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An Interpretation of *G*-Continuity in Neutrosophic Soft Topological Spaces

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Abstract. Scientists have always adopted the concept of sequential continuity as an indispensable subject, not only in Topology but also in some other branches of Mathematics. Connor and Grosse-Erdmann gave this concept for real functions by using an arbitrary linear functional G defined on a linear subspace of the vector space of all real sequences instead of lim. Afterwards, this concept were adapted to a topological group X by replacing a linear functional G with an arbitrary additive function defined on a subgroup of the group of all X-valued sequences. Furthermore, alternative theorems in generalized setting were given and varied theorems that had not been achieved for real functions were presented. In this investigation, we offer neutrosophic soft G-continuity and analyze its nature in neutrosophic soft topological spaces.
Keywords: Neutrosophic soft sequences, neutrosophic soft quasi-coincidence, neutrosophic soft q-neighborhood, neurosophic soft cluster point, neutrosophic soft boundary point, neutrosophic soft sequential closure, neutrosophic soft group, neutrosophic soft method, neutrosophic soft G-sequential continuity, neutrosophic soft function.
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INTRODUCTION

In almost all mathematical disciplines continuity plays a vital role. Any concept connected with it constitutes an integral part in all disciplines that involve Mathematics, such as computer science, information theory, biological science and dynamical systems. Continuity has always been considered as a focus point in numerous investigations. In these investigations, various concepts related to it were introduced and these concepts were used by scholars to further their studies. Sequential continuity has always been adopted as one of major concepts related to continuity. In [3], Connor and Grosse-Erdmann made the convergence of sequences gain a new identity by using the structure of sequential continuity. Furthermore, Cakalli [2] interpreted this identity by using topological group-valued sequences, offered theorems in this generalized setting some of which were not only new for topological groups, but also new for the real case. In this paper, our goal is to make these ideas acquire new notions in neutrosophic soft topological spaces and offer concepts related to them.

Preliminaries

In this section, some fundamental definitions related to neutrosophic set theory are reminded.

Definition 1 ([5]) A neutrosophic set A on the universe set X is defined as:

$$A = \{\langle x, T_A(x), I_A(x), F_A x\rangle : x \in X\},$$

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where

$$T, I, F : X \to]^{-}0, 1^{+}[\text{ and } ^{-}0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+}.$$

Since the supremum of each T, I, F is 1, the inequality

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$$

is obvious.

Definition 2 ([4]) Let X be an initial universe set and E be a set of parameters. Let $P(X)$ denote the set of all neutrosophic sets of X. Then a neutrosophic soft set $\left(\widetilde{H}, E\right)$ over X is a set defined by a set valued function $\widetilde{H}$ representing a mapping $\widetilde{H} : E \to P(X)$, where $\widetilde{H}$ is called the approximate function of the neutrosophic soft set $\left(\widetilde{H}, E\right)$. In other words, the neutrosophic soft set is a parametrized family of some elements of the set $P(X)$ and therefore it can be written as a set of ordered pairs:

$$\left(\widetilde{H}, E\right) = \left\{\left(e, \left\langle x, T_{\widetilde{H}(e)}(x), I_{\widetilde{H}(e)}(x), F_{\widetilde{H}(e)}(x)\right\rangle : x \in X\right) : e \in E\right\},$$

where $T_{\widetilde{H}(e)}(x)$, $I_{\widetilde{H}(e)}(x)$, $F_{\widetilde{H}(e)}(x) \in [0, 1]$ are respectively called the truth-membership, indeterminacy-membership and falsity-membership function of $\widetilde{H}(e)$.

Definition 3 ([1]) Let $NSS(X, E)$ be the family of all neutrosophic soft sets over the universe set X and $\tau \subset NSS(X, E)$. Then τ is said to be a neutrosophic soft topology on X if:

1. $0_{(X,E)}$ and $1_{(X,E)}$ belong to τ,
2. the union of any number of neutrosophic soft sets in τ belongs to τ,
3. the intersection of a finite number of neutrosophic soft sets in τ belongs to τ.

Then (X, τ, E) is said to be a neutrosophic soft topological space over X. Each member of τ is said to be a neutrosophic soft open set.

Some definitions

Definition 4 A neutrosophic soft point $x^{e}_{\alpha,\beta,\gamma}$ is said to be neutrosophic soft quasi-coincident (neutrosophic soft q-coincident, for short) with $\left(\widetilde{F}, E\right)$, denoted by $x^{e}_{\alpha,\beta,\gamma} q \left(\widetilde{F}, E\right)$ if and only if $x^{e}_{\alpha,\beta,\gamma} \nsubseteq \left(\widetilde{F}, E\right)^{c}$. If $x^{e}_{\alpha,\beta,\gamma}$ is not neutrosophic soft quasi-coincident with $\left(\widetilde{F}, E\right)$, we denote by $x^{e}_{\alpha,\beta,\gamma} \tilde{q} \left(\widetilde{F}, E\right)$.

Definition 5 A neutrosophic soft sequence in a neutrosophic soft topological space (X, τ, E) is a function $S : N \to (X, \tau, E)$, where N is the set of naturel numbers. We write $\left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ to denote the sequence of neutrosophic soft points in (X, τ, E) indexed by N.

Definition 6 A neutrosophic soft sequence $\mathbf{x} = \left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ is said to be G-convergent to $x^{e}_{\alpha,\beta,\gamma}$, if $\mathbf{x} \in c_G(X)$ and $G(\mathbf{x}) = x^{e}_{\alpha,\beta,\gamma}$.

Definition 7 A neutrosophic soft subset $\left(\widetilde{H}, E\right)$ of X is called G-sequentially neutrsophic soft compact if for any neutrosophic soft sequence $\mathbf{x} = \left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ a of neutrosophic soft points in $\left(\widetilde{F}, E\right)$, there is a subsequence $\mathbf{y} = \left\{x^{e_{n_k}}_{n_{k r_{n_k}, t_{n_k}, s_{n_k}}}\right\}_{n\in N}$ of $\mathbf{x}$ with $G(\mathbf{y}) \in \left(\widetilde{F}, E\right)$.

Definition 8 A neutrosophic soft function $(\Phi, \Psi) : (X, \tau, E) \to (X, \tau, E)$ is neutrosophic soft G-sequentially continuous at a neutrosophic soft point $u^{e}_{r,t,s}$, if, for any given a sequence $\mathbf{x} = \left\{x^{e_n}_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ of neutrosophic soft points in X, $G(\mathbf{x}) = u^{e}_{r,t,s}$ implies that $G((\Phi, \Psi)(\mathbf{x})) = (\Phi, \Psi)(u^{e}_{r,t,s})$. For a neutrosophic soft subset $\left(\widetilde{D}, E\right)$ of X, (Φ, Ψ) is called neutrosophic soft G-sequentially continuous on $\left(\widetilde{D}, E\right)$, if it is neutrosophic G-sequentially continuous at every $u^{e}_{r,t,s} \in \left(\widetilde{D}, E\right)$ and is neutrosophic soft G-sequentially continuous, if it is neutrosophic soft G-sequentially continuous on X.

Theorem 1 *The image of any neutrosophic soft G-sequentially compact subset of X under a neutrosophic soft G-sequential continuous function is neutrosophic soft G-sequentially compact.*

Conclusion

The concept of neutrosophic G-sequential continuity is presented as a new tool to the scientists to further their studies. Also, we introduced the concepts of neutrosophic soft sequence, neutrosophic soft quasi-coincidence, neutrosophic soft q-neighborhood, neurosophic soft cluster point, neutrosophic soft boundary point, neutrosophic soft sequential closure, neutrosophic soft group, neutrosophic soft method. These concepts constitute a base to define also the concepts of neutrosophic soft G-sequential closure and neutrosophic soft G-sequential derived set. Moreover, the concept of G-sequentially neutrsophic soft compactness is introduced. Its properties are analyzed and some implications are given. It is also shown by counterexamples that the converse statements of these implications are not always true. Since topological structures of neutrosophic soft sets carry great importance for numerous mathmaticians, various concepts related to the other types of topological spaces, which constitute advantageous situations in different fields, have been adapted to neutrosophic topological spaces. Our expectation is that many scientists will take advantage of using these detections to advance in their research not only in mathematics, but also in different disciplines which use mathematical methods. We also hope that these findings may constitute a general framework for their applications in practical life.

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