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Research Article

Julia and Mandelbrot Sets of Transcendental Function via Fibonacci-Mann Iteration

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In this paper, utilizing the Fibonacci-Mann iteration process, we explore Julia and Mandelbrot sets by establishing the escape criteria of a transcendental function, $\sin(z^n) + az + c$, $n \geq 2$; here, z is a complex variable, and a and c are complex numbers. Also, we explore the effect of involved parameters on the deviance of color, appearance, and dynamics of generated fractals. It is well known that fractal geometry portrays the complexity of numerous complicated shapes in our surroundings. In fact, fractals can illustrate shapes and surfaces which cannot be described by the traditional Euclidean geometry.

1. Introduction and Preliminaries

Let us consider the well-known Fibonacci sequence $\{f(n)\}$ defined recursively by

$$f(n+1) = f(n) + f(n-1), n \geq 1, \quad (1)$$

with the initial conditions $f(0) = f(1) = 1$. Recently, a novel iteration process, Fibonacci-Mann iteration, is introduced as

$$z_{n+1} = t_n T^{f(n)}(z_n) + (1 - t_n) z_n, \quad (2)$$

where $t_n \in [0, 1]$ and $n \in \mathbb{N}$ (see [1] for more details). It is worth mentioning here that a fixed point iteration performs a significant role in the generation of geometrical pictures of classical Julia and Mandelbrot sets (for instance, see [2–4], and the references therein). In [2], by establishing the escape criteria for a complex function

$$T(z) = \sin(z^n) + az + c, (n \geq 2), \quad (3)$$

where z is a complex variable and a and c are complex numbers; new Julia sets were studied by providing new algorithms for exploring Julia sets utilizing four distinct iterations (the Picard iteration [5], the Mann iteration [6], the Ishikawa iteration [7], and the Noor-iteration [8]). Also, the effects of change in values of parameters on the deviance of color appearance and dynamics of fractals were investigated in the sequel.

Motivated by these recent studies, our aim in this paper is to develop escape criteria for a function of the form (3) using a new algorithm via the Fibonacci-Mann iteration process (2) for visualizing the stunning fractals. It is well known that the escape criterion [9] is indispensable for exploring the Mandelbrot and Julia sets. We furnish some graphical illustrations of the generated complex fractals using the MATLAB software, algorithm, and colormap to demonstrate the variation in images and explore the effect of the involved parameters on the deviance of color, appearance, and dynamics of generated fractals. Also, we observe that as we zoom in at the edges of the petals of the Mandelbrot set, we come across the Julia set meaning thereby each point of the Mandelbrot set includes massive image data of a Julia set.

A filled Julia set is the set of complex numbers so that the orbits do not converge to a point at infinity ([10, 11]). For the polynomial $T : \mathbb{C} \longrightarrow \mathbb{C}$ of degree ≥2, we denote it by F_T , that is,

$$F_T = \left\{ z \in \mathbb{C} : \{|T(z_k)|\}_{k=0}^{\infty} \text{is bounded} \right\}. \tag{4}$$

The boundary of J_T is the Julia set; that is, $J_T = \partial F_T$.

The set of parameters $c \in \mathbb{C}$ so that the filled Julia set J_{T_c} of the polynomial $T_c(z) = z^2 + c$ is connected is known as the Mandelbrot set ([12, 13]), that is,

$$M = \left\{ c \in \mathbb{C} : J_{T_c} \text{is connected} \right\}, \tag{5}$$

or

$$M = \{ c \in \mathbb{C} : \{|T_c(z_k)|\} \nrightarrow \infty \text{ as } k \longrightarrow \infty \}. \tag{6}$$

2. An Escape Criteria via Fibonacci-Mann Iteration Process

In this section, we establish an escape criterion for the complex transcendental function (3). We take $x_0 = x$, $y_0 = y$, $z_0 = z$, and $T(z)$ as $T_{a,c}(z)$. Suppose that

$$\begin{aligned} \left| 1 - \frac{z^{2n}}{3!} + \frac{z^{4n}}{5!} - \cdots \right| &\geq |u_1|, \\ \left| 1 - \frac{y^{2n}}{3!} + \frac{y^{4n}}{5!} - \cdots \right| &\geq |u_2|, \\ \left| 1 - \frac{x^{2n}}{3!} + \frac{x^{4n}}{5!} - \cdots \right| &\geq |u_3|, \end{aligned} \tag{7}$$

where $|u_i| \in (0, 1]$, $1 \leq i \leq 3$ except for the values of x, y, and z so that $|u_1| = |u_2| = |u_3| = 0$. Then, we have

$$|\sin(z^n)| = \left| z^n - \frac{z^{3n}}{3!} + \frac{z^{5n}}{5!} - \cdots \right| = |z^n| \left| 1 - \frac{z^{2n}}{3!} + \frac{z^{4n}}{5!} - \cdots \right|, \tag{8}$$

and so

$$|\sin(z^n)| \geq |z^n||u_1|, \tag{9}$$

$z \in \mathbb{C}$ except for the value of z so that $|u_1| = 0$, $|u_1| \in (0, 1]$.

Theorem 1. *Let $T_{a,c}(z) = \sin(z^n) + az + c$, $n \geq 2$, $a, c \in \mathbb{C}$, and the sequence of iterates $\{z_k\}_{k \in \mathbb{N}}$ be the Fibonacci-Mann iteration. Suppose $\mathfrak{t} = \inf \{t_n\} > 0$ and*

$$|z| \geq |c| > \left(\frac{2(\mathfrak{T}|a| + 1)}{\mathfrak{t}|u_1|} \right)^{1/(n-1)}, \tag{10}$$

where $\mathfrak{T} = \sup \{t_n\}$. Then, we have $|z_k| \longrightarrow \infty$ as $k \longrightarrow \infty$.

Proof. Let $z_0 = z$, $T_{a,c}(z) = \sin(z^n) + az + c$. Now

$$|z_{k+1}| = \left| t_k T_{a,c}^{f(k)}(z_k) + (1 - t_k) z_k \right|. \tag{11}$$

For $k = 0$, since we have $|z| \geq |c|$ and $f(0) = 1$, considering inequality (9), we get

$$\begin{aligned} |z_1| &= \left| t_0 T_{a,c}^{f(0)}(z) + (1 - t_0) z \right| = |t_0[\sin(z^n) + az + c] \\ &\quad + (1 - t_0) z| \geq t_0 |\sin(z^n)| - t_0|az| - t_0|c| - (1 - t_0)|z| \\ &= t_0 |\sin(z^n)| - t_0|a||z| - t_0|c| - (1 - t_0)|z| \geq t_0|u_1||z^n| \\ &\quad - t_0|a||z| - t_0|z| - (1 - t_0)|z| = t_0|u_1||z^n| - |z|(t_0|a| \\ &\quad + t_0 + (1 - t_0)) = t_0|u_1||z^n| - |z|(t_0|a| + 1) \geq \mathfrak{t}|u_1||z^n| \\ &\quad - |z|(\mathfrak{T}|a| + 1) \geq |z|(\mathfrak{T}|a| + 1) \left(\frac{\mathfrak{t}|u_1||z^{n-1}|}{\mathfrak{T}|a| + 1} - 1 \right). \end{aligned} \tag{12}$$

Hence, we obtain

$$|z_1| \geq \frac{|z_1|}{\mathfrak{T}|a| + 1} \geq |z| \left(\frac{\mathfrak{t}|u_1||z^{n-1}|}{\mathfrak{T}|a| + 1} - 1 \right). \tag{13}$$

Let $k = 1$. Since $f(1) = 1$, following similar steps and using the inequality (13), we obtain

$$\begin{aligned} |z_2| &\geq |z_1| \left(\frac{\mathfrak{t}|u_1||z_1^{n-1}|}{\mathfrak{T}|a| + 1} - 1 \right) \geq |z| \left(\frac{\mathfrak{t}|u_1||z^{n-1}|}{\mathfrak{T}|a| + 1} - 1 \right) \\ &\quad \cdot \left(\frac{\mathfrak{t}|u_1||z_1^{n-1}|}{\mathfrak{T}|a| + 1} - 1 \right) \geq |z| \left(\frac{\mathfrak{t}|u_1||z^{n-1}|}{\mathfrak{T}|a| + 1} - 1 \right)^2, \end{aligned} \tag{14}$$

and so

$$|z_2| \geq |z| \left(\frac{\mathfrak{t}|u_1||z^{n-1}|}{\mathfrak{T}|a| + 1} - 1 \right)^2. \tag{15}$$

Because, by inequality (13) and the fact that $|z| \geq |c| > (2(\mathfrak{T}|a| + 1)/\mathfrak{t}|u_1|)^{1/(n-1)}$, it is easy to see that $|z_1| \geq |z|$, and this implies

$$|z_1| \left(\frac{\mathfrak{t}|u_1||z_1^{n-1}|}{\mathfrak{T}|a| + 1} - 1 \right) \geq |z| \left(\frac{\mathfrak{t}|u_1||z^{n-1}|}{\mathfrak{T}|a| + 1} - 1 \right). \tag{16}$$

Again, using the inequality $|z_1| \geq |z| \geq |c| > (2(\mathfrak{T}|a| + 1)/\mathfrak{t}|u_1|)^{1/(n-1)}$ and (14), we find $|z_2| \geq |z_1|$.

Let $k = 2$ and set $\omega_1 = T_{a,c}(z_2)$. By inequality (10), it is easy to see that

$$|z_2^{n-1}||u_1| \geq |a| + 2. \tag{17}$$

Input: $T(z) = \sin(z^n) + az + c$, where $a, c \in \mathbb{C}$ and $n = 2, 3, \cdots$; $A \subset \mathbb{C}-$ area; $K-$maximum number of iterations; $t_n, u_1 \in (0, 1]-$ Parameters of the generalized Fibonacci-Mann iteration; *colormap*$[0..C-1]$ -color map with C colors.
Output: Julia set for area A.

for $z \in A$ **do**
  $R_1 = (2(\mathfrak{T}|a|+1)/\mathfrak{t}|u_1|)^{1/n-1}$
  $R = \max(|c|, R_1)$
  $n \geq 1$
  $z = 0$
  while $n \leq K$ **do**
    $f(0) = 1$
    $f(1) = 1$
    $f(n+1) = f(n) + f(n-1)$
    $z_{n+1} = t_n T^{f(n)}(z_n) + (1-t_n)z_n$
    if $|z_{n+1}| > R$ **then**
      break
    end if
    $n = n + 1$
  end while
  $i = \lfloor (C-1)(n/K) \rfloor$
  color z with *colormap*$[i]$
end for

Algorithm 1:Geometry of Julia set.

Input: $T(z) = \sin(z^n) + az + c$, where $a, c \in \mathbb{C}$ and $n = 2, 3, \cdots$; $A \subset \mathbb{C}-$ area; $K-$maximum number of iterations; $t_n, u_1 \in (0, 1]-$ Parameters of the generalized Fibonacci-Mann iteration; *colormap*$[0..C-1]$ -color map with C colors.
Output: Mandelbrot set for area A.

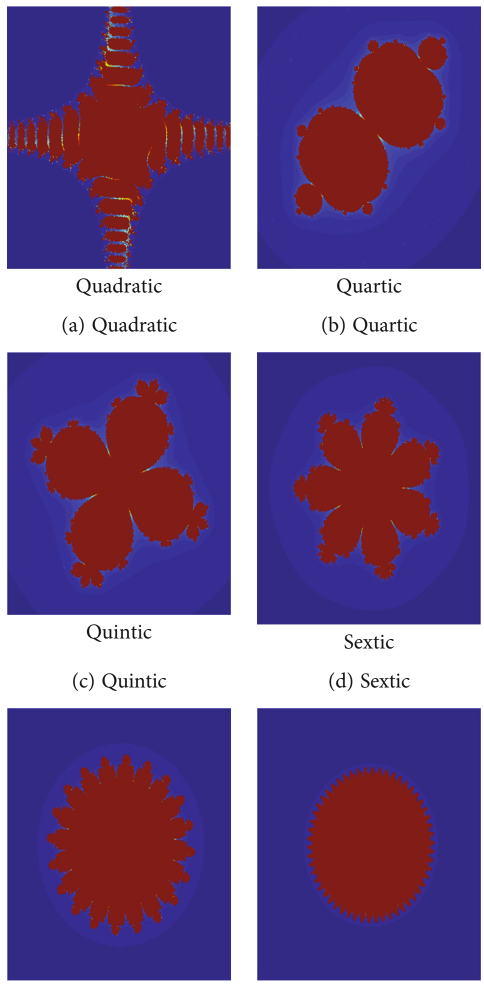
for $c \in A$ **do**
  $R_1 = (2(\mathfrak{T}|a|+1)/\mathfrak{t}|u_1|)^{1/n-1}$
  $R = \max(|c|, R_1)$
  $n \geq 1$
  while $n \leq K$ **do**
    $f(0) = 1$
    $f(1) = 1$
    $f(n+1) = f(n) + f(n-1)$
    $z_{n+1} = t_n T^{f(n)}(z_n) + (1-t_n)z_n$
    if $|z_{n+1}| > R$ **then**
      break
    end if
    $n = n + 1$
  end while
  $i = \lfloor (C-1)(n/K) \rfloor$
  color z with *colormap*$[i]$
end for

Algorithm 2: Geometry of Mandelbrot set.

Figure 1: Colormap used in the graphical examples.

Table 1: Parameters for generation of Julia set for different values of n.

	a	c	t	T	t	u_1	n
(i)	$19i$	$-0.835-0.2321i$	0.0009	0.1	0.1	0.2	2
(ii)	$19i$	$-0.835-0.2321i$	0.0009	0.1	0.1	0.2	4
(iii)	$19i$	$-0.835-0.2321i$	0.0009	0.1	0.1	0.2	5
(iv)	$19i$	$-0.835-0.2321i$	0.0009	0.1	0.1	0.2	6
(v)	$19i$	$-0.835-0.2321i$	0.0009	0.1	0.1	0.2	8
(vi)	$19i$	$-0.835-0.2321i$	0.0009	0.1	0.1	0.2	10



(a) Quadratic (b) Quartic (c) Quintic (d) Sextic (e) Octic (f) Decic

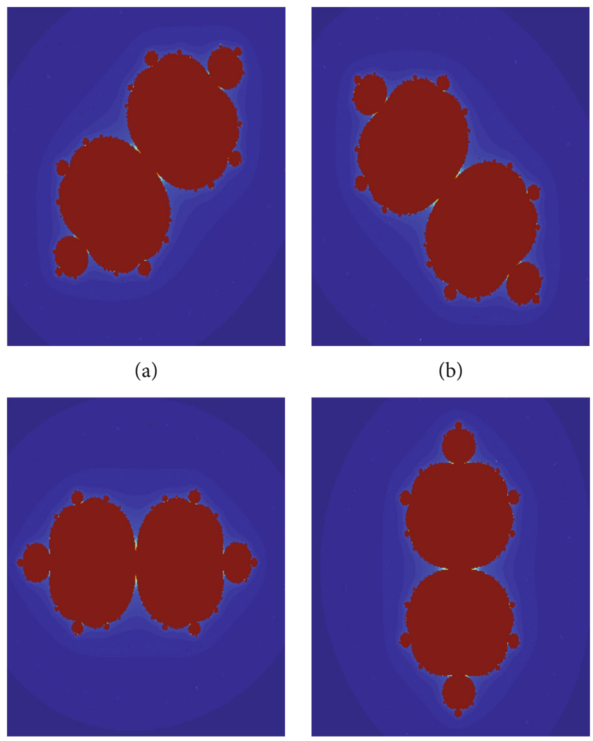
Figure 2: Effect of n on Julia set.

Using this last inequality and inequality (9), we get

$$\frac{|\omega_1|}{|z_2|} = \frac{|\sin(z_2^n) + az_2 + c|}{|z_2|} \geq \frac{|\sin(z_2^n)| - |a||z_2| - |c|}{|z_2|} \geq \frac{|z_2^n||u_1| - |a||z_2| - |z_2|}{|z_2|} \geq |z_2^{n-1}||u_1| - |a| - 1 \geq 1, \tag{18}$$

TABLE 2: Parameters for generation of quartic Julia set for different values of a.

	a	c	t	T	t	u_1	n
(i)	$19i$	$-0.835-0.2321i$	0.0009	0.1	0.1	0.2	4
(ii)	$-19i$	$-0.835-0.2321i$	0.0009	0.1	0.1	0.2	4
(iii)	-19	$-0.835-0.2321i$	0.0009	0.1	0.1	0.2	4
(iv)	19	$-0.835-0.2321i$	0.0009	0.1	0.1	0.2	4

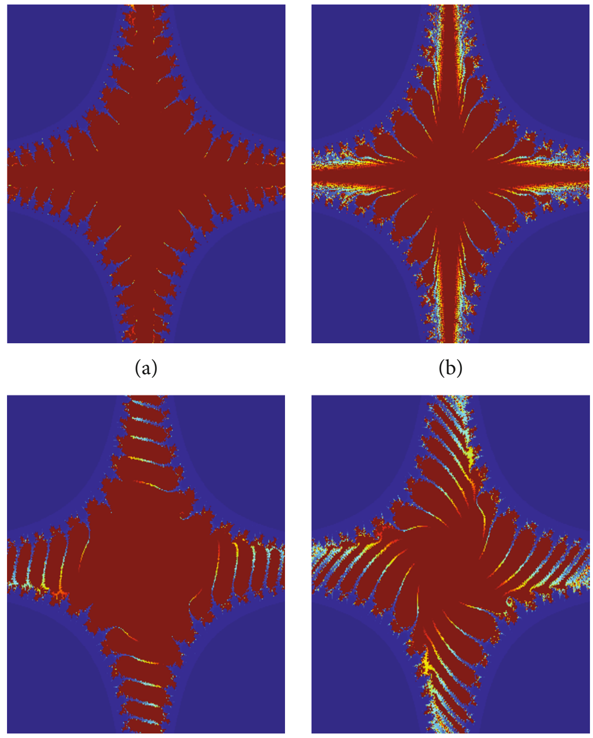


(a) (b) (c) (d)

FIGURE 3: Effect of change in sign in the real and complex parameter a of quartic Julia set.

TABLE 3: Parameters for generation of quadratic Julia set for different values of a.

	a	c	t	T	t	u_1	n
(i)	10	3.14	0.00029901	0.0105	0.0105	0.9	2
(ii)	20	3.14	0.00029901	0.0105	0.0105	0.9	2
(iii)	$-10+50i$	3.14	0.00029901	0.0105	0.0105	0.9	2
(iv)	$50-50i$	3.14	0.00029901	0.0105	0.0105	0.9	2

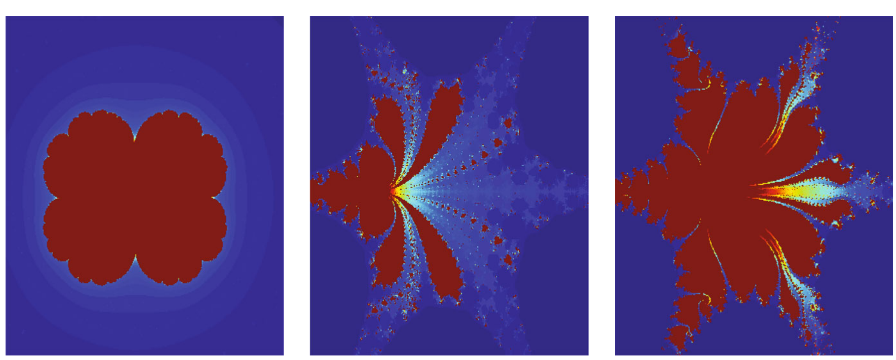


(a) (b) (c) (d)

Figure 4: Effect of increase in the absolute value of a on quadratic Julia set.

Table 4: Parameters for generation of cubic Julia set for different values of a and c.

	a	c	t	T	t	u_1	n
(i)	$40-40i$	$-3.25+3.50i$	0.0019990914	0.0191	0.0191	0.012	3
(ii)	5.7	7.5	0.0019990914	0.0191	0.0191	0.012	3
(iii)	1.8	2.718	0.0019990914	0.0191	0.0191	0.012	3

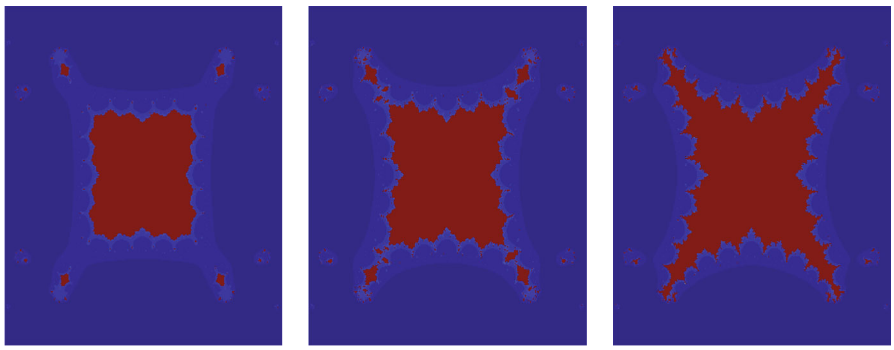


(a) (b) (c)

Figure 5: Effect of decrease in the absolute value of parameters a and c simultaneously on cubic Julia set.

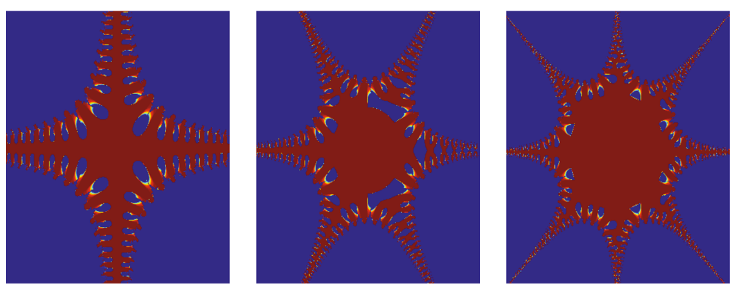
Table 5: Parameters for generation of quintic Julia set for different values of t.

	a	c	t	T	t	u_1	n
(i)	2.2	0.0035	0.35	0.115025	0.115025	0.92	5
(ii)	2.2	0.0035	0.25	0.115025	0.115025	0.92	5
(iii)	2.2	0.0035	0.20	0.115025	0.115025	0.92	5



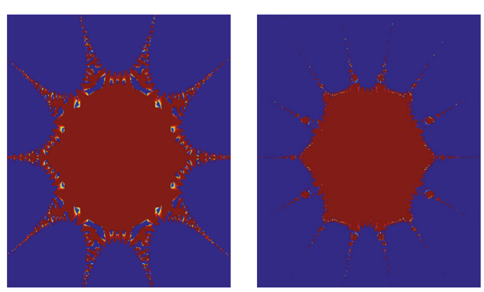
(a) (b) (c)

Figure 6: Effect of decrease in parameter t on quintic Julia set.



Quadratic Cubic Quartic

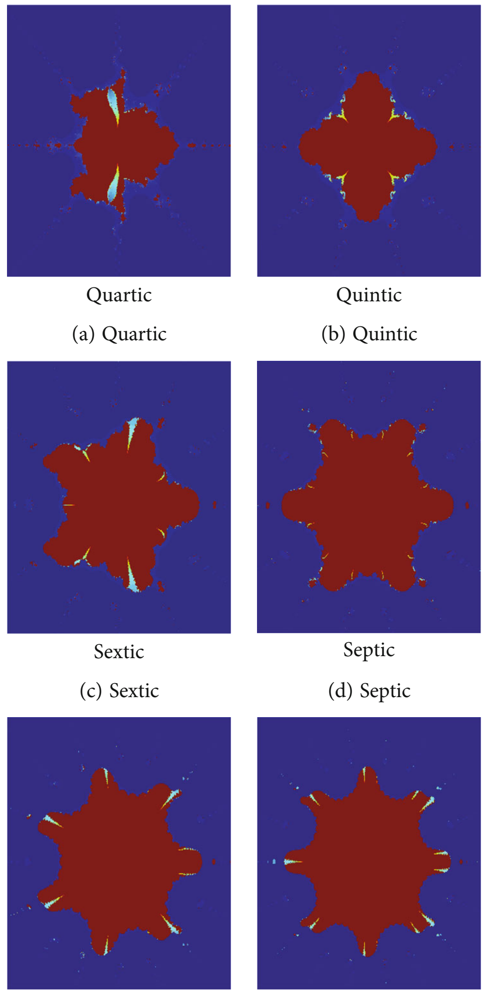
(a) Quadratic (b) Cubic (c) Quartic



Quintic Septic

(d) Quintic (e) Septic

Figure 7: Effect of n on Mandelbrot set.



(a) Quartic (b) Quintic (c) Sextic (d) Septic (e) Octic (f) Nonic

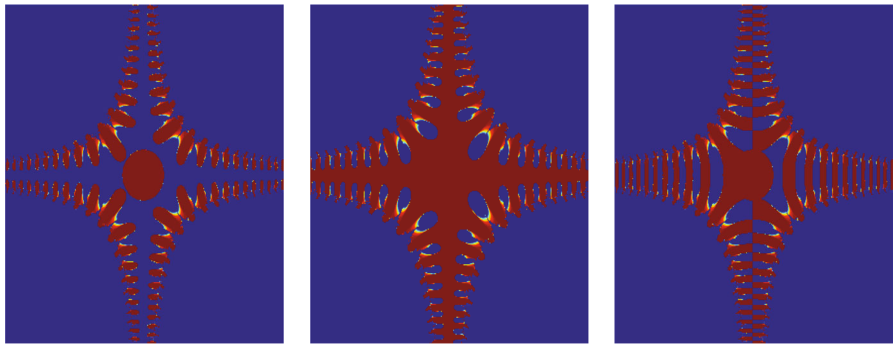
Figure 8: Effect of change in n on Mandelbrot set.

Table 6: Parameters for generation of Mandelbrot set for different values of n.

	a	t	T	t	u_1	n
(i)	-1.87897	0.000026	0.2105	0.2105	0.0932	2
(ii)	-1.87897	0.000026	0.2105	0.2105	0.0932	3
(iii)	-1.87897	0.000026	0.2105	0.2105	0.0932	4
(iv)	-1.87897	0.000026	0.2105	0.2105	0.0932	5
(v)	-1.87897	0.000026	0.2105	0.2105	0.0932	7

Table 7: Parameters for generation of Mandelbrot set for different values of n.

	a	t	T	t	u_1	n
(i)	-2.2	0.1593911	0.115025	0.115025	0.92	4
(ii)	-2.2	0.1593911	0.115025	0.115025	0.92	5
(iii)	-2.2	0.1593911	0.115025	0.115025	0.92	6
(iv)	-2.2	0.1593911	0.115025	0.115025	0.92	7
(v)	-2.2	0.1593911	0.115025	0.115025	0.92	8
(vi)	-2.2	0.1593911	0.115025	0.115025	0.92	9



(a) (b) (c)

Figure 9: Effect of change in sign as well change in real to complex parameter a on quadratic Mandelbrot set.

Table 8: Parameters for generation of quadratic Mandelbrot set for different values of a.

	a	t	T	t	u_1	n
(i)	1.87897	0.000026	0.2105	0.2105	0.0932	2
(ii)	-1.87897	0.000026	0.2105	0.2105	0.0932	2
(iii)	1.87897i	0.000026	0.2105	0.2105	0.0932	2

Table 9: Parameters for generation of cubic Mandelbrot set for different values of t.

	a	t	T	t	u_1	n
(i)	-2.2	0.059	0.115025	0.115025	0.92	3
(ii)	-2.2	0.1593911	0.115025	0.115025	0.92	3
(iii)	-2.2	0.91	0.115025	0.115025	0.92	3

and this implies

$$|\omega_1| \geq |z_2|. \quad (19)$$

Since $f(2) = 2$, we have

$$\begin{aligned} |z_3| &= \left| t_2 T_{a,c}^{f(2)}(z_2) + (1-t_2)z_2 \right| = |t_2[\sin(\omega_1^n) + a\omega_1 + c] \\ &\quad + (1-t_2)z_2| \geq t_2|\sin(\omega_1^n)| - t_2|a\omega_1| - t_2|c| - (1-t_2)|z_2| \\ &= t_2|\sin(\omega_1^n)| - t_2|a||\omega_1| - t_2|c| - (1-t_2)|z_2| \geq t_2|u_1||\omega_1^n| \\ &\quad - t_2|a||\omega_1| - t_2|\omega_1| - (1-t_2)|\omega_1| \geq t_2|u_1||\omega_1^n| \\ &\quad - |\omega_1|(t_2|a|+1) \geq \mathfrak{t}|u_1||\omega_1^n| \\ &\quad - |\omega_1|(\mathfrak{T}|a|+1) \geq |\omega_1|(\mathfrak{T}|a|+1)\left(\frac{\mathfrak{t}|u_1||\omega_1^{n-1}|}{(\mathfrak{T}|a|+1)} - 1\right), \end{aligned} \quad (20)$$

and hence,

$$|z_3| \geq \frac{|z_3|}{\mathfrak{T}|a|+1} \geq |\omega_1|\left(\frac{\mathfrak{t}|u_1||\omega_1^{n-1}|}{\mathfrak{T}|a|+1} - 1\right). \quad (21)$$

Similarly, by inequalities (15), (19), and (21), we get

$$|z_3| \geq |z|\left(\frac{\mathfrak{t}|u_1||z^{n-1}|}{\mathfrak{T}|a|+1} - 1\right)^3. \quad (22)$$

Repeating this process till kth term, we find

$$|z_k| \geq |z|\left(\frac{\mathfrak{t}|u_1||z^{n-1}|}{\mathfrak{T}|a|+1} - 1\right)^k. \quad (23)$$

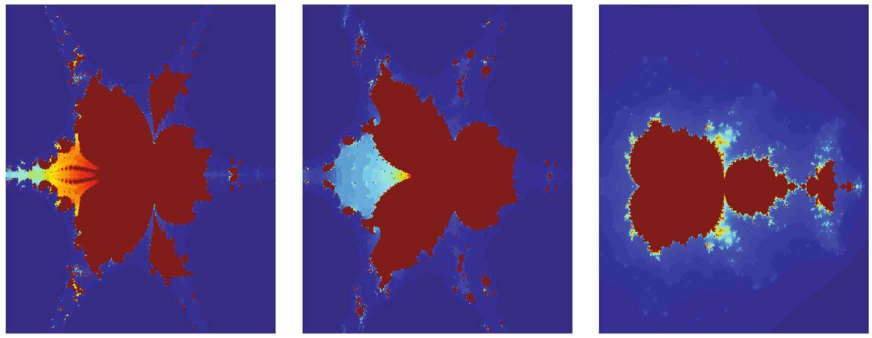
Then, because of inequality (10), we have

$$\frac{\mathfrak{t}|u_1||z^{n-1}|}{\mathfrak{T}|a|+1} - 1 > 1, \quad (24)$$

where $|u_1| \in (0, 1]$. This implies that the orbit of z tends to infinity; that is, we find $|z_k| \longrightarrow \infty$ as $k \longrightarrow \infty$. □

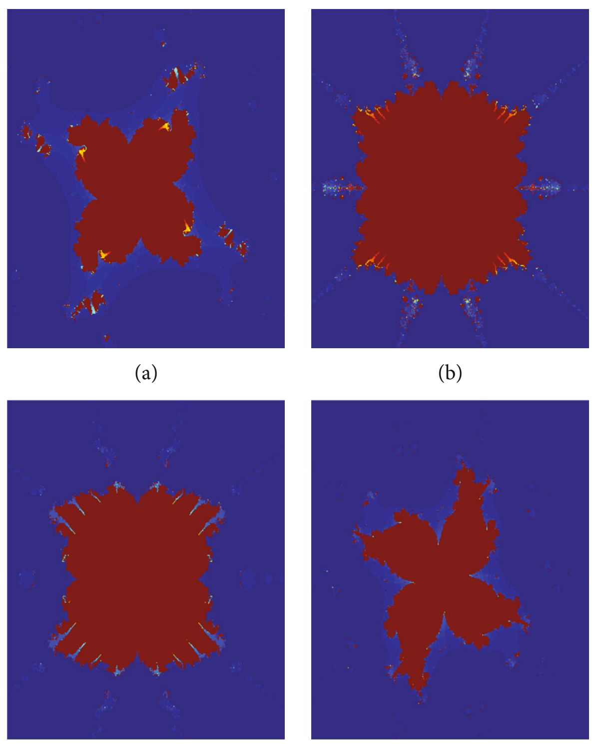
Corollary 2. *If we consider* $|c| > (2(\mathfrak{T}|a|+1)/\mathfrak{t}|u_1|)^{1/(n-1)}$, *then the Fibonacci-Mann orbit escapes to infinity.*

Remark 3. The motivation for choosing the Fibonacci-Mann iteration method in the generation of Julia and Mandelbrot fractal sets is the fact that for $t_n \in (0, .5]$, both Mann



(a) (b) (c)

Figure 10: Effect of change in parameter t on cubic Mandelbrot set.



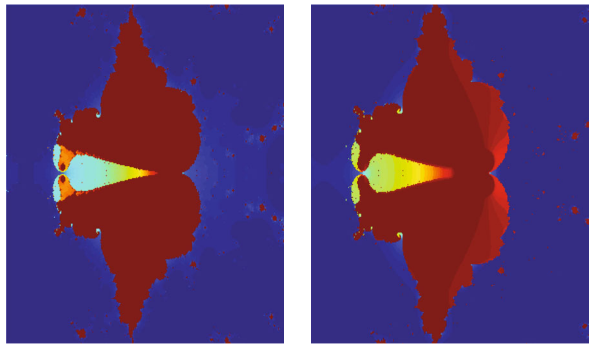
(a) (b) (c) (d)

Figure 11: Effect of change in parameters a and t simultaneously on quintic Mandelbrot set.

Table 10: Parameters for generation of quintic Mandelbrot set for different values of a and t.

	a	t	T	t	u_1	n
(i)	$-2i$	0.13	0.9025	0.9025	0.92	5
(ii)	-0.5	0.1593911	0.9025	0.9025	0.92	5
(iii)	0	0.91	0.9025	0.9025	0.92	5
(iv)	$-2.2i$	0.031	0.9025	0.9025	0.92	5

iteration, as well as Fibonacci-Mann iteration, converge to a fixed point. However, the Fibonacci-Mann iteration converges faster than the Mann iteration. But for $t_n \in (0.5,1)$, Mann iteration needs not converge to a fixed point; however, the Fibonacci-Mann iteration converges for all the initial values. By taking $f(n) = 1$ in inequality (2), we get the Mann iteration [6]. Also, for $f(n) = 1$ and $t_n = 1$, we get the Picard iteration [5]. It neither reduces to Ishikawa-iteration [7], nor



(a) (b)

Figure 12: Effect of change in parameters $\mathfrak{T}$, $\mathfrak{t}$, and u_1 simultaneously on cubic Mandelbrot set.

Table 11: Parameters for generation of cubic Mandelbrot set for different values of $\mathfrak{T}$, $\mathfrak{t}$, and u_1.

	a	t	T	t	u_1	n
(i)	-5	0.0525	0.2	0.01	0.05	3
(ii)	-5	0.0525	0.3	0.05	0.005	3

to Noor-iteration [8] since Ishikawa-iteration is a two-step process and Noor-iteration is a three-step process. On the other hand, Antal et al. [2] used the Picard iteration, the Mann iteration, the Ishikawa iteration, and the Noor-iteration to explore and compare the fractals as Julia sets. It is well known that Banach [14] utilized Picard iteration [5] to approximate a fixed point for underlying contraction mapping. But when we use slightly weaker mapping, then Picard iteration needs not converge. Consequently, Mann iteration [6], Ishikawa iteration [7], Krasnosel'ski iteration [15], modified Mann iteration [16], and so on have been introduced by distinct researchers to solve this issue for different contractions.

Remark 4. In Theorem 1, we proved the conclusion by symmetry by starting with taking $k = 0$, then $k = 1, k = 2$, and repeating the process till the k^{th} term. The parameters selected have not been studied in this point of view till now and are new. We refer the interested reader to [17, 18] for a detailed information about the Fibonacci sequence. It is well-known that the golden ratio and the Fibonacci sequence have numerous applications which range from the description of plant growth, the crystallographic structure of certain solids to music, and the development of computer algorithms for searching data bases. This fascinating sequence of numbers is named after the Italian mathematician Leonardo of Pisa, later known as Fibonacci, who introduced the sequence to Western European mathematics in his 1202 book Liber Abaci. It is interesting to recall that the Fibonacci sequence is initially explored by an ancient Indian mathematician and poet Acharya Pingala (450 BC-200 BC), the author of the Chandaśāstra (the earliest known treatise on Sanskrit prosody).

3. Generation of Julia and Mandelbrot Sets

We use MATLAB 8.5.0 (R2015a) for developing fractals for transcendental complex sine function (3) via the Fibonacci-Mann iteration (2) process. We develop Algorithms 1 and 2 to explore the geometry of Julia and Mandelbrot sets, respectively. It is interesting to notice that the structure of the fractals is very much dependent on the selection of iterative processes. During the simulation process, we have obtained and analyzed many fractals but included a limited number of fractals to discuss the behavior for the different parameter values associated with it. The parameters $a, c, n, u_1, t, \mathfrak{t}$, and $\mathfrak{T}$ perform a very significant role in giving vibrant colors and exploring the characteristics of the associated Julia sets and Mandelbrot sets. Throughout the paper, we use the standard "jet" colormap (as shown in Figure 1).

3.1. Julia Set. As we change the value of n (see Table 1), keeping other parameters fixed, we get amazing fractals, which are visible in Figures 2(a)–2(f). As the value of n increases, the fractal takes a circular shape. For $n = 10$, we obtain a Julia set that is similar to a circular saw or colorful teething ring (Figure 2(f)).

The parameter a gives rotational symmetry when it is purely real (imaginary) and changes the sign. For the same set of parameters and only changing the sign of real and complex parameter a as in Table 2, the resultant fractals can be seen in Figures 3(a)–3(d).

The parameter a also adds beauty to the fractals. As the absolute value of a increases keeping other parameters the same (as in Table 3), the more aesthetic fractals can be seen (Figures 4(a)–4(d)).

The impact of change in the values of parameters a and c simultaneously (see Table 4) on the cubic Julia set can be seen in Figures 5(a)–5(c). Noticeably, cubic Julia set in Figure 5(a) is symmetrical about both the axes; however, in Figures 5(b) and 5(c), it is symmetrical only about x-axis. Changes in the values of a and c from complex to real as well as a decrease in absolute value add beauty to resulting fractals.

The parameter t is responsible for the volume of the fractal (see Table 5). Even a slight decrease in t from 0.35 to 0.20 expands the quintic Julia set which are symmetrical about x-axis as shown in Figures 6(a)–6(c).

3.2. Mandelbrot Set. Like Julia set, Mandelbrot also becomes rounded (see Figures 7 and 8) as n increases (Tables 6 and 7). Noticeably, the number of branches in Figures 7(a)–7(e) is $2n$ while the number of branches in Figures 8(a)–8(f) is $(n-1)$ (unlike Figure 7).

Figure 9 demonstrates the effect of change in sign as well change in real to the complex value of parameter a on quadratic Mandelbrot set (see Table 8).

Lower values (Table 9) of t give more beautiful, artistic, and larger fractals which are symmetrical about x-axis (Figures 10(a)–10(c)).

Figure 11 demonstrates the effect of change in parameters a and t simultaneously on the quintic Mandelbrot set (see Table 10).

Figure 12 demonstrates the effect of change in parameters $\mathfrak{T}, \mathfrak{t}$, and u_1 simultaneously on the cubic Mandelbrot set (see Table 11). Figures 12(a) and 12(b) appear like a pair of duck which are mirror images of each other.

Remark 5.

(i) During the generation of fractals, it is surprising to see that, for the same parameter set values, the effect of even minor changes in one parameter causes a major impact on the appearance of the resultant fractal. Consequently, it is significant to select appropriate parameters to obtain the desired fractal pattern.

(ii) The majority of Julia and Mandelbrot sets generated by the sine function are symmetrical about the x-axis except Figures 2(a)–2(c) and Figures 3(a) and 3(b).

(iii) The change in the sign of the value of parameters a leads to reflexive and rotational symmetry.

(iv) The Julia and Mandelbrot fractals explored in this work are aesthetic, novel, and pleasing because the complex sine function $T(z) = \sin(z^n) + az + c$ contains a lot of attributes in it. The motivation behind this is the fact that on altering the iteration process, the dynamics and behavior of the fractals are also altered, which are significant from the graphical as well as applications viewpoint.

(v) We have displayed just the zoomed kind of fractals since the transcendental function $\sin(z)$ is unbounded so that the fractals which occupy the infinite area may lie in. But due to the unboundedness of $\sin(z)$ only on a real and imaginary axis, it can be observable.

(vi) Almost all the fractals occupy the area from $[-0.1,0.1] \times [-0.1,0.1]$ to $[-10, 10] \times [-10, 10]$.

4. Conclusion

We have generated Mandelbrot and Julia sets of various transcendental complex sine functions to demonstrate the significance of the newly developed Fibonacci-Mann iteration process. We have analyzed the behavior of variants of the Julia and Mandelbrot sets for different parameter values after obtaining fascinating nonclassical variants of classical Mandelbrot and Julia fractals using the MATLAB software. We have noticed that the role of each parameter is distinct. Therefore, we have restricted our discussion to a limited type of combination of parameters. However, we have tried to cover the maximum possible combination of parameters involved in developing the algorithm (escape criterion) in the Corollary 2. Also, we have observed that as we zoom in on the edges of the petals of the Mandelbrot set, we come across the Julia set meaning thereby each point of the Mandelbrot set includes massive image data of a Julia set. Also, the size of fractals relies on the value of parameter n. As the value of n parameter increases, the area captured by the fractals decreases, and its shape becomes circular. On the other hand, the shape as well as the symmetry of each fractal relies on the values of parameters a and c. We have explored a new technique via Fibonacci-Mann iteration for visualizing the filled-in Julia and Mandelbrot sets.

Data Availability

Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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