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On G-Continuity in Neutrosophic Topological Spaces

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Abstract. Continuity is one of most important concepts in many mathematical disciplines. In some situations general notion of continuity is replaced by sequential continuity. Connor and Grosse-Erdmann replaced lim in the definition of sequential continuity of real functions by a linear functional G on a linear subspace of the vector space of all real sequences. Their definition was extended to topological group X by replacing a linear functional G with an additive function defined on a subgroup of the group of all X-valued sequences. In this paper we introduce neutrosophic G-continuity and investigate its properties in neutrosophic topological spaces.
Keywords: Neutrosophic sequential closure, neutrosophic group, neutrosophic method, neutrosophic G-sequential continuity.
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INTRODUCTION

The importance of sequential continuity in mathematics and its applications in other sciences (such as computer science, information theory, biological science, dynamical systems and so on) is well known. In [2], Connor and Grosse-Erdmann changed the definition of the convergence of sequences of real functions by replacing the standard lim operator by a linear functional G. In [1], Cakalli extended this concept to topological group-valued sequences. Our goal in this paper is to extend these ideas to neutrosophic topological spaces. We introduce the neutrosophic regular method and neutrosophic subsequential method and investigate some their properties.

We use standard terminology and notations for neutrosophic set theory and the theory of neutrosophic topological spaces following mainly [3, 4]. Our basic notions are neutrosophic set, neutrosophic point, and neutrosophic topological space.

A *neutrosophic set* A over the universe X is defined as

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\},$$

where $T, I, F : X \to]^-0, 1^+[$ and $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$. T, I and F are called the membership function, indeterminacy function and non-membership function, respectively.

A *neutrosophic point* (with the support x_p) is denoted by $x_{r,t,s}$ and defined by

$$x_{r,t,s}(x_p) = \begin{cases} (r, t, s), & \text{if } x = x_p, \\ (0, 0, 1), & \text{if } x \neq x_p; \end{cases}$$

here r denotes the degree of membership value, t the degree of indeterminacy, and s the degree of non-membership value of $x_{r,t,s}$.

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The collection τ of neutrosophic sets over the universe X is called a *neutrosophic topology* on X, if (i) the null neutrosophic set 0_X and the absolute neutrosophic set 1_X belong to τ, (ii) the union of any subcollection of τ belongs to τ, and (iii) the intersection of a finite number of neutrosophic sets in τ belongs to τ. The pair (X, τ) is called *neutrosophic topologica spaces*, and elements from τ are called *neutrosphic open sets*. Sometime, instead of (X, τ) we simply write X.

Definitions

In this section we introduce several new notions which are fundamental for our study in this article.

Definition 1 (1) A neutrosophic point $x_{r,t,s}$ is said to be *neutrosophic quasi-coincident* (neutrosophic q-coincident, for short) with a neutrosophic set A, denoted by $x_{r,t,s}qA$ if $x_{r,t,s} \nsubseteq A^c$. If $x_{r,t,s}$ is not neutrosophic quasi-coincident with A, we write $x_{r,t,s}\tilde{q}A$.

(2) A neutrosophic set A is said to be *neutrosophic quasi-coincident* (neutrosophic q-coincident, for short) with B, denoted by AqB if $A \nsubseteq B^c$. If A is not neutrosophic quasi-coincident with B, we denote it by $A\tilde{q}B$.

Definition 2 Let (X, τ) be a neutrosophic topological space. Then:

(1) A neutrosophic set A in (X, τ) is said to be a *neutrosophic q-neighborhood* of a neutrosophic point $x_{r,t,s}$ if there exists a neutrosophic open set B such that $x_{r,t,s}qB \subset A$.

(2) A neutrosophic point $x_{r,t,s}$ in (X, τ) is said to be a *neurosophic cluster point* of a neutrosophic set A if every neutrosophic open q-neighborhood B of $x_{r,t,s}$ is q-coincident with A. The union of all neutrosophic cluster points of A is called the *neutrosophic closure* of A and is denoted by $\overline{A}$.

Definition 3 Let (X, τ) be a neutrosophic topological space.

(1) A *neutrosophic sequence* in (X, τ) is a function $S : \mathbb{N} \to (X, \tau)$ from the set $\mathbb{N}$ of natural numbers to (X, τ). We write $\mathbf{x} = \{x_{n_{rn,tn,sn}}\}_{n\in\mathbb{N}}$ to denote a sequence in (X, τ).

(2) A neutrosophic sequence $\mathbf{x} = \{x_{n_{rn,tn,sn}}\}_{n\in\mathbb{N}}$ in (X, τ) *converges to a neutrosophic point* $x_{r,t,s}$ in (X, τ) (written $x_{n_{rn,tn,sn}} \to x_{r,t,s}$) if for each neutrosophic q-neighbourhood U of $x_{r,t,s}$ there exists $n_0 \in \mathbb{N}$ such that $x_{n_{rn,tn,sn}}qU$ for all $n \geq n_0$.

(3) $\mathsf{s(X)}$ and $\mathsf{c(X)}$ denote the set of all neutrosophic sequences in (X, τ) and the set of all convergent neutrosophic sequences in (X, τ), respectively.

Definition 4 Let (X, τ) be a neutrosophic topological space.

(1) (X, Δ) is a *neutrosophic group* on X if Δ is a binary operation defined on X such that the following conditions hold:

(a) Associativity: For all neutrosophic points $x1_{r_1,t_1,s_1}, x2_{r_2,t_2,s_2}, x3_{r_3,t_3,s_3}$ in (X, τ), we have $x1_{r_1,t_1,s_1} \Delta \left(x2_{r_2,t_2,s_2} \Delta\, x3_{r_3,t_3,s_3}\right)$.

(b) Identity: There exists an identity neutrosophic point $e_{\alpha,\beta,\gamma}$ in (X, τ) such that $x1_{r_1,t_1,s_1} \Delta\, e_{\alpha,\beta,\gamma} = e_{\alpha,\beta,\gamma} \Delta\, x1_{r_1,t_1,s_1} = x1_{r_1,t_1,s_1}$ for any neutrosophic point $x1_{r_1,t_1,s_1}$ in (X, τ).

(c) Inverse: For any neutrosophic point $x1_{r_1,t_1,s_1}$ in (X, τ), there exists an inverse neutrosophic point $\left(x1_{r_1,t_1,s_1}\right)^{-1}$ in (X, τ) such that $x1_{r_1,t_1,s_1} \Delta \left(x1_{r_1,t_1,s_1}\right)^{-1} = e_{\alpha,\beta,\gamma}$ and $\left(x1_{r_1,t_1,s_1}\right)^{-1} \Delta x1_{r_1,t_1,s_1} = e_{\alpha,\beta,\gamma}$.

(2) Let $(\mathsf{s(X)}, *)$ be the group of neutrosophic sequences in X, and (X, Δ) be a neutrosophic group. A *neutrosophic method* is a function G defined on a subgroup $(\mathsf{c_G(X)}, *)$ of $(\mathsf{s(X)}, *)$ such that $G(\mathbf{x}*\mathbf{y}) = G(\mathbf{x})\Delta G(\mathbf{y})$ for all neutrosophic sequences $\mathbf{x}$, $\mathbf{y}$ in (X, τ).

(3) A neutrosophic sequence $\mathbf{x} = \left\{x_{n_{rn,tn,sn}}\right\}_{n\in N}$ is said to be *G-convergent* to $x_{r,t,s}$, if $\mathbf{x} \in \mathsf{c_G(X)}$ and $G(\mathbf{x}) = x_{r,t,s}$.

(4) A neutrosophic method G is called *neutrosophic regular* if every convergent neutrosophic sequence $\mathbf{x} = \{x_{n_{rn,tn,sn}}\}_{n\in\mathbb{N}}$ is G-convergent with $G(\mathbf{x}) = x_{r,t,s}$, where $\mathbf{x}$ converges to $x_{r,t,s}$.

G-sequential continuity

In this section we introduce G-continuity and G-sequential continuity and explore some their properties.

Definition 5 Let A be a neutrosophic set and $x_{r,t,s}$ be a neutrosophic point in a neutrosophic topological space (X,τ). Then $x_{r,t,s}$ is in the *neutrosophic G-sequential closure of A* (or in the *neutrosophic G-hull of A*), if there is a neutrosophic sequence $\mathbf{x} = \{x_{n_{r_n,t_n,s_n}}\}_{n\in\mathbb{N}}$ in A such that $G(\mathbf{x}) = x_{r,t,s}$. We denote neutrosophic G-sequential closure of A by $\overline{A}^G$. We say that a neutrosophic set is *G-sequentially neutrosophic closed* if it contains all the neutrosophic points in its neutrosophic G-closure.

If G is a neutrosophic regular method, then

$$A \subset \overline{A} \subset \overline{A}^G.$$

Hence, A is G-sequentially closed if $\overline{A}^G = A$. Even for neutrosophic regular methods, it is not always true that $\overline{(\overline{A}^G)}^G = \overline{A}^G$ as the following example shows.

Example 1 Let $X = [0,1]$ and let τ be a neiutrosophic topology on X. Consider the neutrosophic method G defined as

$$G(\mathbf{x}) = u_{0.5,0.5,0.5}, \text{ where } u = \lim_{n\to\infty} \frac{x_n + x_{n+1}}{2}$$

for a neutrosophic sequence $\mathbf{x} = \left\{x_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ in X. Further, consider a neutrosophic set $A = 0_{0.5,0.5,0.5} \cup 1_{0.5,0.5,0.5}$. Then,

$$\overline{A}^G = 0_{0.5,0.5,0.5} \cup 0.5_{0.5,0.5,0.5} \cup 1_{0.5,0.5,0.5}$$

and

$$\overline{\left(\overline{A}^G\right)}^G = 0_{0.5,0.5,0.5} \cup 0.25_{0.5,0.5,0.5} \cup 0.5_{0.5,0.5,0.5} \cup 0.75_{0.5,0.5,0.5} \cup 1_{0.5,0.5,0.5}.$$

So, $\overline{\left(\overline{A}^G\right)}^G \neq \overline{A}^G$.

One of properties of neutrosophic G-sequential closure is given in the following lemma.

Lemma 1 *Let (X,τ) be a neutrosophic topological space, A and B neutrosophic subsets of (X,τ), G a neutrosophiv regular method. Then*

$$\overline{A}^G \cup \overline{B}^G \subset \overline{A \cup B}^G.$$

The next example shows that the converse inclusion is not always satisfied.

Example 2 Take the neutrosophic topological space (X,τ) from the previous example. Consider neutrosophic sets $A = 0_{0.5,0.5,0.5}$ and $B = 1_{0.5,0.5,0.5}$. Then

$$\overline{A}^G \cup \overline{B}^G = 0_{0.5,0.5,0.5} \cup 1_{0.5,0.5,0.5}$$

and

$$\overline{A \cup B}^G = 0_{0.5,0.5,0.5} \cup 0,5_{0.5,0.5,0.5} \cup 1_{0.5,0.5,0.5}.$$

Definition 6 A subset A of X is called *G-sequentially neutrsophic compact* if for any neutrosophic sequence $\mathbf{x} = \{x_{n_{r_n,t_n,s_n}}\}_{n\in\mathbb{N}}$ in A there is a subsequence $\mathbf{y} = \left\{x_{n_{r_{n_k},t_{n_k},s_{n_k}}}\right\}_{k\in\mathbb{N}}$ of $\mathbf{x}$ with $G(\mathbf{y}) \in A$.

Definition 7 A function $f : X \to Y$ is *neutrosophic G-sequentially continuous at a neutrosophic point $u_{r,t,s}$*, if for any sequence $\mathbf{x} = \{x_{n_{r_n,t_n,s_n}}\}_{n\in\mathbb{N}}$ in X , $G(\mathbf{x}) = u_{r,t,s}$ implies $G(f(\mathbf{x})) = f(u_{r,t,s})$. We say that f is G-sequentially continuous on a neutrosophic subset D of X, if it is neutrosophic G-sequentially continuous at every $u_{r,t,s} \in D$ and is neutrosophic G-sequentially continuous if it is neutrosophic G-sequentially continuous on X.

Theorem 1 *The image of any neutrosophic G-sequentially compact subset of X under a neutrosophic G-sequentially continuous function is neutrosophic G-sequentially compact.*

Definition 8 A neutrosophic method G is called *neutrosophic subsequential* if for any neutrosophic sequence $\mathbf{x}$ such that $G(\mathbf{x}) = x_{r,t,s}$, there exists a subsequence $\left\{x_{n_{r_{n_k}},t_{n_k},s_{n_k}}\right\}_{k\in\mathbb{N}}$ of $\mathbf{x}$ that converges to $x_{r,t,s}$.

We give some results involving a neutrosophic G-subsequential method.

Theorem 2 *Let G be a neutrosophic regular method and A be any neutrosophic subset of X. Then, $\overline{A}^G = \overline{A}$ if and only if G is a neutrosophic subsequential method, where $\overline{A}$ denotes the usual closure of the neutrosophic set A.*

Theorem 3 *Let G be a neutrosophic regular method. Then G is a neutrosophic subsequential method if and only if $A^b = (A^b)^G$ for every neutrosophic subset A of X, where A^b and $(A^b)^G$ are neutrosophic boundary and neutrosophic G-sequential boundary of A.*

Theorem 4 *(1) Let G be a neutrosophic regular subsequential method. Then every neutrosophic G-sequentially continuous function is neutrosophic continuous in the ordinary sense.*

(2) Let G be a neutrosophic regular method. If every continuous function is neutrosophic G-sequentially continuous, then G is a neutrosophic subsequential method.

We define now neutrosophic G-sequentially closed mappings and and give one their charcaterization.

Definition 9 A function $f X \to Y$ is called *neutrosophic G-sequentially closed* if $f(K)$ is neutrosophic G-sequentially closed in Y for every neutrosophic G-sequentially closed neutrosophic subset K of X.

Theorem 5 *Let G be a neutrosophic regular method. A function f is G-sequentially closed if and only if $\overline{(f(A))}^G \subset f(\overline{A}^G)$ for every neutrosophic subset A of X.*

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