FAN, D. ZHAO, *On the spaces $L^{p(x)}(\Omega)$ and $W^{m,p(x)}(\Omega)$*, J. Math. Anal. Appl. 263 (2001), 424–446.
[11] A. GUVEN, *Trigonometric approximation of functions in weighted L^p spaces*, Sarajevo J. Math 5 (17) (2009), 99–108.
[12] D. M. ISRAFILOV, V. KOKILASHVILI, S. SAMKO, *Approximation in weighted Lebesgue and smirnov Spaces with variable exponents*, Proc. A. Razmadze Math. Inst. 143 (2007), 25–35.
[13] O. KOVACIK, J. RAKOSNIK, *On spaces $L^{p(x)}$ and $W^{k,p(x)}$*, Czechoslovak Math. J. 41 (1991), 592–618.
[14] N. X. KY, *Moduli of Mean Smoothness and Approximation with A_p-weights*, Annales Univ. Sci. Budapest 40 (1997), 37–48.
[15] L. LEINDLER, *Trigonometric approximation in L_p-norm*, J. Math. Anal. Appl. 302 (2005), 129–136.
[16] R. N. MOHAPATRA, D. C. RUSSELL, *Some direct and inverse theorems in approximation of functions*, J. Austral. Math. Soc. (Ser. A) 34 (1983), 143–154.
[17] A. NEKVINDA, *Hardy-Littlewood maximal operator on $L^{p(x)}(\mathbb{R})$*, Math. Inequal. Appl. 7 (2004), 255–265.
[18] L. PICK, M. RUZICKA, *An example of a space $L^{p(x)}$ on which the Hardy-Littlewood maximal operator is not bounded*, Expo. Math. 19 (2001), 369–371.

Paper JMI-04-25

[19] E. S. QUADE, *Trigonometric approximation in the mean*, Duke Math. J. 3 (1937), 529–542.
[20] I. I. SHARAPUDINOV, *Uniform boundedness in L^p $(p = p(x))$ of some families of convolution operators*, Math. Notes 59 (1996), 205–212.
[21] I. I. SHARAPUDINOV, *Some problems in approximation theory in the spaces $L^{p(x)}$*, (Russian), Analysis Mathematica 33 (2007), 135–153.
[22] A. ZYGMUND, *Trigonometric Series, Vol I*, Cambridge Univ. Press, 2nd edition, (1959).