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A Study on Connectedness in Neutrosophic Topological Spaces

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Abstract. In this study, we introduce the concept of neutrosophic connectedness and give some of its characterizations. Additionally, we present neutrosophic product space and show that this type of connectedness is not preserved under neutrosophic product spaces. We also introduce the notions of neutrosophic super-connected spaces, neutrosophic strongly connected spaces and study their properties.
Keywords: Neutrosophic connectedness, neutrosophic super-connectedness, neutrosophic strong connectedness, neutrosophic interior point, neutrosophic regular open set, neutrosophic regular closed set, neutrosophic semi-open set, neutrosophic semi-closed set, neutrosophic function, neutrosophic base, neutrosophic subbase, neutrosophic product topology.
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INTRODUCTION

In [3], Smarandache presented the concept of neutrosophic set as a new instrument to the world of Mathematics. Salama and Alblowi [2] studied the topological structure of the family of neutrosophic sets and introduced the concept of neutrosophic topological space by using membership functions, indeterminacy functions and non-membership functions, each of which have one-to-one correspondence between the members of X and $]^-0, 1^+[$. This new type of topological spaces has been adopted as a more advanced tool than general topological spaces. The concept of neutrosophic topology has attracted the attention of scientists and there are still many studies on this new type of topology. In this paper, the concept of neutrosophic connectedness is presented and its properties are investigated. Also, we present neutrosophic super-connectedness, neutrosophic strong connectedness and focus on their characterizations in neutrosophic topological spaces.

Definition 1 ([3]) A neutrosophic set A on the universe set X is defined as:

$$A = \{\langle x, T_A(x), I_A(x), H_A(x)\rangle : x \in X\},$$

where $T, I, H : X \to]^-0, 1^+[$ and $^-0 \leq T_A(x) + I_A(x) + H_A(x) \leq 3^+$.

Membership functions T, indeterminacy functions I and non-membership functions F of a neutrosophic set take value from real standard or nonstandard subsets of $]^-0, 1^+[$. However, these subsets are sometimes inconvenient to be used in real life applications such as economical and engineering problems. On account of this fact, we consider the neutrosophic sets, whose membership function, indeterminacy function and non-membership function take values from subsets of $[0, 1]$.

Definition 2 ([1]) Let X be a nonempty set. If r, t, s are real standard or nonstandard subsets of $]^-0, 1^+[$, then the neutrosophic set $x_{r,t,s}$ given by

$$x_{r,t,s}(x_p) = \begin{cases} (r, t, s), & \text{if } x = x_p, \\ (0, 0, 1), & \text{if } x \neq x_p; \end{cases}$$

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is called a neutrosophic point in X. $x_p \in X$ is called the support of $x_{r,t,s}$, where r denotes the degree of membership value, t denotes the degree of indeterminacy, and s is the degree of non-membership value of $x_{r,t,s}$.

It is clear that every neutrosophic set is the union of its neutrosophic soft points.

Definition 3 ([2]) Let A_1 and A_2 be two neutrosophic soft sets over the universe set X. Then, their union is denoted by $A_1 \cup A_2 = A_3$ and is defined by $A_3 = \{\langle x, T_{A_3}(x), I_{A_3}(x), F_{A_3}(x)\rangle : x \in X\}$, where

$$T_{A_3}(x) = \max\{T_{A_1}x), T_{A_2}(x)\},$$
$$I_{A_3}(x) = \max\{I_{A_1}(x), I_{A_2}(x)\},$$
$$F_{A_3}(x) = \min\{F_{A_1}(x), F_{A_2}(x)\}.$$

Definition 4 ([2]) Let A_1 and A_2 be two neutrosophic soft sets over the universe set X. Then, their intersection is denoted by $A_1 \cap A_2 = A_3$ and defined by $A_3 = \{\langle x, T_{A_3}(x), I_{A_3}(x), F_{A_3}(x)\rangle : x \in X\}$, where

$$T_{A_3}(x) = \min\{T_{A_1}(x), T_{A_2}(x)\},$$
$$I_{A_3}(x) = \min\{I_{A_1}(x), I_{A_2}(x)\},$$
$$F_{A_3}(x) = \max\{F_{A_1}(x), F_{A_2}(x)\}.$$

Definition 5 ([2]) A neutrosophic set A over the universe set X is said to be a null neutrosophic set if $T_A(x) = 0,\ I_A(x) = 0,\ F_A(x) = 1,\ \forall x \in X$. It is denoted by 0_X .

Definition 6 ([2]) A neutrosophic set A over the universe set X is said to be an absolute neutrosophic set if $T_A(x) = 1,\ I_A(x) = 1,\ F_A(x) = 0,\ \forall x \in X$. It is denoted by 1_X.

Definition 7 ([2]) Let $NS(X)$ be the family of all neutrosophic sets over the universe set X and $\tau \subset NS(X)$. Then, τ is said to be a neutrosophic topology on X, if:
1. 0_X and 1_X belong to τ,
2. The union of any number of neutrosophic sets in τ belongs to τ,
3. The intersection of a finite number of neutrosophic sets in τ belongs to τ.

Then, (X, τ) is said to be a neutrosophic topological space over X. Each member of τ is said to be a neutrosophic open set.

Definition 8 ([2]) Let (X, τ) be a neutrosophic topological space over X and A be a neutrosophic set over X. Then A is said to be a neutrosophic closed set iff its complement is a neutrosophic open set.

Definitions

Definition 9 A neutrosophic point $x_{r,t,s}$ is said to be neutrosophic quasi-coincident (neutrosophic q-coincident, for short) with a neutrosophic set H, denoted by $x_{r,t,s}qH$ if and only if $x_{r,t,s} \not\subseteq H^c$. If $x_{r,t,s}$ is not neutrosophic quasi-coincident with H, we denote by $x_{r,t,s}\tilde{q}H$.

Definition 10 A neutrosophic set G is said to be neutrosophic quasi-coincident (neutrosophic q-coincident, for short) with H, denoted by GqH if and only if $G \not\subseteq H^c$. If G is not neutrosophic quasi-coincident with H, we denote by $G\tilde{q}H$.

Definition 11 A neutrosophic point $x_{r,t,s}$ is said to be a neurosophic interior point of a neutrosophic set H if and only if there exists a neutrosophic open q-neighborhood G of $x_{r,t,s}$ such that $G \subset H$. The union of all neutrosophic interior points of H is called the neutrosophic interior of H and denoted by H°.

Definition 12 A neutrosophic point $x_{r,t,s}$ is said to be a neurosophic cluster point of a neutrosophic set H if and only if every neutrosophic open q-neighborhood G of $x_{r,t,s}$ is q-coincident with H. The union of all neutrosophic cluster points of H is called the neutrosophic closure of H and denoted by $\overline{H}$.

Definition 13 A neutrosophic set H in a neutrosophic topological space (X, τ) is called a neutrosophic regular open (neutrosophic regular closed) set if and only if $H = \left(\overline{H}\right)^{\circ}$ $\left(H = \overline{(H^{\circ})}\right)$. The complement of a neutrosophic regular open set is a neutrosophic regular closed set.

Definition 14 A neutrosophic set H in a neutrosophic topological space (X, τ) is called a neutrosophic semi-open set if and only if there exists a neutrosophic open set K such that $K \subset H \subset \overline{K}$. A neutrosophic set H is neutrosophic semi-open if and only if $H \subset \overline{(H^{\circ})}$.The complement of a neutrosophic semi-open open set is called a neutrosophic semi-closed set. Equivalently, a neutrosophic set H in a neutrosophic topological space (X, τ) is called a neutrosophic semi-closed set if and only if there exists a neutrosophic closed set G such that $G^{\circ} \subset H \subset G$. A neutrosophic set H is neutrosophic semi-closed if and only if $\left(\overline{H}\right)^{\circ} \subset H$.

Let (X, τ) be a neutrosophic topological space and $A \subset X$. In this paper, we denote any neutrosophic set, whose supports are members of A, by μ_A. It means that, if $x \in X - A$, then $T_{\mu_A}(x) = 0$, $I_{\mu_A}(x) = 0$ and $F_{\mu_A}(x) = 1$. Otherwise, $0 \leq T_{\mu_A}(x) \leq 1$, $0 \leq I_{\mu_A}(x) \leq 1$ and $0 \leq F_{\mu_A}(x) \leq 1$.

Neutrosophic connectedness

Definition 15 A neutrosophic topological space (X, τ) said to be neutrosophic connected, if it has no proper neutrosophic clopen set (neutrsphic closed and open).(A neutrosophic set μ is in (X, τ) is said to be proper, if it is neither the null neutrosophic set, nor the absolute neutrosophic set.)

Theorem 1 *A neutrosophic topological space (X, τ) is neutrosophic connected if and only if there are no any neutrosophic open sets A and B such that $T_A(x) = H_B(x), H_A(x) = T_B(x)$ and $I_A(x) + I_B(x) = 1$.*

Corollary 1 *A neutrosophic topological space (X, τ) is neutrosophic connected if and only if it does not contain any neutrosophic open sets A and B such that $T_A(x) = H_{\overline{B}}(x), H_A(x) = T_{\overline{B}}(x)$ and $I_A(x) + I_{\overline{B}}(x) = 1$.*

Neutrosophic connected subsets in a neutrosophic topological space

Definition 16 Let (X, τ) be a neutrosophic topological space and $Y \subseteq X$. Let H be a neutrosophic set over Y such that

$$T_H(x) = \begin{cases} 1, & x \in Y \\ 0, & x \notin Y \end{cases}, \quad I_H(x) = \begin{cases} 1, & x \in Y \\ 0, & x \notin Y \end{cases}, \quad F_H(x) = \begin{cases} 0, & x \in Y \\ 1, & x \notin Y \end{cases}$$

Let $\tau_Y = \{H \cap F : F \in \tau\}$, then (Y, τ_Y) is called neutrosophic subspace of (X, τ). If $H \in \tau$ (resp., $H^c \in \tau$), then (Y, τ_Y) is called neutrosophic open (resp., closed) subspace of (X, τ). And, the restriction of a neutrosophic set H of Y is denoted as H/Y

Definition 17 If $A \subset X$, (X, τ) is a neutrosophic topological space, then A is said to be a neutrosophic connected subset of X if A is a neutrosophic connected space as a neutrosophic subspace of X. Clearly, if $A \subset Y \subset X$, then A is a neutrosophic connected subset of the neutrosophic topological space X if and only if it is a neutrosophic connected subset of the neutrosophic subspace Y of X.

Theorem 2 *If (X, τ) is a neutrosophic topological space and Y is a neutrosophic connected subset of X. For any non-null neutrosophic open sets A and B in (X, τ) satisfying $T_A(x) = H_B(x), H_A(x) = T_B(x)$ and $I_A(x) + I_B(x) = 1$ for all $x \in X$, either $T_{A/Y}(x) = 1, I_{A/Y}(x) = 1, H_{A/Y}(x) = 0$ or $T_{B/Y}(x) = 1, I_{B/Y}(x) = 1, H_{B/Y}(x) = 0$.*

Definition 18 Let (X, τ) be a neutrosophic topological space. Then (X, τ) is said to be a neutrosophic super-connected space, if there is no any proper neutrosophic regular open set in (X, τ).

Theorem 3 *A neutrosophic topological space (X, τ) is neutrosophic super-connected if and only if does not exist any proper neutrosophic open set which is also neutrosophic semi-closed or, equivalently, if and only if does not exist any proper neutrosophic closed set which is also neutrosophic semi-open.*

Neutrosophic strong connectedness

Definition 19 Let (X, τ) be a neutrosophic topological space. (X, τ) is said to be neutrosophic strongly connected if it has no non-null neutrosophic closed sets L and K such that $K \subset L^c$. If (X, τ) is not neutrosophic strongly connected then it will be called neutrosophic weakly disconnected.

Theorem 4 *Let (X, τ) be a neutrosophic topological space. Then, it is neutrosophic strongly connected if and only if there do not exist non-absolute neutrosophic open sets λ and δ such that $\delta^c \subset \lambda$.*

Theorem 5 *Let (X, τ) be a neutrosophic topological space and H be a subset of X such that μ_H is neutrosophic closed in (X, τ). If (X, τ) is neutrosophic strongly connected then H is a neutrosophic strongly connected subset of X.*

Conclusion

We have introduced a new direction to the world of topology in adherence to neutrosofic topological spaces. We also presented the neutrosophic product space and explored the features of neutrosophic connectedness and some of its different forms presented in this article. We hope that these new concepts will be very useful, especially for other mathematical studies in topology.

REFERENCES

[1] R. Dhavaseelan, S. Jafari, C. Özel and M.A. Al Shumrani, Generalized neutrosophic contra-continuity, submitted.

[2] A.A. Salama, S.A. Alblowi, Neutrosophic set and neutrosophic topological spaces, *IOSR J. Math.* 3:4, 31–35 (2012).

[3] F. Smarandache, Neutrosophic set - a generalisation of the intuitionistic fuzzy sets, *Intern. J. Pure Appl. Math.* 24, 187–297 (2005).