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A Look on Separation Axioms in Neutrosophic Topological Spaces

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Abstract. This study is dedicated to make an attempt to define different types of separation axioms in neutrosophic topological spaces. The relationships among them are shown with a diagram and counterexamples. We also introduce some new notions, such as neutrosophic quasi-coincidence, neutrosophic q-neighborhood, neurosophic cluster point, and give a new definition for neutrosophic function.
Keywords: Neutrosophic separation axioms, neutrosophic quasi-coincidence, neutrosophic q-neighborhood, neurosophic cluster point, neutrosophic closure, neutrosophic function.
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INTRODUCTION

Undoubtedly, the concept of separation axioms has always been an indispensable subject in topology. This concept formed the basis of many valuable researches in general topology. And, these researches played very important roles in many parts of real life and the findings of these researches came to life in many applications. But, as technology advances and the industry evolves, people's needs have changed and general topology has become inadequate in the real life. So, the impact of these findings on real life has diminished. Then, scientists went on to find different types of topological spaces, and separation axioms occupied an important place in these topological spaces. In [2], Salama and Alblowi introduced the theory of neutrosophic topological spaces by using neutrosophic sets that had been defined in [3]. In this study, we present different types of separation axioms in neutrosophic topological spaces as a new instrument for the real life applications and new notions that we think benefit in other investigations.

Definition 1 ([3]) A neutrosophic set A on the universe set X is defined as:

$$A = \{\langle x, T_A(x), I_A(x), H_A(x)\rangle : x \in X\},$$

where $T, I, H : X \to]^-0, 1^+[$ and $^-0 \leq T_A(x) + I_A(x) + H_A(x) \leq 3^+$.

Membership functions T, indeterminacy functions I and non-membership functions F of a neutrosophic set take value from real standard or nonstandard subsets of $]^-0, 1^+[$. However, these subsets are sometimes inconvenient to be used in real life applications such as economical and engineering problems. On account of this fact, we consider the neutrosophic sets, whose membership functions, indeterminacy functions and non-membership functions take values from subsets of $[0, 1]$.

Definition 2 ([1]) Let X be a nonempty set. If r, t, s are real standard or nonstandard subsets of $]^-0, 1^+[$, then the neutrosophic set $x_{r,t,s}$ given by

$$x_{r,t,s}(x_p) = \begin{cases} (r, t, s), & \text{if } x = x_p, \\ (0, 0, 1), & \text{if } x \neq x_p; \end{cases}$$

is called a neutrosophic point in X, and $x_p \in X$ is called the support of $x_{r,t,s}$. Here r denotes the degree of membership value, t denotes the degree of indeterminacy, and s is the degree of non-membership value of $x_{r,t,s}$.

Clearly, every neutrosophic set is the union of its neutrosophic soft points.

Definition 3 ([2]) Let A and B be two neutrosophic soft sets over the universe set X. Then, their union is denoted by $A \cup B = C$ and is defined by $C = \{\langle x, T_C(x), I_C(x), F_C(x)\rangle : x \in X\}$, where

$$T_C(x) = \max\{T_A x), T_B(x)\},$$
$$I_C(x) = \max\{I_A(x), I_B(x)\},$$
$$F_C(x) = \min\{F_A(x), F_B(x)\}.$$

Definition 4 ([2]) Let A_1 and A_2 be two neutrosophic soft sets over the universe set X. Then the intersection of A_1 and A_2, denoted by $A_1 \cap A_2 = A_3$ is defined by $A_3 = \{\langle x, T_{A_3}(x), I_{A_3}(x), F_{A_3}(x)\rangle : x \in X\}$, where

$$T_{A_3}(x) = \min\{T_{A_1}(x), T_{A_2}(x)\},$$
$$I_{A_3}(x) = \min\{I_{A_1}(x), I_{A_2}(x)\},$$
$$F_{A_3}(x) = \max\{F_{A_1}(x), F_{A_2}(x)\}.$$

Definition 5 ([2]) A neutrosophic set A over the universe set X is said to be a null neutrosophic set if $T_A(x) = 0,\ I_A(x) = 0,\ F_A(x) = 1,\ \forall x \in X$.
It is denoted by 0_X.

Definition 6 ([2]) A neutrosophic set A over the universe set X is said to be an absolute neutrosophic set if $T_A(x) = 1,\ I_A(x) = 1,\ F_A(x) = 0,\ \forall x \in X$.
It is denoted by 1_X.

Definition 7 ([2]) Let $NS(X)$ be the family of all neutrosophic sets over the universe set X and $\tau \subset NS(X)$. Then, τ is said to be a neutrosophic topology on X, if:

1. 0_X and 1_X belong to τ,
2. The union of any number of neutrosophic sets in τ belongs to τ ,
3. The intersection of a finite number of neutrosophic sets in τ belongs to τ.

Then (X, τ) is said to be a neutrosophic topological space over X. Each member of τ is said to be a neutrosophic open set.

Definition 8 ([2]) Let (X, τ) be a neutrosophic topological space over X and A be a neutrosophic set over X. Then A is said to be a neutrosophic closed set if its complement is a neutrosophic open set.

Some definitions

Definition 9 A neutrosophic point $x_{r,t,s}$ is said to be neutrosophic quasi-coincident (neutrosophic q-coincident, for short) with H, denoted by $x_{r,t,s}qH$ if and only if $x_{r,t,s} \nsubseteq H^c$. If $x_{r,t,s}$ is not neutrosophic quasi-coincident with H, we denote by $x_{r,t,s}\tilde{q}H$.

Definition 10 A neutrosophic set H in a neutrosophic topological space (X, τ) is said to be a neutrosophic q-neighborhood of a neutrosophic point $x_{r,t,s}$ if and only if there exists a neutrosophic open set G such that $x_{r,t,s}qG \subset H$.

Definition 11 A neutrosophic set G is said to be neutrosophic quasi-coincident (neutrosophic q-coincident, for short) with H, denoted by GqH if and only if $G \nsubseteq H^c$. If G is not neutrosophic quasi-coincident with H, we denote by $G\tilde{q}H$.

Definition 12 A neutrosophic point $x_{r,t,s}$ is said to be a neurosophic cluster point of a neutrosophic set H if and only if every neutrosophic open q-neighborhood G of $x_{r,t,s}$ is q-coincident with H. The union of all neutrosophic cluster points of H is called the neutrosophic closure of H and is denoted by $\overline{H}$.

Definition 13 A neutrosophic topological space (X,τ) is said to be a neutrosophic T_0-space (resp. neutrosophic T_1-space) if for every pair of neutrosophic points $x_{\alpha,\beta,\gamma}$, $y_{\alpha',\beta',\gamma'}$, whose supports are different, there exist neutrosophic open sets H, G such that $x_{\alpha,\beta,\gamma} \in H$, $y_{\alpha',\beta',\gamma'} \in H^c$ (resp. and $x_{\alpha,\beta,\gamma} \in G^c$, $y_{\alpha',\beta',\gamma'} \in G$).

Example 1 Consider the set $X = \{x,y\}$ and the family $\tau = \left\{\left\{x_{\alpha,\alpha,1-\alpha}, y_{\beta,\beta,1-\beta}\right\} : \alpha,\beta \in [0,1]\right\}$. Then τ is a neutrosophic topology over X. It is easily seen that (X,τ) is a neutrosophic T_1-space. But, the neutrosophic point $x_{0,2,0,2,0,7}$ is not closed in τ. Because, $x_{0,2,0,2,0,7} \neq \overline{x_{0,2,0,2,0,7}}$.

Definition 14 A neutrosophic topological space τ is said to be a neutrosophic T_2-space if for every pair of neutrosophic points $x_{\alpha,\beta,\gamma}, y_{\alpha',\beta',\gamma'}$, whose supports are different, there exists neutrosophic open sets H, G such that $x_{\alpha,\beta,\gamma} \in H$, $y_{\alpha',\beta',\gamma'} \in H^c$, $y_{\alpha',\beta',\gamma'} \in G$, $x_{\alpha,\beta,\gamma} \in G^c$ and $H\tilde{q}G$.

For a neutrosophic topological space (X,τ) we have the following diagram:

$$\begin{array}{c} \text{neutrosophic } T_2\text{-space} \\ \downarrow \\ \text{neutrosophic } T_1\text{-space} \\ \downarrow \\ \text{neutrosophic } T_0\text{-space} \end{array}$$

Converse statements may not be true as shown in the examples below;

Example 2 Consider the set $X = \{x,y\}$ and the family

$$\tau = \left\{\left\{x_{\alpha,\alpha,1-\alpha}, y_{\beta,\beta,1-\beta}\right\} : \alpha \in [0,1], \beta \in [0,1)\right\}.$$

Then, τ is a neutrosophic topology over X. It is easily seen that (X,τ) is a neutrosophic T_0-space. But, it is not a neutrosophic T_1-space, because, $x_{1,1,0}$ and $y_{1,1,0}$ are neutrosophic points in (X,τ) with different supports and the only neutrosophic open set that contains $y_{1,1,0}$ is 1_X.

Example 3 Let $X = N$ be the set of naturel numbers. For any $n \in N$, $n_{1,1,0}$ is a neutrosophic point. Clearly, there is a one-to-one compability between N and $\{n_{1,1,0} : n \in N\}$. Then, we can define a cofinite topology on $\{n_{1,1,0} : n \in N\}$. That is, a neutrosophic set H is neutrosophic open if and only if it is constituted by discarding a finite number of elements from $\{n_{1,1,0} : n \in N\}$. Hence, this cofinite topological space is a neutrosophic T_1-space. But, it is not a neutrosophic T_2-space.

Neutrosophic R_i-spaces, i=0, 1

Definition 15 A neutrosophic topological space (X,τ) is said to be a neutrosophic R_0-space if and only if for any two neutrosophic points $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, if $x_{\alpha,\beta,\gamma}\tilde{q}y_{\alpha',\beta',\gamma'}$ then $x_{\alpha,\beta,\gamma}\tilde{q}\overline{y_{\alpha',\beta',\gamma'}}$.

Definition 16 A neutrosophic topological space (X,τ) is said to be a neutrosophic R_1-space if and only if for any two neutrosophic points $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ if $x_{\alpha,\beta,\gamma}\tilde{q}\overline{y_{\alpha',\beta',\gamma'}}$, then there exist two neutrosophic open sets H and G in (X,τ) such that $x_{\alpha,\beta,\gamma} \in H$, $y_{\alpha',\beta',\gamma'} \in G$ and $H\tilde{q}G$.

Neutrosophic regular, normal and T_i-Spaces, $i = 3, 4$

Definition 17 A neutrosophic topological space (X,τ) is said to be a neutrosophic regular (neutrosophic R_2-space, for short) space if and only if, for any neutrosophic points $x_{\alpha,\beta,\gamma}$ and any neutrosophic closed set K in (X,τ) such that $x_{\alpha,\beta,\gamma}\tilde{q}K$, there exist two neutrosophic open sets H and G in (X,τ) such that $x_{\alpha,\beta,\gamma} \in H$, $K \subset G$ and $H\tilde{q}G$.

Definition 18 A neutrosophic topological space (X,τ) is said to be a neutrosophic normal (neutrosophic R_3-space, for short) space if and only if for any two neutrosophic closed sets P and K in (X,τ) such that $P\tilde{q}K$, there exists two neutrosophic open sets H and G in (X,τ) such that $P \subset H$, $K \subset G$ and $H\tilde{q}G$.

Definition 19 A neutrosophic topological space (X,τ) is said to be a neutrosophic T_3-space if and only if it is both a neutrosophic R_2 and neutrosophic T_1-space.

Definition 20 A neutrosophic topological space (X,τ) is said to be a neutrosophic T_4-space if and only if it is both a neutrosophic R_3 and neutrosophic T_1-space.

Theorem 1 *Let (X,τ) be a neutrosophic topological space. If (X,τ) is a neutrosophic T_4-space, then it is a neutrosophic T_3-space.*

Theorem 2 *Let (X,τ) be a neutrosophic topological space. If (X,τ) is a neutrosophic T_3- space, then it is a neutrosophic T_2-space.*

Conclusion

We have brought a new perspective to the world of topology on separation axioms in neutrosophic topological spaces. In addition, we have given a new definition for neutrosophic function that we think will benefit the other mathematical studies especially in topology. It is our wish that this study and the new notions and concepts we offer will help other scientists around the world to create new fields of work and make inventions that will benefit people.

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