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Research Article

Wardowski Type Contractions and the Fixed-Circle Problem on S-Metric Spaces

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In this paper, we present new fixed-circle theorems for self-mappings on an S-metric space using some Wardowski type contractions, ψ-contractive, and weakly ψ-contractive self-mappings. The common property in all of the obtained theorems for Wardowski type contractions is that the self-mapping fixes both the circle and the disc with the center x_0 and the radius r.

1. Introduction

Fixed-point theory has many applications in different fields; see [1–10]. Recently, using Wardowski's technique, some new fixed-point theorems on S-metric spaces [11] and some new fixed-circle theorems on metric spaces [12, 13] have been obtained. Our aim in this paper is to obtain various fixed-circle results using this technique. In Section 2, we recall some necessary background on S-metric spaces and give new examples. In Section 3, we introduce the notion of an F_c^S-contraction to obtain fixed-circle theorems. By means of this notion, we define new types of an F_c^S-contraction such as Hardy-Rogers type F_c^S-contraction and Reich type F_c^S-contraction and present some fixed-circle results on S-metric spaces. Also, we give an illustrative example of a self-mapping satisfying all of the conditions of the obtained theorems. In Section 4, we prove the existence along with the conditions that give us uniqueness of a fixed circle for ψ-contractive and weakly ψ-contractive self-mappings on S-metric spaces. In Section 5, we give an application of fixed-circle results obtained by Wardowski technique to integral type contractive self-mappings.

2. Preliminaries

In this section, we recall some necessary notions, relations, and results about S-metric spaces.

Definition 1 (see [14]). Let X be a nonempty set and $\mathcal{S} : X \times X \times X \longrightarrow [0, \infty)$ be a function satisfying the following conditions for all $x, y, z, a \in X$:

(S1) $\mathcal{S}(x, y, z) = 0$ if and only if $x = y = z$

(S2) $\mathcal{S}(x, y, z) \leq \mathcal{S}(x, x, a) + \mathcal{S}(y, y, a) + \mathcal{S}(z, z, a)$

Then $\mathcal{S}$ is called an S-metric on X and the pair $(X, \mathcal{S})$ is called an S-metric space.

Example 2 (see [15]). Let $X = \mathbb{R}$ (or $\mathbb{C}$) and the function $\mathcal{S} : X \times X \times X \longrightarrow [0, \infty)$ be defined as

$$\mathcal{S}(x, y, z) = |x - z| + |y - z|, \quad (1)$$

for all $x, y, z \in \mathbb{R}$ (or $\mathbb{C}$). Then the function $\mathcal{S}$ is an S-metric on $\mathbb{R}$ (or $\mathbb{C}$). This S-metric is called the usual S-metric on $\mathbb{R}$ (or $\mathbb{C}$).

Lemma 3 (see [14]). *Let $(X, \mathcal{S})$ be an S-metric space and $x, y \in X$. Then we have*

$$\mathcal{S}(x, x, y) = \mathcal{S}(y, y, x). \tag{2}$$

The relationships between a metric and an S-metric were studied in different papers (see [16–18] for more details). In [17], a formula of an S-metric space which is generated by a metric d was investigated as follows.

Let (X, d) be a metric space. Then the function $\mathcal{S}_d : X \times X \times X \longrightarrow [0, \infty)$ defined by

$$\mathcal{S}_d(x, y, z) = d(x, z) + d(y, z), \tag{3}$$

for all $x, y, z \in X$, is an S-metric on X. The S-metric $\mathcal{S}_d$ is called the S-metric generated by d [18]. We note that there exists an S-metric which is not generated by any metric d as seen in the following example.

Example 4. Let X be a nonempty set, the function $d : X \times X \longrightarrow [0, \infty)$ be any metric on X, and the function $\mathcal{S} : X \times X \times X \longrightarrow [0, \infty)$ be defined by

$$\begin{aligned}\mathcal{S}(x, y, z) = \min\{1, d(x, y)\} + \min\{1, d(y, z)\} \\ + \min\{1, d(x, z)\},\end{aligned} \tag{4}$$

for all $x, y, z \in X$. Then the function $\mathcal{S}$ is an S-metric and $(X, \mathcal{S})$ is an S-metric space. Indeed,

($S1$) for $x, y, z \in X$, we have

$$\begin{aligned}&\mathcal{S}(x, y, z) = 0 \Longleftrightarrow \\ &\min\{1, d(x, y)\} + \min\{1, d(y, z)\} + \min\{1, d(x, z)\} \\ &\quad = 0 \Longleftrightarrow \\ &d(x, y) = d(y, z) = d(x, z) = 0 \Longleftrightarrow \\ &x = y = z\end{aligned} \tag{5}$$

($S2$) using Table 1, we can easily see that the condition ($S2$) is satisfied.

Also the S-metric $\mathcal{S}$ is not generated by any metric m. Conversely, suppose that there exists a metric m such that

$$\mathcal{S}(x, y, z) = m(x, z) + m(y, z), \tag{6}$$

for all $x, y, z \in X$. Then we get

$$\begin{aligned}\mathcal{S}(x, x, z) = 2m(x, z) \\ \text{and so } m(x, z) = \min\{1, d(x, z)\}\end{aligned} \tag{7}$$

and

$$\begin{aligned}\mathcal{S}(y, y, z) = 2m(y, z) \text{ and so} \\ m(y, z) = \min\{1, d(y, z)\}.\end{aligned} \tag{8}$$

Therefore, we obtain

$$\begin{aligned}&\min\{1, d(x, y)\} + \min\{1, d(y, z)\} + \min\{1, d(x, z)\} \\ &\quad = \min\{1, d(x, z)\} + \min\{1, d(y, z)\},\end{aligned} \tag{9}$$

which is a contradiction. Consequently, $\mathcal{S}$ is not generated by any metric m.

In [19] and [14], a circle and a disc are defined on an S-metric space as follows, respectively:

$$C^S_{x_0,r} = \{x \in X : \mathcal{S}(x, x, x_0) = r\} \tag{10}$$

and

$$D^S_{x_0,r} = \{x \in X : \mathcal{S}(x, x, x_0) \leq r\}. \tag{11}$$

We give an example.

Example 5. Let X be a nonempty set, the function $d : X \times X \longrightarrow [0, \infty)$ be any metric on X, and the S-metric space be defined as Example 4. Let us consider the circle $C^S_{x_0,r}$ according to the S-metric:

$$\begin{aligned}&C^S_{x_0,r} \\ &\quad = \{x \in X : \mathcal{S}(x, x, x_0) = 2\min\{1, d(x, x_0)\} = r\}.\end{aligned} \tag{12}$$

Then we have the following cases:

Case 1. If $r = 2$ then $C^S_{x_0,r} = \{x \in X : d(x, x_0) \geq 1\}$.

Case 2. If $r > 2$ then $C^S_{x_0,r} = \emptyset$.

Case 3. If $r < 2$ then $C^S_{x_0,r} = C_{x_0,r/2}$, where $C_{x_0,r/2} = \{x \in X : d(x, x_0) = r/2\}$.

Definition 6 (see [19]). Let $(X, \mathcal{S})$ be an S-metric space, $C^S_{x_0,r}$ be a circle, and $T : X \longrightarrow X$ be a self-mapping. If $Tx = x$ for every $x \in C^S_{x_0,r}$ then the circle $C^S_{x_0,r}$ is called the fixed circle of T.

3. F^S_c-Contraction and Hardy-Rogers Type F^S_c-Contraction on S-Metric Spaces

At first, we recall the definition of the following family of functions which was introduced by Wardowski in [20].

Definition 7 (see [20]). Let $\mathbb{F}$ be the family of all functions $F : (0, \infty) \longrightarrow \mathbb{R}$ such that

(F_1) F is strictly increasing

(F_2) for each sequence $\{\alpha_n\}$ in $(0, \infty)$ the following holds: $\lim \alpha_n = 0$ if and only if $\lim F(\alpha_n) = -\infty$

(F_3) there exists $k \in (0, 1)$ such that $\lim_{\alpha \longrightarrow 0^+} \alpha^k F(\alpha) = 0$.

The following is an example of some functions that satisfies conditions (F_1), (F_2), and (F_3) of Definition 7.

Example 8 (see [20]). (1) $F : (0, \infty) \longrightarrow \mathbb{R}$ defined by $F(x) = \ln(x)$.

(2) $F : (0, \infty) \longrightarrow \mathbb{R}$ defined by $F(x) = \ln(x) + x$.

(3) $F : (0, \infty) \longrightarrow \mathbb{R}$ defined by $F(x) = -1/\sqrt{x}$.

(4) $F : (0, \infty) \longrightarrow \mathbb{R}$ defined by $F(x) = \ln(x^2 + x)$.

Note that these four functions satisfy conditions (F_1), (F_2), and (F_3) of Definition 7.

Table 1

1	1	1	≤	2	2	2				
1	1	1	≤	2	2	$2d(z,a)$				
1	1	1	≤	1	1	$d(y,z)$	≤	2	$2d(y,a)$	$2d(z,a)$
1	1	1	≤	$d(x,y)$	$d(y,z)$	$d(x,z)$	≤	$2d(x,a)$	$2d(y,a)$	$2d(z,a)$
1	1	$d(y,z)$	≤	1	1	1	≤	2	2	2
1	1	$d(y,z)$	≤	1	1	1	≤	2	2	$2d(z,a)$
1	1	$d(y,z)$	≤	2	$d(y,a)$	$d(z,a)$	≤	2	$2d(y,a)$	$2d(z,a)$
1	1	$d(y,z)$	≤	1	$d(x,z)$	1	≤	2	$2d(y,a)$	$2d(z,a)$
1	1	$d(y,z)$	≤	$d(x,y)$	1	1	≤	2	$2d(y,a)$	$2d(z,a)$
1	1	$d(y,z)$	≤	$d(x,y)$	$d(x,z)$	$d(y,z)$	≤	$2d(x,a)$	$2d(y,a)$	$2d(z,a)$
1	$d(x,z)$	$d(y,z)$	≤	1	1	1	≤	2	2	2
1	$d(x,z)$	$d(y,z)$	≤	1	1	1	≤	2	2	$2d(z,a)$
1	$d(x,z)$	$d(y,z)$	≤	1	1	$d(y,z)$	≤	2	$2d(y,a)$	$2d(z,a)$
1	$d(x,z)$	$d(y,z)$	≤	$d(x,y)$	1	1	≤	$2d(x,a)$	$2d(y,a)$	2
1	$d(x,z)$	$d(y,z)$	≤	$d(x,y)$	$d(x,z)$	$d(y,z)$	≤	$2d(x,a)$	$2d(y,a)$	$2d(z,a)$
$d(x,y)$	$d(x,z)$	$d(y,z)$	≤	1	1	1	≤	2	2	2
$d(x,y)$	$d(x,z)$	$d(y,z)$	≤	1	1	1	≤	2	2	$2d(z,a)$
$d(x,y)$	$d(x,z)$	$d(y,z)$	≤	1	1	$d(y,z)$	≤	2	$2d(y,a)$	$2d(z,a)$
$d(x,y)$	$d(x,z)$	$d(y,z)$	≤	$2d(x,a)$	$2d(y,a)$	$2d(z,a)$				

Other possibilities can be proved like this table.

Now we introduce the following new contraction type using this family of functions.

Definition 9. Let $(X,\mathcal{S})$ be an S-metric space. A self-mapping T on X is said to be an F_c^S-contraction if there exist $F\in\mathbb{F}$, $t>0$, and $x_0\in X$ such that for all $x\in X$ the following holds:

$$\mathcal{S}(Tx,Tx,x)>0 \Longrightarrow t+F(\mathcal{S}(Tx,Tx,x))\leq F(\mathcal{S}(x,x,x_0)). \tag{13}$$

Now, we present the following proposition.

Proposition 10. *Let $(X,\mathcal{S})$ be an S-metric space. If a self-mapping T on X is an F_c^S-contraction with $x_0\in X$, then we have $Tx_0=x_0$.*

Proof. Assume that $Tx_0\neq x_0$. From the definition of an F_c^S-contraction, we get

$$\mathcal{S}(Tx_0,Tx_0,x_0)>0 \Longrightarrow t+F(\mathcal{S}(Tx_0,Tx_0,x_0))\leq F(\mathcal{S}(x_0,x_0,x_0)). \tag{14}$$

Inequality (14) contradicts with the definition of F since $F:(0,\infty)\longrightarrow\mathbb{R}$ and $\mathcal{S}(x_0,x_0,x_0)=0$. Therefore, it should be $Tx_0=x_0$. □

Using this new type contraction, we give the following fixed-circle theorem.

Theorem 11. *Let $(X,\mathcal{S})$ be an S-metric space, T be an F_c^S-contractive self-mapping with $x_0\in X$, and $r=\min\{\mathcal{S}(Tx,Tx,x):Tx\neq x\}$. Then $C_{x_0,r}^S$ is a fixed circle of T. T especially fixes every circle $C_{x_0,\rho}^S$ where $\rho<r$.*

Proof. Let $x\in C_{x_0,r}^S$. If $Tx\neq x$, by the definition of r we have $\mathcal{S}(Tx,Tx,x)\geq r$. Hence, using the F_c^S-contractive property and the fact that F is increasing, we obtain

$$F(r)\leq F(\mathcal{S}(Tx,Tx,x))\leq F(\mathcal{S}(x,x,x_0))-t < F(\mathcal{S}(x,x,x_0))=F(r), \tag{15}$$

which also lead to a contradiction. Therefore, $\mathcal{S}(Tx,Tx,x)=0$ and that is $Tx=x$. Consequently, $C_{x_0,r}^S$ is a fixed circle of T.

Now we show that T also fixes any circle $C_{x_0,\rho}^S$ with $\rho<r$. Let $x\in C_{x_0,\rho}^S$ and assume that $\mathcal{S}(Tx,Tx,x)>0$. By the F_c^S-contractive property, we have

$$F(\mathcal{S}(Tx,Tx,x))\leq F(\mathcal{S}(x,x,x_0))-t<F(\rho). \tag{16}$$

Since F is increasing, then we find

$$\mathcal{S}(Tx,Tx,x)<\rho<r. \tag{17}$$

But $r=\min\{\mathcal{S}(Tx,Tx,x):\text{for all } Tx\neq x\}$, which leads us to a contradiction. Thus, $\mathcal{S}(Tx,Tx,x)=0$ and $Tx=x$. Hence, $C_{x_0,\rho}^S$ is a fixed circle of T. □

Remark 12. Notice that, in Theorem 11, the F_c^S-contractive self-mapping T fixes the disc with the center x_0 and the radius r. Therefore, the center of any fixed circle is also fixed by T.

In the following example, we see that the converse statement of Theorem 11 is not always true.

Example 13. Let $(X, \mathcal{S})$ be an S-metric space, $x_0 \in X$ be any point, and the self-mapping $T : X \longrightarrow X$ be defined as

$$Tx = \begin{cases} x & \text{if } \mathcal{S}(x, x, x_0) \le r \\ x_0 & \text{if } \mathcal{S}(x, x, x_0) > r, \end{cases} \tag{18}$$

for all $x \in X$ with $r > 0$. Then it can be easily seen that T is not an F_c^S-contractive self-mapping. Indeed, if $\mathcal{S}(x, x, x_0) > r$ for $x \in X$, then, using Lemma 3 and the F_c^S-contractive property, we get

$$\begin{gathered} \mathcal{S}(Tx, Tx, x) = \mathcal{S}(x_0, x_0, x) > 0 \Longrightarrow \\ t + F(\mathcal{S}(x_0, x_0, x)) \le F(\mathcal{S}(x, x, x_0)) \Longrightarrow \\ t \le 0, \end{gathered} \tag{19}$$

which is a contradiction since $t > 0$. Hence T is not an F_c^S-contractive self-mapping. But T fixes every circle $C_{x_0,\rho}^S$ where $\rho \le r$.

Related to the number of the elements of the set X, the number of the fixed circles of an F_c^S-contractive self-mapping T can be infinite as seen in the following example.

Example 14. Let $X = \{x \in \mathbb{Q} : 0 \le x \le 2\}$, the metric $d : X \times X \longrightarrow [0, \infty)$ be defined as

$$d(x, y) = \left| \frac{x}{1 + |x|} - \frac{y}{1 + |y|} \right|, \tag{20}$$

for all $x, y \in X$, and the S-metric be defined as in Example 4. Let us define the self-mapping $T : X \longrightarrow X$ as

$$Tx = \begin{cases} \frac{1}{8} & \text{if } x = 0 \\ x & \text{otherwise}, \end{cases} \tag{21}$$

for all $x \in X$. Then the self-mapping T is an F_c^S-contractive self-mapping with $F = \ln x + x$, $t = \ln 3$, and $x_0 = 1/2$. Indeed, we get

$$\begin{gathered} \mathcal{S}(Tx, Tx, x) = \frac{2}{9} > 0 \Longrightarrow \\ \mathcal{S}(Tx, Tx, x) = \frac{2}{9} < \mathcal{S}(x, x, x_0) = \frac{2}{3} \Longrightarrow \\ \ln\left(\frac{2}{9}\right) < \ln\left(\frac{2}{3}\right) \Longrightarrow \\ \ln\left(\frac{2}{9}\right) + \frac{2}{9} < \ln\left(\frac{2}{3}\right) + \frac{2}{3} \Longrightarrow \\ \ln 3 + \ln\left(\frac{2}{9}\right) + \frac{2}{9} \le \ln\left(\frac{2}{3}\right) + \frac{2}{3} \Longrightarrow \\ t + F(\mathcal{S}(Tx, Tx, x)) \le F(\mathcal{S}(x, x, x_0)). \end{gathered} \tag{22}$$

Using Theorem 11, we have

$$r = \min\{\mathcal{S}(Tx, Tx, x) : Tx \ne x\} = \frac{2}{9}. \tag{23}$$

Therefore, T fixes the circle $C_{1/2,2/9}^S = \{2/7, 4/5\}$ and the disc $D_{1/2,2/9}^S = \{x \in X : \mathcal{S}(x, x, 1/2) \le 2/9\}$. Evidently, the number of the fixed circles of T is infinite.

In the following definition, we introduce the notion of a Hardy-Rogers type F_c^S-contraction.

Definition 15. Let $(X, \mathcal{S})$ be an S-metric space and T be a self-mapping on X. If there exist $F \in \mathbb{F}$, $t > 0$, and $x_0 \in X$ such that for all $x \in X$ the following holds:

$$\begin{aligned} &\mathcal{S}(Tx, Tx, x) > 0 \Longrightarrow \\ &t + F(\mathcal{S}(Tx, Tx, x)) \le F(\alpha\mathcal{S}(x, x, x_0) \\ &\quad + \beta\mathcal{S}(Tx, Tx, x) + \gamma\mathcal{S}(Tx_0, Tx_0, x_0) \\ &\quad + \delta\mathcal{S}(Tx_0, Tx_0, x) + \eta\mathcal{S}(Tx, Tx, x_0)), \end{aligned} \tag{24}$$

where

$$\begin{gathered} \alpha + \beta + \gamma + \delta + \eta = 1, \\ \alpha, \beta, \gamma, \delta, \eta \ge 0 \\ \text{and } \alpha \ne 0, \end{gathered} \tag{25}$$

then the self-mapping T is called a Hardy-Rogers type F_c^S-contraction on X.

Proposition 16. *Let $(X, \mathcal{S})$ be an S-metric space. If a self-mapping T on X is a Hardy-Rogers type F_c^S-contraction with $x_0 \in X$ then we have $Tx_0 = x_0$.*

Proof. Suppose that $Tx_0 \ne x_0$. Using the hypothesis, we obtain

$$\begin{aligned} &\mathcal{S}(Tx_0, Tx_0, x_0) > 0 \Longrightarrow \\ &t + F(\mathcal{S}(Tx_0, Tx_0, x_0)) \\ &\quad \le F(\alpha\mathcal{S}(x_0, x_0, x_0) + \beta\mathcal{S}(Tx_0, Tx_0, x_0) + \gamma\mathcal{S}(Tx_0, Tx_0, x_0) + \delta\mathcal{S}(Tx_0, Tx_0, x_0) + \eta\mathcal{S}(Tx_0, Tx_0, x_0)) \\ &\quad = F((\beta + \gamma + \delta + \eta)\mathcal{S}(Tx_0, Tx_0, x_0)) < F(\mathcal{S}(Tx_0, Tx_0, x_0)), \end{aligned} \tag{26}$$

which is a contradiction since $t > 0$. Therefore, we get $Tx_0 = x_0$. □

Remark 17. Using Proposition 16, a Hardy-Rogers type F_c^S-contraction condition can be changed as follows:

$$\begin{aligned} &\mathcal{S}(Tx,Tx,x) > 0 \Longrightarrow \\ &t + F(\mathcal{S}(Tx,Tx,x)) \leq F(\alpha\mathcal{S}(x,x,x_0) \\ &\quad + \beta\mathcal{S}(Tx,Tx,x) + \delta\mathcal{S}(Tx_0,Tx_0,x) \\ &\quad + \eta\mathcal{S}(Tx,Tx,x_0)), \end{aligned} \tag{27}$$

where

$$\begin{gathered} \alpha+\beta+\delta+\eta \leq 1, \\ \alpha,\beta,\delta,\eta \geq 0 \\ \text{and } \alpha \neq 0. \end{gathered} \tag{28}$$

Now using the Hardy-Rogers type F_c^S-contraction condition, we prove the following fixed-circle theorem.

Theorem 18. *Let $(X,\mathcal{S})$ be an S-metric space, T be a Hardy-Rogers type F_c^S-contractive self-mapping with $x_0 \in X$, and r be defined as in Theorem 11. If $\mathcal{S}(Tx,Tx,x_0) = r$, then $C_{x_0,r}^S$ is a fixed circle of T. T especially fixes every circle $C_{x_0,\rho}^S$ where $\rho < r$.*

Proof. Let $x \in C_{x_0,r}^S$ and $Tx \neq x$. Using the Hardy-Rogers type F_c^S-contraction property, Proposition 16, Lemma 3, and the fact that F is increasing, we get

$$\begin{aligned} F(r) &\leq F(\mathcal{S}(Tx,Tx,x)) \leq F(\alpha\mathcal{S}(x,x,x_0) + \beta\mathcal{S}(Tx,Tx,x) + \delta\mathcal{S}(Tx_0,Tx_0,x) + \eta\mathcal{S}(Tx,Tx,x_0)) - t \\ &< F(\alpha\mathcal{S}(x,x,x_0) + \beta\mathcal{S}(Tx,Tx,x) + \delta\mathcal{S}(Tx_0,Tx_0,x) + \eta\mathcal{S}(Tx,Tx,x_0)) = F((\alpha+\delta+\eta)r + \beta\mathcal{S}(Tx,Tx,x)) \\ &\leq F((\alpha+\beta+\delta+\eta)\mathcal{S}(Tx,Tx,x)) \leq F(\mathcal{S}(Tx,Tx,x)), \end{aligned} \tag{29}$$

which is a contradiction. Hence $\mathcal{S}(Tx,Tx,x) = 0$ and so $Tx = x$. Consequently, $C_{x_0,r}^S$ is a fixed circle of T. By the similar arguments used in the proof of Theorem 11, T also fixes any circle $C_{x_0,\rho}^S$ where $\rho < r$. □

Corollary 19. *(1) Let $(X,\mathcal{S})$ be an S-metric space, T be a Hardy-Rogers type F_c^S-contractive self-mapping with $x_0 \in X$, and r be defined as in Theorem 11. If $\mathcal{S}(Tx,Tx,x_0) = r$ for all $x \in C_{x_0,r}^S$ then T fixes the disc $D_{x_0,r}^S$.*

(2) If we consider $\alpha = 1$ and $\beta = \gamma = \delta = \eta = 0$ in Definition 15, then we obtain the concept of an F_c^S-contractive mapping.

In Definition 15, if we get $\delta = \eta = 0$ then we have the following definition.

Definition 20. Let $(X,\mathcal{S})$ be an S-metric space and T be a self-mapping on X. If there exist $F \in \mathbb{F}$, $t > 0$, and $x_0 \in X$ such that for all $x \in X$ the following holds:

$$\begin{aligned} &\mathcal{S}(Tx,Tx,x) > 0 \Longrightarrow \\ &t + F(\mathcal{S}(Tx,Tx,x)) \leq F(\alpha\mathcal{S}(x,x,x_0) \\ &\quad + \beta\mathcal{S}(Tx,Tx,x) + \gamma\mathcal{S}(Tx_0,Tx_0,x_0)), \end{aligned} \tag{30}$$

where

$$\begin{gathered} \alpha+\beta+\gamma < 1 \\ \text{and } \alpha,\beta,\gamma \geq 0, \end{gathered} \tag{31}$$

then the self-mapping T is called a Reich type F_c^S-contraction on X.

Proposition 21. *Let $(X,\mathcal{S})$ be an S-metric space. If a self-mapping T on X is a Reich type F_c^S-contraction with $x_0 \in X$ then we get $Tx_0 = x_0$.*

Proof. The proof follows easily since $\beta + \gamma < 1$. □

Remark 22. Using Proposition 21, a Reich type F_c^S-contraction condition can be changed as follows:

$$\begin{aligned} &\mathcal{S}(Tx,Tx,x) > 0 \Longrightarrow \\ &t + F(\mathcal{S}(Tx,Tx,x)) \\ &\quad \leq F(\alpha\mathcal{S}(x,x,x_0) + \beta\mathcal{S}(Tx,Tx,x)), \end{aligned} \tag{32}$$

where

$$\begin{gathered} \alpha+\beta < 1 \\ \text{and } \alpha,\beta \geq 0. \end{gathered} \tag{33}$$

Theorem 23. *Let $(X,\mathcal{S})$ be an S-metric space, T be a Reich type F_c^S-contractive self-mapping with $x_0 \in X$, and r be defined as in Theorem 11. Then $C_{x_0,r}^S$ is a fixed circle of T. Also, T fixes every circle $C_{x_0,\rho}^S$ where $\rho < r$. In other words, T fixes the disc $D_{x_0,r}^S$.*

Proof. The proof follows easily since

$$\begin{aligned} F(r) &\leq F(\mathcal{S}(Tx,Tx,x)) \leq F((\alpha+\beta)\mathcal{S}(Tx,Tx,x)) \\ &< F(\mathcal{S}(Tx,Tx,x)). \end{aligned} \tag{34}$$

□

In Definition 15, if we get $\alpha = \beta = \gamma = 0$ and $\delta = \eta$, then we have the following definition.

Definition 24. Let $(X, \mathcal{S})$ be an S-metric space and T be a self-mapping on X. If there exist $F \in \mathbb{F}$, $t > 0$, and $x_0 \in X$ such that for all $x \in X$ the following holds:

$$\begin{aligned} &\mathcal{S}(Tx, Tx, x) > 0 \Longrightarrow \\ &t + F(\mathcal{S}(Tx, Tx, x)) \\ &\quad \leq F(\eta(\mathcal{S}(Tx_0, Tx_0, x) + \mathcal{S}(Tx, Tx, x_0))), \end{aligned} \tag{35}$$

where

$$\eta \in \left(0, \frac{1}{2}\right), \tag{36}$$

then the self-mapping T is called a Chatterjea type F_c^S-contraction on X.

Proposition 25. *Let $(X, \mathcal{S})$ be an S-metric space. If a self-mapping T on X is a Chatterjea type F_c^S-contraction with $x_0 \in X$ then we get $Tx_0 = x_0$.*

Proof. The proof follows easily. □

Theorem 26. *Let $(X, \mathcal{S})$ be an S-metric space, T be a Chatterjea type F_c^S-contractive self-mapping with $x_0 \in X$, and r be defined as in Theorem 11. If $\mathcal{S}(Tx, Tx, x_0) = r$ for all $x \in C_{x_0,r}^S$ then $C_{x_0,r}^S$ is a fixed circle of T. Also, T fixes every circle $C_{x_0,\rho}^S$ where $\rho < r$. In other words, T fixes the disc $D_{x_0,r}^S$.*

Proof. The proof follows easily by the similar arguments used in the proofs of Theorems 11 and 18. □

Now we give the following illustrative example.

Example 27. Let $\mathbb{C}$ be the set of all complex numbers. Consider the set

$$X_z = \{0, 4, z, z^2, z^4, z^8, z^8 - 2, z^8 + 2, z^{16}, z^{16} - 2, z^{16} + 2\} \subset \mathbb{C}, \tag{37}$$

where z is any complex number with $|z| = 2$ and the S–metric is defined as in [18] such that

$$\mathcal{S}(x, y, t) = |x - t| + |x + t - 2y|, \tag{38}$$

for all $x, y, t \in X_z$. Let us define the self-mapping $T : X_z \longrightarrow X_z$ as

$$Tx = \begin{cases} z & \text{if } x = 0 \\ x & \text{otherwise,} \end{cases} \tag{39}$$

for all $x \in X_z$. Then the self-mapping T is an F_c^S-contractive self-mapping with $F = -1/\sqrt{x}$, $t = 1/2^8$ and $x_0 = z^{16}$. Indeed, we obtain

$$\mathcal{S}(Tx, Tx, x) = 4 > 0, \tag{40}$$

for $x = 0$, and

$$\mathcal{S}(x, x, x_0) = 2^{17}. \tag{41}$$

Then we have

$$t + \mathcal{S}(Tx, Tx, x) = \frac{1}{2^8} - \frac{1}{2} \leq -\frac{1}{2^8\sqrt{2}}. \tag{42}$$

Also we obtain

$$r = \min\{\mathcal{S}(Tx, Tx, x) : Tx \neq x\} = 4. \tag{43}$$

Therefore, the self-mapping T fixes the circle $C_{z^{16},4}^S = \{z^{16} - 2, z^{16} + 2\}$ and the disc $D_{z^{16},4}^S = \{z^{16} - 2, z^{16}, z^{16} + 2\}$.

Also the self-mapping T is a Hardy-Rogers type F_c^S-contractive self-mapping (resp., a Reich type F_c^S-contractive self-mapping and a Chatterjea type F_c^S-contractive self-mapping) on X_z with $\alpha = 1$, $\beta = \delta = \eta = 0$ (resp., $\alpha = (2^{16} - 2^{14} + 2^8)/2^{17}(2^{14} - 2^8 + 1)$, $\beta = 1/4$ and $\eta = 5/(2^{17} + 4(1 - 2^{15}))$).

4. ψ-Contractive and Weakly ψ-Contractive Self-Mappings on S-Metric Spaces

First, in this section we present this well-known interesting class of functions.

Definition 28. Denote by Ψ the family of nondecreasing functions

$$\begin{gathered} \psi : [0, +\infty) \longrightarrow [0, +\infty) \\ \text{such that } \sum_{n=1}^{+\infty} \psi^n(t) < +\infty \quad \text{for each } t > 0, \end{gathered} \tag{44}$$

where ψ^n is the n-th iterate of ψ.

Lemma 29. *For every function $\psi : [0, +\infty) \longrightarrow [0, +\infty)$ the following holds: if ψ is nondecreasing, then, for each $t > 0$, $\lim_{n\longrightarrow+\infty}\psi^n(t) = 0$ implies that $\psi(t) < t$.*

Now, we define the ψ-contractive self-mapping in an S-metric space.

Definition 30. Let T be a self-mapping on an S-metric space $(X, \mathcal{S})$. We say that T is ψ-contractive self-mapping if there exist $x_0 \in X$ and $\psi \in \Psi$ such that for all $x, y, z \in X$ we have

$$\begin{aligned} &\mathcal{S}(Ty, Tz, x) \\ &\quad \leq \psi(\mathcal{S}(x, x, x_0)) \\ &\qquad - \min\{\psi(\mathcal{S}(Ty, Ty, x_0)), \psi(\mathcal{S}(Tz, Tz, x_0))\}. \end{aligned} \tag{45}$$

Theorem 31. *Let T be a ψ-contractive self-mapping with $x_0 \in X$ on an S-metric space $(X, \mathcal{S})$, and consider the circle $C_{x_0,r}^S$. Thus, for every $x \in C_{x_0,r}^S$, T either fixes x or maps x to the interior of $C_{x_0,r}^S$. Moreover, if for every $x \in C_{x_0,r}^S$ we have $\mathcal{S}(Tx, Tx, x_0) = r$, then $C_{x_0,r}^S$ is a unique fixed circle of T in X.*

Proof. If $x \in C^{S}_{x_0,r}$, then since T is ψ-contractive we have

$$\begin{aligned}\mathcal{S}(Tx,Tx,x) &\leq \psi(\mathcal{S}(x,x,x_0)) - \psi(\mathcal{S}(Tx,Tx,x_0)) \\ &= \psi(r) - \psi(\mathcal{S}(Tx,Tx,x_0)).\end{aligned} \tag{46}$$

If $\mathcal{S}(Tx,Tx,x_0) < r$, then we are in the case where T maps x to the interior of $C^{S}_{x_0,r}$. If $\mathcal{S}(Tx,Tx,x_0) \geq r$, then by using the fact that ψ is a nondecreasing function we have

$$\mathcal{S}(Tx,Tx,x) \leq \psi(r) - \psi(\mathcal{S}(Tx,Tx,x_0)). \tag{47}$$

Now, if $\mathcal{S}(Tx,Tx,x_0) > r$, then the above inequality implies that $\mathcal{S}(Tx,Tx,x) < 0$ which leads to a contradiction. Hence, in this case we must have $\mathcal{S}(Tx,Tx,x_0) = r$. Thus,

$$\begin{aligned}\mathcal{S}(Tx,Tx,x) &\leq \psi(r) - \psi(\mathcal{S}(Tx,Tx,x_0)) \\ &= \psi(r) - \psi(r) = 0,\end{aligned} \tag{48}$$

and that is $Tx = x$.

Therefore, T either fixes x or maps x to the interior of $C^{S}_{x_0,r}$ as required.

To prove the second part of our theorem, we may assume that $\mathcal{S}(Tx,Tx,x_0) = r$, for all $x \in C^{S}_{x_0,r}$. Now, we only need to show that if there exists $x \in X$ where $Tx = x$, then $x \in C^{S}_{x_0,r}$, and that will prove the uniqueness. So, first let $x \in C^{S}_{x_0,r}$, and that is $Tx = x$, and also let $y \in X$ be an arbitrary fixed point of T (i.e., $Ty = y$) we have two cases.

Case 1. If $\mathcal{S}(y,y,x_0) \geq r$ then by using the fact that ψ is a nondecreasing function we have

$$\begin{aligned}\mathcal{S}(y,y,x) &= \mathcal{S}(Ty,Ty,x) \\ &\leq \psi(r) - \psi(\mathcal{S}(Ty,Ty,x_0)).\end{aligned} \tag{49}$$

Now, if $\mathcal{S}(Ty,Ty,x_0) > r$ then the above inequality implies that $\mathcal{S}(y,y,x) < 0$ which leads to a contradiction. Hence, in this case we must have $\mathcal{S}(y,y,x_0) = r$.

$$\begin{aligned}\mathcal{S}(y,y,x) &= \mathcal{S}(Ty,Ty,x) \\ &\leq \psi(\mathcal{S}(x,x,x_0)) - \psi(\mathcal{S}(Ty,Ty,x_0)) \\ &= \psi(r) - \psi(r) = 0,\end{aligned} \tag{50}$$

and that is $x = y$.

Case 2. If $\mathcal{S}(y,y,x_0) < r$ then once again by using the fact that ψ is a nondecreasing function we have

$$\begin{aligned}\mathcal{S}(x,x,y) &\leq \psi(\mathcal{S}(y,y,x_0)) - \psi(\mathcal{S}(Tx,Tx,x_0)) \\ &= \psi(\mathcal{S}(y,y,x_0)) - \psi(\mathcal{S}(x,x,x_0)) \\ &= \psi(\mathcal{S}(y,y,x_0)) - \psi(r) < \psi(r) - \psi(r) \\ &= 0,\end{aligned} \tag{51}$$

which leads us to a contradiction.

Therefore, $C^{S}_{x_0,r}$ is the unique fixed circle of T in X as desired. □

Next, we give the definition of a weakly ψ-contractive self-mapping.

Definition 32. Let T be a self-mapping on an S-metric space $(X,\mathcal{S})$. We say that T is a weakly ψ-contractive self-mapping with $x_0 \in X$ if there exist $x_0 \in X$ and $\psi \in \Psi$ such that for all $x,y,z \in X$ we have

$$\begin{aligned}&\mathcal{S}(Ty,T^2z,x) \\ &\quad\leq \psi(\mathcal{S}(x,x,x_0)) \\ &\qquad - \min\{\psi(\mathcal{S}(Ty,Ty,x_0)),\psi(\mathcal{S}(Tz,Tz,x_0))\}.\end{aligned} \tag{52}$$

Theorem 33. *Let T be a weakly ψ-contractive self-mapping with $x_0 \in X$ on an S-metric space $(X,\mathcal{S})$ and consider the circle $C^{S}_{x_0,r}$. Thus, for every $x \in C^{S}_{x_0,r}$ T either fixes x or maps x to the interior of $C^{S}_{x_0,r}$. Moreover, if for every $x \in C^{S}_{x_0,r}$, we have $\mathcal{S}(Tx,Tx,x_0) = r$, then $C^{S}_{x_0,r}$ is a unique fixed circle of T in X.*

Proof. If $x \in C^{S}_{x_0,r}$, then since T is weakly ψ-contractive we have

$$\begin{aligned}\mathcal{S}(Tx,T^2x,x) &\leq \psi(\mathcal{S}(x,x,x_0)) \\ &\quad - \psi(\mathcal{S}(Tx,Tx,x_0)) \\ &= \psi(r) - \psi(\mathcal{S}(Tx,Tx,x_0)).\end{aligned} \tag{53}$$

If $\mathcal{S}(Tx,Tx,x_0) < r$, then we are in the case where T maps x to the interior of $C^{S}_{x_0,r}$. If $\mathcal{S}(Tx,Tx,x_0) \geq r$, then by using the fact that ψ is a nondecreasing function we have

$$\mathcal{S}(Tx,T^2x,x) \leq \psi(r) - \psi(\mathcal{S}(Tx,Tx,x_0)). \tag{54}$$

Now, if $\mathcal{S}(Tx,Tx,x_0) > r$, then the above inequality implies that $\mathcal{S}(Tx,T^2x,x) < 0$ which leads to a contradiction. Hence, in this case we must have $\mathcal{S}(Tx,Tx,x_0) = r$. Thus,

$$\begin{aligned}\mathcal{S}(Tx,T^2x,x) &\leq \psi(r) - \psi(\mathcal{S}(Tx,Tx,x_0)) \\ &= \psi(r) - \psi(r) = 0,\end{aligned} \tag{55}$$

and that is $Tx = x$.

Therefore, T either fixes x or maps x to the interior of $C^{S}_{x_0,r}$ as required.

To prove the second part of our theorem, we may assume that $\mathcal{S}(Tx,Tx,x_0) = r$, for all $x \in C^{S}_{x_0,r}$. Now, we only need to show that if there exists $x \in X$, where $Tx = x$, then $x \in C^{S}_{x_0,r}$, and that will prove the uniqueness. So, first let $x \in C^{S}_{x_0,r}$, and that is $Tx = x$, and also let $y \in X$ be an arbitrary fixed point (i.e., $Ty = y$) we have two cases.

Case 1. If $\mathcal{S}(y,y,x_0) \geq r$ then by using the fact that ψ is a nondecreasing function we have

$$\begin{aligned}\mathcal{S}(y,y,x) &= \mathcal{S}(Ty,T^2y,x) \\ &\leq \psi(r) - \psi(\mathcal{S}(Ty,Ty,x_0)).\end{aligned} \tag{56}$$

Now, if $\mathcal{S}(Ty, Ty, x_0) > r$, then the above inequality implies that $\mathcal{S}(y, y, x) < 0$ which leads to a contradiction. Hence, in this case we must have $\mathcal{S}(y, y, x_0) = r$.

$$\begin{aligned} \mathcal{S}(y, y, x) &= \mathcal{S}(Ty, T^2 y, x) \\ &\leq \psi(\mathcal{S}(x, x, x_0)) - \psi(\mathcal{S}(Ty, Ty, x_0)) \\ &= \psi(r) - \psi(r) = 0, \end{aligned} \tag{57}$$

and that is $x = y$.

Case 2. If $\mathcal{S}(y, y, x_0) < r$ then once again by using the fact that ψ is a nondecreasing function we have

$$\begin{aligned} \mathcal{S}(x, x, y) &= \mathcal{S}(Tx, T^2 x, y) \\ &\leq \psi(\mathcal{S}(y, y, x_0)) - \psi(\mathcal{S}(Tx, Tx, x_0)) \\ &= \psi(\mathcal{S}(y, y, x_0)) - \psi(\mathcal{S}(x, x, x_0)) \\ &= \psi(\mathcal{S}(y, y, x_0)) - \psi(r) < \psi(r) - \psi(r) \\ &= 0, \end{aligned} \tag{58}$$

which leads us to a contradiction.

Therefore, $C^S_{x_0,r}$ is the unique fixed circle of T in X as desired. □

5. An Application to Integral Type Contractive Self-Mappings

We assume that $\varphi : [0,\infty) \longrightarrow [0,\infty)$ is a Lebesgue-integrable mapping which is summable (that is, with finite integral) on each compact subset of $[0,\infty)$, nonnegative, and such that, for each $\varepsilon > 0$,

$$\int_0^{\varepsilon} \varphi(t)\, dt > 0. \tag{59}$$

Now we give the following definition.

Definition 34. Let $(X, \mathcal{S})$ be an S-metric space and $\varphi : [0,\infty) \longrightarrow [0,\infty)$ be defined as in (59). A self-mapping T on X is said to be an integral type F^S_c-contraction if there exist $F \in \mathbb{F}$, $t > 0$, and $x_0 \in X$ such that for all $x \in X$ the following holds:

$$\begin{aligned} &\mathcal{S}(Tx, Tx, x) > 0 \Longrightarrow \\ &t + \int_0^{F(\mathcal{S}(Tx,Tx,x))} \varphi(t)\, dt \leq \int_0^{F(\mathcal{S}(x,x,x_0))} \varphi(t)\, dt. \end{aligned} \tag{60}$$

Proposition 35. *Let $(X, \mathcal{S})$ be an S-metric space and $\varphi : [0,\infty) \longrightarrow [0,\infty)$ be defined as in (59). If a self-mapping T on X is an integral type F^S_c-contraction with $x_0 \in X$ then we get $Tx_0 = x_0$.*

Proof. Suppose that $Tx_0 \neq x_0$. From the definition of an integral type F^S_c-contraction, we have

$$\begin{aligned} &\mathcal{S}(Tx_0, Tx_0, x_0) > 0 \Longrightarrow \\ &t + \int_0^{F(\mathcal{S}(Tx_0,Tx_0,x_0))} \varphi(t)\, dt \leq \int_0^{F(\mathcal{S}(x_0,x_0,x_0))} \varphi(t)\, dt. \end{aligned} \tag{61}$$

Inequality (61) contradicts with the definition of F since $F : (0,\infty) \longrightarrow \mathbb{R}$ and $\mathcal{S}(x_0, x_0, x_0) = 0$. Hence, it should be $Tx_0 = x_0$. □

Using this new definition, we get the following fixed-circle result.

Theorem 36. *Let $(X, \mathcal{S})$ be an S-metric space, $\varphi : [0,\infty) \longrightarrow [0,\infty)$ be defined as in (59), T be an integral type F^S_c-contraction with $x_0 \in X$, and r be defined as in Theorem 11. Then $C^S_{x_0,r}$ is a fixed circle of T.*

Proof. Let $x \in C^S_{x_0,r}$. Assume that $Tx \neq x$. Then, by the definition of r, we get

$$r \leq \mathcal{S}(Tx, Tx, x). \tag{62}$$

Using the fact that F is increasing property, we have

$$F(r) \leq F(\mathcal{S}(Tx, Tx, x)) \tag{63}$$

and

$$\int_0^{F(r)} \varphi(t)\, dt \leq \int_0^{F(\mathcal{S}(Tx,Tx,x))} \varphi(t)\, dt. \tag{64}$$

From inequality (64) and the definition of integral type F^S_c-contractivity, we obtain

$$\begin{aligned} \int_0^{F(r)} \varphi(t)\, dt &\leq \int_0^{F(\mathcal{S}(Tx,Tx,x))} \varphi(t)\, dt \\ &\leq \int_0^{F(\mathcal{S}(x,x,x_0))} \varphi(t)\, dt - t \\ &< \int_0^{F(\mathcal{S}(x,x,x_0))} \varphi(t)\, dt = \int_0^{F(r)} \varphi(t)\, dt, \end{aligned} \tag{65}$$

which is a contradiction. Therefore, we find $Tx = x$. Consequently, $C^S_{x_0,r}$ is a fixed circle of T. □

Remark 37. (1) An integral type F^S_c-contractive self-mapping T fixes also the disc $D^S_{x_0,r}$.

(2) If we set the function $\varphi : [0,\infty) \longrightarrow [0,\infty)$ in Theorem 36 as $\varphi(t) = 1$ for all $t \in [0,\infty)$, then we get Theorem 11.

(3) By the similar argument used in Definition 34, the notions of an integral Hardy-Rogers type F^S_c-contractive self-mapping, an integral Reich type F^S_c-contractive self-mapping, an integral Chatterjea type F^S_c-contractive self-mapping, and obtained corresponding fixed-circle theorems can be defined.

Finally, we give the following example.

Example 38. Let $X = \{e, 2e, e+1/2, 2e-1/2, 2e+1/2\} \subset \mathbb{R}$ be the S-metric space with the usual S-metric and the function $\varphi : [0,\infty) \longrightarrow [0,\infty)$ be defined by

$$\varphi(t) = 2t + 1, \tag{66}$$

for all $t \in [0,\infty)$. Let us define the self-mapping $T : X \longrightarrow X$ as

$$Tx = \begin{cases} e + \dfrac{1}{2} & \text{if } x = e \\ x & \text{otherwise,} \end{cases} \tag{67}$$

for all $x \in X$. The self-mapping T is an integral type F_c^S-contractive self-mapping with $F = \ln x$, $t = 1$, and $x_0 = 2e$. Indeed, we get

$$\begin{aligned} \mathcal{S}(Tx, Tx, x) &= \mathcal{S}\left(e + \frac{1}{2}, e + \frac{1}{2}, e\right) = 2\left|e + \frac{1}{2} - e\right| \\ &= 1 > 0, \end{aligned} \tag{68}$$

for $x = e$. Then we have

$$\begin{gathered} \mathcal{S}(Tx, Tx, x) = 1 < \mathcal{S}(x, x, x_0) = 2e \Longrightarrow \\ \ln 1 = 0 < \ln(2e) = \ln 2 + 1 \Longrightarrow \\ \int_0^0 (2t+1)\,dt = 0 < \int_0^{\ln 2+1} (2t+1)\,dt \\ = \ln^2 2 + \ln 8 + 2 \Longrightarrow \\ 1 \le \ln^2 2 + \ln 8 + 2. \end{gathered} \tag{69}$$

Also we obtain

$$r = \min\{\mathcal{S}(Tx, Tx, x) : Tx \neq x\} = 1. \tag{70}$$

Consequently, T fixes the circle $C_{2e,1}^S = \{2e - 1/2, 2e + 1/2\}$ and the disc $D_{2e,1}^S = \{2e - 1/2, 2e, 2e + 1/2\}$.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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