SYSTEM RESPONSE OF AN ALCOHOLISM MODEL UNDER THE EFFECT OF IMMIGRATION VIA NON-SINGULAR KERNEL DERIVATIVE

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Abstract. In this study, we aim to comprehensively investigate a drinking model connected to immigration in terms of Atangana-Baleanu derivative in Caputo type. To do this, we firstly extend the model describing drinking model by changing the derivative with time fractional derivative having Mittag-Leffler kernel. The existence and uniqueness of the drinking model solutions together with the stability analysis is shown by the help of Banach fixed point theorem. The special solution of the model is investigated using the Sumudu transformation and then, we present some numerical simulations for the different fractional orders to emphasize the effectiveness of the used derivative.

1. **Introduction.** Mathematical modeling is needed to better understand the causes and effects of many physical, chemical or biological events in our environment and world. Especially, the biological models related to diseases came to the forefront due to the more direct effects on human health and life.

From this point of view, scientists from many fields have been developing various mathematical models for such events in recent years. For instance, Castillo-Chavez et al. [8] given a comprehensive review on the control and dynamics of the tuberculosis model. Mulone et al. [22] designed a two-stage model for youths with serious drinking problems. In [16], for different latent stages and treatment, an HIV/AIDS epidemic model is constructed and stability conditions are investigated. Celik et al. [9, 20] studied the stability of the birth-death process in the relation to the Keller-Segel model and the optimal control problem for a Schröndinger equation with complex potential, respectively. Rahman et al. [28] introduced a giving up smoking model with the continuous age-structure in the chain smokers class. By these suggested mathematical models, the results of the handled problems under different factors are investigated with various numerical simulations and can be

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easily adapted to the technology and real life. Furthermore, it can be lead to the building and testing of new theories in this field.

In recent years, the concept of fractional calculus has also been a subject of increasing interest to researchers. Many authors, in different disciplines, reported that fractional derivative reflect the system behavior more accurately and efficiently than the integer derivative [4, 5, 6, 7, 11, 12, 10, 21, 24, 25, 26, 30, 31, 33, 36]. With this motivation, the development of new fractional derivative definitions has became one of the most important research subjects in this field. For this purpose, various fractional derivative definitions are introduced to the literature. Riemann-Liouville (RL), and Caputo fractional derivatives are the most classic ones. But the fact that these derivative definitions have singular points is seen as their biggest shortcoming. In order to eliminate this deficiency, Atangana and Baleanu [2] have introduced the new fractional derivative definition in Riemann-Liouville and Caputo type by adopting the Mittag-Leffler function as a non-local kernel.

The studies conducted over the last years according to this improvement show us that the new fractional derivative with Mittag-Leffler kernel can be used as an effective mathematical tool for modeling the complex real-life problems. In this sense, Atangana and Koca [3] analyzed a nonlinear chaotic system and demonstrated new chaotic behavior under nonlocal derivative, Alkahtani et al. [1] redefined N1H1 spread model by replacing the time derivative by nonlocal fractional derivative. Koca [18, 19] handled rubella and Ebola disease models under the Mittag-Leffler function as a non-local kernel and investigated system response. Toufik and Atangana [32] observed fractional nonlinear chaotic models with a newly defined numerical approximation method. Gómez-Aguilar et al. [15] also described and analyzed the electrical series circuits using Atangana-Baleanu (AB) fractional derivative. For computer viruses, Singh et al. [29] presented a fractional epidemiological model. The nonlinear Baggs-Freedman model is adapted to the AB fractional derivative by Gómez-Aguilar and Atangana [14]. Jarad et al. [17] investigated existence theory for a class of ordinary differential equations by newly established Gronwall inequality for AB fractional integrals. The dynamics of hepatitis E virus is analyzed by Prakasha et al. [27] considering Atangana-Baleanu derivative. Finally, Ucar et al. [34] have interested in a smoking mathematical model for analyzing the system response in the sense of AB fractional derivative.

In the other hand, alcoholism has been one of the most important problems that societies have not been able to find an exact solution for a long time. It is estimated that over 2 billion people consume alcohol in the world and about 76 million of them are addicted to alcohol. In addition, the age of starting to alcohol consumption in the world is decreasing day by day and this increases the risk of being addicted to alcohol at later ages. Also, people who consume alcohol are at risk of being exposed to a number of deadly diseases such as esophageal, throat, stomach and pancreatic cancers. Moreover, according to a study by the World Health Organization (WHO), alcohol was defined as the cause of many violent acts such as crime .

Therefore, by the above motivations, this article is devoted to analyzing the dynamics of the alcohol consumption system modeled with the AB fractional derivative in the Caputo type sense. For this purpose, the drinking model presented by Xiang et al. [35] is handled by the following integer order form:

$$
\begin{aligned}
\frac{dP(t)}{dt} &= \Lambda + (1 - q_1 - q_2)\Pi - \varepsilon P(t) - \xi P(t) L(t) \\
&\quad -\overline{\alpha} P(t) S(t) - \beta P(t) Q(t) - \mu P(t),
\end{aligned}
$$

$$\begin{aligned}
\frac{dL(t)}{dt} &= q_1\Pi + \varepsilon P(t) + \xi P(t)L(t) + \overline{\alpha}P(t)S(t) \\
&\quad + \beta P(t)Q(t) + \rho Q(t) - (\mu + d_1 + w)L(t), \qquad (1) \\
\frac{dS(t)}{dt} &= q_2\Pi + (1-p)wL(t) - (\mu + d_2 + \phi)S(t), \\
\frac{dQ(t)}{dt} &= pwL(t) + \phi S(t) - (\mu + d_3 + \rho)Q(t), \\
N(t) &= P(t) + L(t) + S(t) + Q(t).
\end{aligned}$$

The description of model components and the transfer diagram has been given by:

$P(t)$: The moderate alcoholics number at time t,

$L(t)$: The light drinkers number at time t,

$S(t)$: The heavy drinkers number at time t,

$Q(t)$: The treated drinkers number at time t,

$\quad \Lambda$: The ratio of recruitment in the population,

$\quad \Pi$: The number of immigrants in the population,

$\quad q_i$: $(i = 1, 2)$ The proportion of the number of immigrants who enter light and heavy drinkers, respectively,

$\quad \rho$: The failure ratio of the treatment,

$\quad \varepsilon$: The increase rate in the alcohol consumption of moderate drinkers,

$\quad \xi$: The ratio that moderate alcoholics interact with light drinkers,

$\quad \overline{\alpha}$: The ratio that moderate alcoholics contact with heavy drinkers,

$\quad \beta$: The ratio that moderate alcoholics interact with those under treatment,

$\quad \phi$: The ratio of treating in heavy alcoholics,

$\quad w$: The ratio of alcoholics who depart from light drinkers and enter into the group of heavy alcoholics or treatment,

$\quad \mu$: The ratio of departure from the drinking environment per-person,

$\quad d_i$: $(i = 1, 2, 3)$ The ratio of death related to drinking,

$\quad p$: The ratio of treating in light alcoholics,
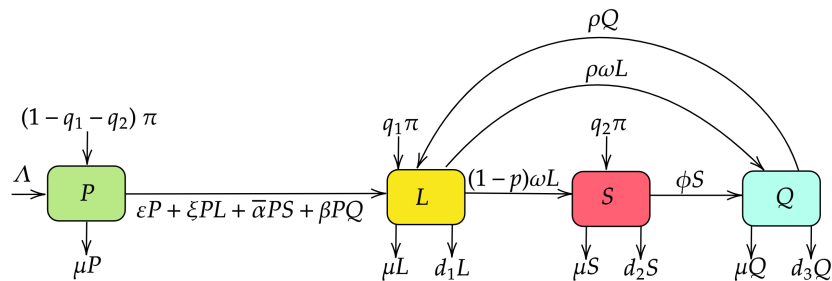
FIGURE 1. The transfer diagram of the alcoholism model under the effect of immigration.

In the literature, Gómez-Aguilar [13] analyzed a different drinking model under the effect on Twitter via Liouville-Caputo and AB fractional derivatives. Also, existence of the unique solutions are shown by the fixed point theory.

Inspired by the above study, the drinking model in (1) is modeled with AB fractional derivative and the existence and uniqueness of the solutions are demonstrated by the fixed point theorem.

In order to show the results properly, the next part of the study is divided into five sections. In Section 2, some basic definitions and theorems of the AB fractional derivative are summarized. The drinking model with AB fractional derivative is introduced and the existence of the solutions are proved by the Picard-Lindelof approach, in Section 3. In Section 4, the specific solution of the model is investigated by the iterative method created with the Sumudu transformation. Furthermore, the stability analysis of the method is handled with the help of fixed point postulate, in Section 5. In Section 6, the theoretical knowledge obtained is supported by numerical simulations. Finally, in the last chapter, we summarized our findings.

2. **Basic tools.** In this part, we will briefly summarize some basic definitions and properties of the AB derivative which will be useful in the following chapters.

Definition 2.1. The Sobolev space of order 1 in (a, b) is given by

$$H^1(a, b) = \{g \in L^2(a, b) : u' \in L^2(a, b)\}.$$

Definition 2.2. Let $g \in H^1(a, b)$, $a < b$ be a function and $\eta \in [0, 1]$. The AB derivative in Caputo type of order η of g is given by [2]

$$
{}^{ABC}_{a}D^{\eta}_t [g(t)] = \frac{B(\eta)}{1 - \eta} \int_a^t g'(x) E_\eta \left[-\eta \frac{(t - x)^\eta}{1 - \eta} \right] dx, \tag{2}
$$

where $B(\eta)$ is a normalization function with $B(0) = B(1) = 1$ and is of the following form

$$B(\eta) = 1 - \eta + \frac{\eta}{\Gamma(\eta)},$$

E_η is the Mittag-Leffler function, defined by its series representation as

$$E_{\eta, \beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\eta k + \beta)}, \eta, \beta > 0.$$

Definition 2.3. Let $g \in H^1(a, b)$, $a < b$ be a function and $\eta \in [0, 1]$. The AB derivative in RL type of order η of g is given by [2]:

$$
{}^{ABR}_{a}D^{\eta}_t [g(t)] = \frac{B(\eta)}{1 - \eta} \frac{d}{dt} \int_a^t g(x) E_\eta \left[-\eta \frac{(t - x)^\eta}{1 - \eta} \right] dx. \tag{3}
$$

Definition 2.4. The fractional integral associated to the AB fractional derivative is defined by [2]:

$$
{}^{AB}_{a}I^{\eta}_t [g(t)] = \frac{1 - \eta}{B(\eta)} g(t) + \frac{\eta}{B(\eta)\Gamma(\eta)} \int_a^t g(\lambda)(t - \lambda)^{\eta - 1} d\lambda. \tag{4}
$$

3. **Existence of solutions by means of Picard-Lindelof method.** Let us consider the drinking model (1) with the AB fractional derivative in Caputo type:

$$
\begin{aligned}
{}^{ABC}_{0}D^{\eta}_t (P(t)) &= \Lambda + (1 - q_1 - q_2)\Pi - \varepsilon P(t) - \xi P(t) L(t) \\
&\quad - \overline{\alpha} P(t) S(t) - \beta P(t) Q(t) - \mu P(t),
\end{aligned}
$$

$$\begin{aligned}
{}_0^{ABC}D_t^\eta\left(L\left(t\right)\right) &= q_1\Pi + \varepsilon P\left(t\right) + \xi P\left(t\right)L\left(t\right) + \overline{\alpha}P\left(t\right)S\left(t\right) \\
&\quad + \beta P\left(t\right)Q\left(t\right) + \rho Q\left(t\right) - \left(\mu + d_1 + w\right)L\left(t\right), \\
{}_0^{ABC}D_t^\eta\left(S\left(t\right)\right) &= q_2\Pi + \left(1-p\right)wL\left(t\right) - \left(\mu + d_2 + \phi\right)S\left(t\right), \\
{}_0^{ABC}D_t^\eta\left(Q\left(t\right)\right) &= pwL\left(t\right) + \phi S\left(t\right) - \left(\mu + d_3 + \rho\right)Q\left(t\right),
\end{aligned} \tag{5}$$

with the following initial conditions

$$P\left(0\right) = \overline{n}_1, L\left(0\right) = \overline{n}_2,$$
$$S\left(0\right) = \overline{n}_3, Q\left(0\right) = \overline{n}_4,$$

where ${}_0^{ABC}D_t^\eta$ is AB derivative in Caputo type and $\eta \in [0,1]$.

In order to show existence of solution, we state the operator as:

$$\begin{aligned}
m_1\left(t,P\right) &= \Lambda + \left(1 - q_1 - q_2\right)\Pi - \varepsilon P\left(t\right) - \xi P\left(t\right)L\left(t\right) \\
&\quad - \overline{\alpha}P\left(t\right)S\left(t\right) - \beta P\left(t\right)Q\left(t\right) - \mu P\left(t\right), \\
m_2\left(t,L\right) &= q_1\Pi + \varepsilon P\left(t\right) + \xi P\left(t\right)L\left(t\right) + \overline{\alpha}P\left(t\right)S\left(t\right) \\
&\quad + \beta P\left(t\right)Q\left(t\right) + \rho Q\left(t\right) - \left(\mu + d_1 + w\right)L\left(t\right), \\
m_3\left(t,S\right) &= q_2\Pi + \left(1 - \rho\right)wL\left(t\right) - \left(\mu + d_2 + \phi\right)S\left(t\right), \\
m_4\left(t,Q\right) &= pwL\left(t\right) + \phi S\left(t\right) - \left(\mu + d_3 + \rho\right)Q\left(t\right).
\end{aligned} \tag{6}$$

It can be easily seen that m_1, m_2, m_3, m_4 are contraction in accordance with the functions P, L, S, Q respectively. Let

$$\begin{aligned}
N_1 &= \sup_{C[a,b_1]}\left\|m_1\left(t,P\right)\right\|, \\
N_2 &= \sup_{C[a,b_2]}\left\|m_2\left(t,L\right)\right\|, \\
N_3 &= \sup_{C[a,b_3]}\left\|m_3\left(t,S\right)\right\|, \\
N_4 &= \sup_{C[a,b_4]}\left\|m_1\left(t,Q\right)\right\|,
\end{aligned} \tag{7}$$

where

$$\begin{aligned}
C\left[d,e_1\right] &= \left[t-d,t+d\right] \times \left[x-e_1,x+e_1\right] = D \times E_1, \\
C\left[d,e_2\right] &= \left[t-d,t+d\right] \times \left[x-e_2,x+e_2\right] = D \times E_2, \\
C\left[d,e_3\right] &= \left[t-d,t+d\right] \times \left[x-e_3,x+e_3\right] = D \times E_3, \\
C\left[d,e_4\right] &= \left[t-d,t+d\right] \times \left[x-e_4,x+e_4\right] = D \times E_4.
\end{aligned}$$

Here, we benefit from the Banach fixed point theorem using the metric on $C\left[a,b_i\right]$, $\left(i=1,2,3,4\right)$ induced by the norm given as:

$$\left\|f\left(t\right)\right\| = \sup_{t\in[t-d,t+d]}\left|f\left(t\right)\right| \tag{8}$$

Taking into consideration Picard's operator

$$\theta : C\left(D, E_1, E_2, E_3, E_4\right) \to C\left(D, E_1, E_2, E_3, E_4\right) \tag{9}$$

given the following definition:

$$\begin{aligned}
\theta X\left(t\right) &= X_0\left(t\right) + X\left(t\right)\frac{1-\eta}{B\left(\eta\right)} \\
&\quad + \frac{\eta}{B\left(\eta\right)\Gamma\left(\eta\right)}\int_0^t \left(t-y\right)^{\eta-1}F\left(y, X\left(y\right)\right)dy,
\end{aligned} \tag{10}$$

where

$$X\left(t\right) = \left[P\left(t\right)\ L\left(t\right)\ S\left(t\right)\ Q\left(t\right)\right]^{T},$$

$$X_0\left(t\right) = \left[P\left(0\right)\ L\left(0\right)\ S\left(0\right)\ Q\left(0\right)\right]^{T},$$

$$F\left(t, X\left(t\right)\right) = \left[m_1\left(t, P\left(t\right)\right) m_2\left(t, L\left(t\right)\right) m_3\left(t, S\left(t\right)\right) m_4\left(t, Q\left(t\right)\right)\right].$$

Assume that the problem under examined satisfies

$$\left\|X\left(t\right)\right\| \leq \max\left\{e_1, e_2, e_3, e_4\right\}. \tag{11}$$

Now we consider

$$\left\|\theta X\left(t\right) - X_0\left(t\right)\right\|$$

$$= \left\|\frac{1-\eta}{B\left(\eta\right)} F\left(t, X\left(t\right)\right) + \frac{\eta}{B\left(\eta\right)\Gamma\left(\eta\right)} \int_0^t \left(t-y\right)^{\eta-1} F\left(y, X\left(y\right)\right) dy\right\|$$

$$\leq \frac{1-\eta}{B\left(\eta\right)} \left\|F\left(t, X\left(t\right)\right)\right\| + \frac{\eta}{B\left(\eta\right)\Gamma\left(\eta\right)} \int_0^t \left(t-y\right)^{\eta-1} \left\|F\left(y, X\left(y\right)\right)\right\| dy$$

$$\leq \frac{1-\eta}{B\left(\eta\right)} N + \frac{\eta}{B\left(\eta\right)} N a^{\eta},$$

where $N = \max\left\{N_1, N_2, N_3, N_4\right\}$. Let $d < \frac{e}{N}$ and $e = \max\left\{e_1, e_2, e_3, e_4\right\}$. Then, we have

$$\left\|\theta X\left(t\right) - \theta X_0\left(t\right)\right\| < dN < e. \tag{12}$$

Also, we evaluate the following:

$$\left\|\theta X_1 - \theta X_2\right\| = \sup_{t \in A} \left|X_1 - X_2\right|. \tag{13}$$

Here, we get

$$\left\|\theta X_1 - \theta X_2\right\| = \left\|\frac{1-\eta}{B\left(\eta\right)} \left(F\left(t, X_1\left(t\right)\right) - F\left(t, X_2\left(t\right)\right)\right)\right.$$

$$\left. + \frac{\eta}{B\left(\eta\right)\Gamma\left(\eta\right)} \int_0^t \left(t-y\right)^{\eta-1} \left(F\left(y, X_1\left(y\right)\right) - F\left(y, X_2\left(y\right)\right)\right) dy\right\|$$

$$\leq \frac{1-\eta}{B\left(\eta\right)} \left\|F\left(t, X_1\left(t\right)\right) - F\left(t, X_2\left(t\right)\right)\right\|$$

$$+ \frac{\eta}{B\left(\eta\right)\Gamma\left(\eta\right)} \int_0^t \left(t-y\right)^{\eta-1} \left\|F\left(y, X_1\left(y\right)\right) - F\left(y, X_2\left(y\right)\right)\right\| dy$$

$$\leq \frac{1-\eta}{B\left(\eta\right)} q \left\|X_1\left(t\right) - X_2\left(t\right)\right\|$$

$$+ \frac{\eta q}{B\left(\eta\right)\Gamma\left(\eta\right)} \int_0^t \left(t-y\right)^{\eta-1} \left\|X_1\left(y\right) - X_2\left(y\right)\right\| dy$$

$$\leq \left(\frac{1-\eta}{B\left(\eta\right)} q + \frac{\eta q a^{\eta}}{B\left(\eta\right)\Gamma\left(\eta\right)}\right) \left\|X_1\left(t\right) - X_2\left(t\right)\right\|$$

$$\leq dq \left\|X_1\left(t\right) - X_2\left(t\right)\right\|, \tag{14}$$

with $q < 1$. By using contraction properties of F, we obtain $dq < 1$. Thereby θ is a contraction. This gives the drinking model with AB derivative in Caputo type given in (5) has a unique set of solution.

4. **Derivation of special solution with iterative method.** We aim to show a specific solution of the model by applying Sumudu transform to the Eq. (5) with a recursive formula in this section. The Sumudu transform for AB fractional derivative is introduced by Atangana and Koca [3] as follows:

Theorem 4.1. *Let $\eta \in [0, 1]$, $a < b$ and $f \in H^1(a, b)$. The Sumudu transform for AB derivative in the Caputo type sense is presented by*

$$ST\left\{{}_0^{ABC}D_t^\eta\left[f(t)\right]\right\} = \frac{B(\eta)}{1-\eta}\left(\eta\Gamma(\eta+1)E_\eta\left(-\frac{1}{1-\eta}u^\eta\right)\right)$$
$$\times\left(ST\left(f(t)\right) - f(0)\right). \tag{15}$$

Applying Sumudu transform to the Eq. (5), we find

$$\frac{B(\eta)}{1-\eta}\left(\eta\Gamma(\eta+1)E_\eta\left(-\frac{1}{1-\eta}u^\eta\right)\right)\left(ST\left(P(t)\right) - P(0)\right)$$
$$= ST\left\{\Lambda + (1-q_1-q_2)\Pi - \varepsilon P(t) - \xi P(t)L(t) - \overline{\alpha}P(t)S(t)\right.$$
$$\left. -\beta P(t)Q(t) - \mu P(t)\right\},$$

$$\frac{B(\eta)}{1-\eta}\left(\eta\Gamma(\eta+1)E_\eta\left(-\frac{1}{1-\eta}u^\eta\right)\right)\left(ST\left(L(t)\right) - L(0)\right)$$
$$= ST\left\{q_1\Pi + \varepsilon P(t) + \xi P(t)L(t) + \overline{\alpha}P(t)S(t)\right.$$
$$\left. +\beta P(t)Q(t) + \rho Q(t) - (\mu + d_1 + w)L(t)\right\},$$

$$\frac{B(\eta)}{1-\eta}\left(\eta\Gamma(\eta+1)E_\eta\left(-\frac{1}{1-\eta}u^\eta\right)\right)\left(ST\left(S(t)\right) - S(0)\right)$$
$$= ST\left\{q_2\Pi + (1-\rho)wL(t) - (\mu + d_2 + \phi)S(t)\right\},$$

$$\frac{B(\eta)}{1-\eta}\left(\eta\Gamma(\eta+1)E_\eta\left(-\frac{1}{1-\eta}u^\eta\right)\right)\left(ST\left(Q(t)\right) - Q(0)\right)$$
$$= ST\left\{pwL(t) + \phi S(t) - (\mu + d_3 + \rho)Q(t)\right\}. \tag{16}$$

Regulating the Eq. (16), we have

$$ST\left(P(t)\right) = P(0) + \psi$$
$$\times ST\left\{\begin{array}{c}\Lambda + (1-q_1-q_2)\Pi - \varepsilon P(t) - \xi P(t)L(t) \\ -\overline{\alpha}P(t)S(t) - \beta P(t)Q(t) - \mu P(t)\end{array}\right\},$$

$$ST\left(L(t)\right) = L(0) + \psi$$
$$\times ST\left\{\begin{array}{c}q_1\Pi + \varepsilon P(t) + \xi P(t)L(t) + \overline{\alpha}P(t)S(t) \\ +\beta P(t)Q(t) + \rho Q(t) - (\mu + d_1 + w)L(t)\end{array}\right\},$$

$$ST\left(S(t)\right) = S(0) + \psi$$
$$\times ST\left\{q_2\Pi + (1-\rho)wL(t) - (\mu + d_2 + \phi)S(t)\right\},$$

$$ST\left(Q(t)\right) = Q(0) + \psi$$
$$\times ST\left\{pwL(t) + \phi S(t) - (\mu + d_3 + \rho)Q(t)\right\}, \tag{17}$$

where

$$\psi = \frac{1 - \eta}{B\left(\eta\right)\left(\eta\Gamma\left(\eta + 1\right)E_\eta\left(-\frac{1}{1-\eta}u^\eta\right)\right)}. \tag{18}$$

Then, we have the following iterative formula

$$
\begin{aligned}
P_{n+1}\left(t\right) &= P_n\left(0\right) \\
&+ ST^{-1}\left\{ \begin{array}{c} \psi \times ST\left\{\Lambda + \left(1 - q_1 - q_2\right)\Pi - \varepsilon P_n\left(t\right) - \xi P_n\left(t\right)L_n\left(t\right) \\ -\overline{\alpha}P_n\left(t\right)S_n\left(t\right) - \beta P_n\left(t\right)Q_n\left(t\right) - \mu P_n\left(t\right)\right\} \end{array} \right\}, \\
L_{n+1}\left(t\right) &= L_n\left(0\right) \\
&+ ST^{-1}\left\{ \begin{array}{c} \psi \times ST\left\{q_1\Pi + \varepsilon P_n\left(t\right) + \xi P_n\left(t\right)L_n\left(t\right) + \overline{\alpha}P_n\left(t\right)S_n\left(t\right) \\ +\beta P_n\left(t\right)Q_n\left(t\right) + \rho Q_n\left(t\right) - \left(\mu + d_1 + w\right)L_n\left(t\right)\right\} \end{array} \right\}, \\
S_{n+1}\left(t\right) &= S_n\left(0\right) \\
&+ ST^{-1}\left\{\psi \times ST\left\{q_2\Pi + \left(1 - p\right)wL_n\left(t\right) - \left(\mu + d_2 + \phi\right)S_n\left(t\right)\right\}\right\}, \\
Q_{n+1}\left(t\right) &= Q_n\left(0\right) \\
&+ ST^{-1}\left\{\psi \times ST\left\{pwL_n\left(t\right) + \phi S_n\left(t\right) - \left(\mu + d_3 + \rho\right)Q_n\left(t\right)\right\}\right\}. \tag{19}
\end{aligned}
$$

Thus, solution of the Eq. (19) is obtained when n tends to infinitiy:

$$
\begin{aligned}
P\left(t\right) &= \lim_{n\to\infty}P_n\left(t\right),\ L\left(t\right) = \lim_{n\to\infty}L_n\left(t\right), \\
S\left(t\right) &= \lim_{n\to\infty}S_n\left(t\right),\ Q\left(t\right) = \lim_{n\to\infty}Q_n\left(t\right). \tag{20}
\end{aligned}
$$

5. **Stability analysis of iteration method using fixed point theory.** Let $(E, \|.\|)$ be Banach space, $W : E \to E$ be a map and $h_{n+1} = \varphi\left(W, h_n\right)$ be a special recursive procedure. Assume that $H\left(W\right)$ is a fixed point set of W with at least one element h_n which converges to a point $w \in H\left(W\right)$. Also suppose that $\{x_n\} \subset X$ be sequence and $e_n = \|x_{n+1} - \varphi\left(W, x_n\right)\|$. The iteration method $h_{n+1} = \varphi\left(W, h_n\right)$ is said to be H-stable, if $\lim_{n\to\infty}e_n = 0$ induces that $\lim_{n\to\infty}x_n = w$.

Theorem 5.1. *Let $(E, \|.\|)$ be Banach space and $W : E \to E$ be a map satisfying*

$$\|W_x - W_y\| \le K\|x - W_x\| + k\|x - y\|,$$

for all $x, y \in E$, where $0 \le K$, $0 \le k < 1$. Suppose that W is Picard W-stable [23].

Theorem 5.2. *Let M be a self map defined as*

$$
\begin{aligned}
M\left(P_n\left(t\right)\right) &= P_{n+1}\left(t\right) \\
&= P_n\left(t\right) + ST^{-1}\left\{ \begin{array}{c} \psi \times ST\left\{\Lambda + \left(1 - q_1 - q_2\right)\Pi - \varepsilon P\left(t\right) - \xi P\left(t\right)L\left(t\right) \\ -\overline{\alpha}P\left(t\right)S\left(t\right) - \beta P\left(t\right)Q\left(t\right) - \mu P\left(t\right)\right\} \end{array} \right\}, \\
M\left(L_n\left(t\right)\right) &= L_{n+1}\left(t\right) \\
&= L_n\left(t\right) + ST^{-1}\left\{ \begin{array}{c} \psi \times ST\left\{q_1\Pi + \varepsilon P\left(t\right) + \xi P\left(t\right)L\left(t\right) + \overline{\alpha}P\left(t\right)S\left(t\right) \\ +\beta P\left(t\right)Q\left(t\right) + \rho Q\left(t\right) - \left(\mu + d_1 + w\right)L\left(t\right)\right\} \end{array} \right\}, \\
M\left(S_n\left(t\right)\right) &= S_{n+1}\left(t\right) \\
&= S_n\left(t\right) + ST^{-1}\left\{\psi \times ST\left\{q_2\Pi + \left(1 - p\right)wL\left(t\right) - \left(\mu + d_2 + \phi\right)S\left(t\right)\right\}\right\}, \\
M\left(Q_n\left(t\right)\right) &= Q_{n+1}\left(t\right) \\
&= Q_n\left(t\right) + ST^{-1}\left\{\psi \times ST\left\{pwL\left(t\right) + \phi S\left(t\right) - \left(\mu + d_3 + \rho\right)Q\left(t\right)\right\}\right\}. \tag{21}
\end{aligned}
$$

Then, the iteration is M-stable in $L^1(a,b)$ if

$$1 - (\varepsilon + \mu)f(\gamma) - \xi(b+e)g(\gamma) - \overline{\alpha}(c+e)h(\gamma) - \beta(q+e)i(\gamma) \quad < \quad 1,$$

$$1 + \varepsilon j(\gamma) + \xi(a+o)l(\gamma) + \overline{\alpha}(a+r)m(\gamma)$$
$$+ \beta(a+s)n(\gamma) + \rho\overline{k}(\gamma) - (\mu+d_1+w)v(\gamma) \quad < \quad 1,$$

$$1 + (1-p)w\overline{f}_1(\gamma) - (\mu+d_2+\phi)\overline{f}_2(\gamma) \quad < \quad 1,$$

$$1 + pw\overline{f}_3(\gamma) + \phi\overline{f}_4(\gamma) - (\mu+d_2+\phi)\overline{f}_4(\gamma) \quad < \quad 1,$$

where $f, g, h, i, j, l, m, n, v, \overline{k}, \overline{f}_1, \overline{f}_2, \overline{f}_3, \overline{f}_4, \overline{f}_5$ are functions $ST^{-1}\{\psi ST\}$.

Proof. First of all, we demonstrate that W has a fixed point. For this purpose, we consider the followings for all $(n,m) \in \mathbb{N} \times \mathbb{N}$.

$$M(P_n(t)) - M(P_m(t)) = P_n(t) - P_m(t)$$
$$+ ST^{-1}\left\{\psi \times ST\left\{\begin{array}{c} \Lambda + (1-q_1-q_2)\Pi - \varepsilon P_n(t) - \xi P_n(t)L_n(t) \\ -\overline{\alpha}P_n(t)S_n(t) - \beta P_n(t)Q_n(t) - \mu P_n(t) \end{array}\right\}\right\}$$
$$- ST^{-1}\left\{\psi \times ST\left\{\begin{array}{c} \Lambda + (1-q_1-q_2)\Pi - \varepsilon P_m(t) - \xi P_m(t)L_m(t) \\ -\overline{\alpha}P_m(t)S_m(t) - \beta P_m(t)Q_m(t) - \mu P_m(t) \end{array}\right\}\right\}. \quad (22)$$

Applying norm, the Eq. (22) is converted to

$$\|M(P_n(t)) - M(P_m(t))\| = \|P_n(t) - P_m(t)$$
$$+ ST^{-1}\left\{\psi \times ST\left\{\begin{array}{c} \Lambda + (1-q_1-q_2)\Pi - \varepsilon P_n(t) - \xi P_n(t)L_n(t) \\ -\overline{\alpha}P_n(t)S_n(t) - \beta P_n(t)Q_n(t) - \mu P_n(t) \end{array}\right\}\right\}$$
$$+ ST^{-1}\left\{\psi \times ST\left\{\begin{array}{c} \Lambda + (1-q_1-q_2)\Pi - \varepsilon P_m(t) - \xi P_m(t)L_m(t) \\ -\overline{\alpha}P_m(t)S_m(t) - \beta P_m(t)Q_m(t) - \mu P_m(t) \end{array}\right\}\right\}\|. \quad (23)$$

Using norm properties, we get

$$\|M(P_n(t)) - M(P_m(t))\|$$
$$\leq \|P_n(t) - P_m(t)\| + \|ST^{-1}\{\psi \times ST\{(-\varepsilon - \mu)(P_n(t) - P_m(t))$$
$$- \xi(L_n(t)(P_n(t) - P_m(t)) + P_m(t)(L_n(t) - L_m(t)))$$
$$- \overline{\alpha}(S_n(t)(P_n(t) - P_m(t)) + P_m(t)(S_n(t) - S_m(t)))$$
$$- \beta(Q_n(t)(P_n(t) - P_m(t)) + P_m(t)(Q_n(t) - Q_m(t)))\}\}\|. \quad (24)$$

The above can be transformed as follows:

$$\|M(P_n(t)) - M(P_m(t))\| \leq \|P_n(t) - P_m(t)\|$$
$$+ ST^{-1}\{\Psi \times ST\{\|-(\varepsilon+\mu)(P_n(t) - P_m(t))\|\}\}$$
$$+ ST^{-1}\{\Psi \times ST\{\|-\xi(L_n(t)(P_n(t) - P_m(t)) + P_m(t)(L_n(t) - L_m(t)))\|\}\}$$
$$+ ST^{-1}\{\Psi \times ST\{\|-\overline{\alpha}(S_n(t)(P_n(t) - P_m(t)) + P_m(t)(S_n(t) - S_m(t)))\|\}\}$$
$$+ ST^{-1}\{\Psi \times ST\{\|-\beta(Q_n(t)(P_n(t) - P_m(t)) + P_m(t)(Q_n(t) - Q_m(t)))\|\}\}. \quad (25)$$

Because the solutions play the same role, we assume that

$$\|P_n(t) - P_m(t)\| \quad \cong \quad \|L_n(t) - L_m(t)\|,$$
$$\|P_n(t) - P_m(t)\| \quad \cong \quad \|S_n(t) - S_m(t)\|,$$
$$\|P_n(t) - P_m(t)\| \quad \cong \quad \|Q_n(t) - Q_m(t)\|.$$

Writing these in Eq. (25), we have

$$
\begin{aligned}
&\|M\left(P_n\left(t\right)\right) - M\left(P_m\left(t\right)\right)\| \le \|P_n\left(t\right) - P_m\left(t\right)\| \\
&+ST^{-1}\left\{\Psi \times ST\left\{\|-\left(\varepsilon + \mu\right)\left(P_n\left(t\right) - P_m\left(t\right)\right)\|\right\}\right\} \\
&+ST^{-1}\left\{\Psi \times ST\left\{\|-\xi\left(L_n\left(t\right)\left(P_n\left(t\right) - P_m\left(t\right)\right) + P_m\left(t\right)\left(P_n\left(t\right) - P_m\left(t\right)\right)\right)\|\right\}\right\} \\
&+ST^{-1}\left\{\Psi \times ST\left\{\|-\overline{\alpha}\left(S_n\left(t\right)\left(P_n\left(t\right) - P_m\left(t\right)\right) + P_m\left(t\right)\left(P_n\left(t\right) - P_m\left(t\right)\right)\right)\|\right\}\right\} \\
&+ST^{-1}\left\{\Psi \times ST\left\{\|-\beta\left(Q_n\left(t\right)\left(P_n\left(t\right) - P_m\left(t\right)\right) + P_m\left(t\right)\left(P_n\left(t\right) - P_m\left(t\right)\right)\right)\|\right\}\right\}.
\end{aligned}
\tag{26}
$$

Because $P_n\left(t\right)$, $L_n\left(t\right)$, $S_n\left(t\right)$ and $Q_n\left(t\right)$ are bounded functions, we can find distinct constants a, b, c, e, o, r, s, q such that

$$
\begin{aligned}
\|P_n\left(t\right)\| &\le a, \|P_m\left(t\right)\| \le e, \\
\|L_n\left(t\right)\| &\le b, \|L_m\left(t\right)\| \le o, \\
\|S_n\left(t\right)\| &\le c, \|S_m\left(t\right)\| \le r, \\
\|Q_n\left(t\right)\| &\le q, \|Q_m\left(t\right)\| \le s.
\end{aligned}
\tag{27}
$$

Here thinking the Eqs. (26) and (27), we obtain

$$
\begin{aligned}
&\|M\left(P_n\left(t\right)\right) - M\left(P_m\left(t\right)\right)\| \le \|P_n\left(t\right) - P_m\left(t\right)\| \\
&\times \left\{ \begin{array}{c} 1 - \left(\varepsilon + \mu\right)f\left(\gamma\right) - \xi\left(b + e\right)g\left(\gamma\right) \\ -\overline{\alpha}\left(c + e\right)h\left(\gamma\right) - \beta\left(q + e\right)i\left(\gamma\right) \end{array} \right\},
\end{aligned}
\tag{28}
$$

where f, g, h, i are functions from $ST^{-1}\left\{\psi ST\right\}$. By the similar way, we have

$$
\begin{aligned}
&\|M\left(L_n\left(t\right)\right) - M\left(L_m\left(t\right)\right)\| \le \|L_n\left(t\right) - L_m\left(t\right)\| \\
&\times \left\{ \begin{array}{c} 1 + \varepsilon j\left(\gamma\right) + \xi\left(a + o\right)l\left(\gamma\right) + \overline{\alpha}\left(a + r\right)m\left(\gamma\right) \\ +\beta\left(a + s\right)n\left(\gamma\right) - \left(\mu + d_1 + w\right)v\left(\gamma\right) \end{array} \right\},
\end{aligned}
$$

$$
\begin{aligned}
&\|M\left(S_n\left(t\right)\right) - M\left(S_m\left(t\right)\right)\| \le \|S_n\left(t\right) - S_m\left(t\right)\| \\
&\times \left\{1 + \left(1 - p\right)w\overline{f}_1\left(\gamma\right) - \left(\mu + d_2 + \phi\right)\overline{f}_2\left(\gamma\right)\right\},
\end{aligned}
$$

and

$$
\begin{aligned}
&\|M\left(Q_n\left(t\right)\right) - M\left(Q_m\left(t\right)\right)\| \le \|Q_n\left(t\right) - Q_m\left(t\right)\| \\
&\times \left\{1 + pw\overline{f}_3\left(\gamma\right) + \phi\overline{f}_4\left(\gamma\right) - \left(\mu + d_2 + \phi\right)\overline{f}_5\left(\gamma\right)\right\},
\end{aligned}
$$

for

$$
\begin{aligned}
\left\{ \begin{array}{c} 1 - \left(\varepsilon + \mu\right)f\left(\gamma\right) - \xi\left(b + e\right)g\left(\gamma\right) \\ -\overline{\alpha}\left(c + e\right)h\left(\gamma\right) - \beta\left(q + e\right)i\left(\gamma\right) \end{array} \right\} &< 1, \\
\left\{ \begin{array}{c} 1 + \varepsilon j\left(\gamma\right) + \xi\left(a + o\right)l\left(\gamma\right) + \overline{\alpha}\left(a + r\right)m\left(\gamma\right) \\ +\beta\left(a + s\right)n\left(\gamma\right) + \rho\overline{k}\left(\gamma\right) - \left(\mu + d_1 + w\right)v\left(\gamma\right) \end{array} \right\} &< 1, \\
1 + \left(1 - p\right)w\overline{f}_1\left(\gamma\right) - \left(\mu + d_2 + \phi\right)\overline{f}_2\left(\gamma\right) &< 1, \\
1 + pw\overline{f}_3\left(\gamma\right) + \phi\overline{f}_4\left(\gamma\right) - \left(\mu + d_2 + \phi\right)\overline{f}_5\left(\gamma\right) &< 1.
\end{aligned}
\tag{29}
$$

Then the nonlinear M-self mapping has a fixed point. Let

$$k = (0, 0, 0, 0),$$

$$K = \left\{ \begin{array}{c} 1 - (\varepsilon + \mu) f(\gamma) - \xi(b + e) g(\gamma) \\ -\overline{\alpha}(c + e) h(\gamma) - \beta(q + e) i(\gamma), \\ 1 + \varepsilon j(\gamma) + \xi(a + o) l(\gamma) + \overline{\alpha}(a + r) m(\gamma) \\ +\beta(a + s) n(\gamma) + \rho \overline{k}(\gamma) - (\mu + d_1 + w) v(\gamma), \\ 1 + (1 - p) w \overline{f}_1(\gamma) - (\mu + d_2 + \phi) \overline{f}_2(\gamma), \\ 1 + pw \overline{f}_3(\gamma) + \phi \overline{f}_4(\gamma) - (\mu + d_2 + \phi) \overline{f}_4(\gamma). \end{array} \right\}.$$

(30)

Using the Eqs. (28), (29) and (30), M satisfies the conditions in Theorem 5.1. Then M is Picard M-stable. □

6. **Numerical results.** In this part, some numerical simulations for the studied drinking model (5) are presented to show how fractional order η affects the components behavior of the fractional model. For this purpose, a numerical schemes based on the method given by Toufik and Atangana [32] is handled. We choose the parameters $\Lambda = 0.12$, $q_1 = 0.4$, $q_2 = 0.3$, $\Pi = 0.3$, $\varepsilon = 0.1$, $\xi = 0.6$, $\overline{\alpha} = 0.3$, $\beta = 0.2$, $\mu = 0.1595$, $p = 0.3$, $\rho = 0.8$, $d_1 = 0.02$, $w = 0.583$, $d_2 = 0.038$, $\phi = 0.226$, $d_3 = 0.03$ and the initial conditions $P(0) = 1.3$, $L(0) = 0.8$, $S(0) = 0.52$, $Q(0) = 0.11$ as seen Fig. 4 in [35]. The numerical simulations of the special solutions respect to the model (5) for different values of η are performed by Matlab software. One can see from the Figs. 2-5, as fractional order η increases, the number of light, moderate and heavy alcoholics decreases compared to the integer model components in [35].
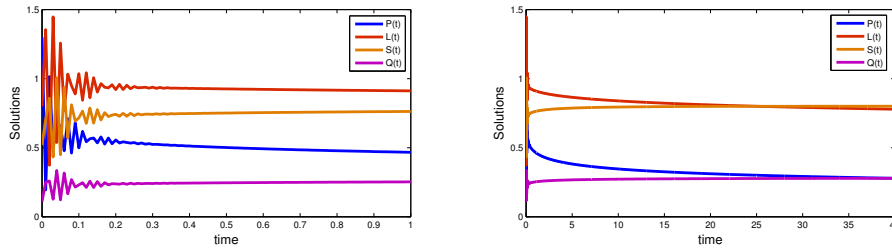
FIGURE 2. System behavior of the fractional drinking model (5) with order $\eta = 0.3$ in respect to time $t = 1$ and $t = 40$.

7. **Conclusions.** Nowadays, alcohol consumption has been observed as one of the critical problems which affect personal health and lead to a wide range of negative social effects. For this reason, many researchers in various disciplines have been studied mathematical model of the alcohol consumption. In our work, firstly, we extend one of the drinking model (5) by using Atangana-Baleanu (AB) fractional derivative. Then, we prove that the solutions of this extended model is unique by using Banach fixed point postulate and derive special solution with the help of Sumudu transform. Additionally, we confirm stability analysis via the Picard W-stable approach. Finally, the numerical simulations in the Figs. 2-5 show that the number of light, medium and heavy drinkers decreases according to the integer-order model components in [35] as the fractional order η increase. We believe that our
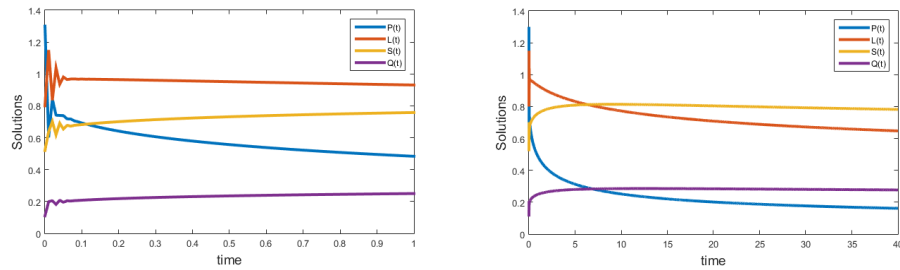
FIGURE 3. System behavior of the fractional drinking model (5) with order $\eta = 0.5$ in respect to time $t = 1$ and $t = 40$.
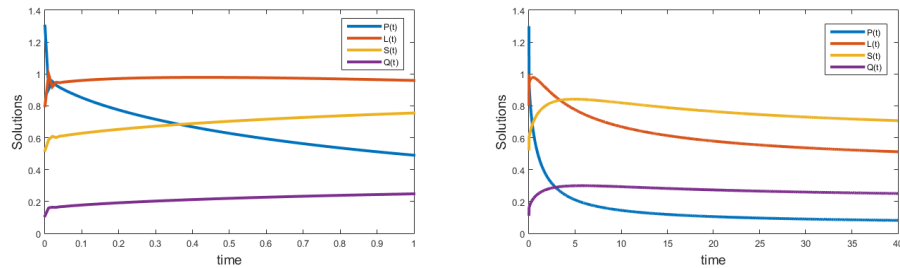

FIGURE 4. System behavior of the fractional drinking model (5) with order $\eta = 0.7$ in respect to time $t = 1$ and $t = 40$.
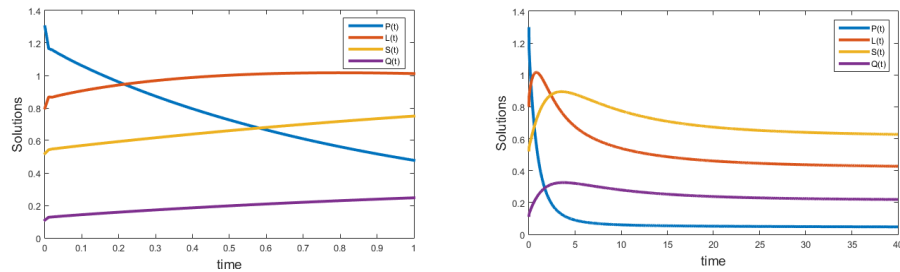

FIGURE 5. System behavior of the fractional drinking model (5) with order $\eta = 0.9$ in respect to time $t = 1$ and $t = 40$.

results are very helpful in the description of drinking matter in biological, medical and social processes.

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