DISCRETE AND CONTINUOUS
DYNAMICAL SYSTEMS SERIES S
Volume **14**, Number **7**, July **2021**

doi:10.3934/dcdss.2020178

pp. **2571–2589**

EXISTENCE AND UNIQUENESS RESULTS FOR A SMOKING MODEL WITH DETERMINATION AND EDUCATION IN THE FRAME OF NON-SINGULAR DERIVATIVES

Sümeyra Uçar*

Department of Mathematics, Balıkesir University
Balıkesir 10145, Turkey

Abstract. These days, it is widely known that smoking causes numerous diseases, as well as resulting in many avoidable losses of life globally, and therefore encumbers the society with enormous unnecessary burdens. The aim of this study is to examine in-depth a smoking model that is mainly influenced by determination and educational actions via CF and AB derivatives. For both fractional order models, the fixed point method is used, which allows us to follow the proof of existence and the results of uniqueness. The effective properties of the above-mentioned fractional models are theoretically exhibited, their results are confirmed by numerical graphs by various fractional orders.

1. **Introduction.** Today, it is widely recognized that smoking does not bring about any benefits; on the contrary, each and every segment of the society is now aware of the numerous hazards that smoking engenders. For instance, your skin becomes deprived of moisture and elasticity, you become more likely to suffer from hypertension, your DNA becomes prone to detrimental effects, your immune system is rendered weaker, and you come face to face with worrisome issues involving economy, pregnancy, overall health, risk of untimely death, and so on. Moreover, many types of cancer, such as lung, mouth, and throat cancers take their source from the hazardous but often-unseen impact of smoking on our health according to researchers. In brief, the hazards originating from the smoking habit result in serious complications in both individual and social spheres. It has been formidably anticipated that no fewer than 7 million people lose their lives due to smoking-related problems every year worldwide. Taking all of these unfavorable facts into consideration, scientists from a multitude of fields seek to vanquish this dangerous habit so that they may expect to extend the health span of human beings. And many of those researchers concentrate on mathematical models to demonstrate the most fitting representation the dynamics of smoking, and thus assist in diminishing the number of smokers or prevent many others from starting smoking (for more details one can see [20], [14], [22], [13] and references therein).

Researchers from various disciplines including physics, neural networks, biology and health sciences have taken extensive interest in the fractional calculus with the features of memorial construction and hereditary properties [24], [29], [30], [11], [12],

2020 *Mathematics Subject Classification.* Primary: 47H10, 26A33, 34A12; Secondary: 00A71.
Key words and phrases. Caputo-Fabrizio derivative, Atangana-Baleanu derivative, fixed point theory, existence and uniqueness theory, mathematical model.

* Corresponding author: Sümeyra Uçar.

[16], [17], [36], [31], [28], [38], [23]. This is why there are different fractional derivatives; Riemann-Liouville (RL) and Caputo operators are the most conventional examples in established usage. Although the said derivatives show more accurate in characterizing real phenomena compared to the integer order derivatives, their kernel functions generate singularities which result in a multitude of computational deficiencies. A new derivative that is non-singular and incorporates exponential law kernel has latterly been brought into operation by Caputo and Fabrizio [15], and aptly titled after them as Caputo-Fabrizio (CF) derivative. In this direction, Atangana and Baleanu [6] have presented a novel version of non-singular derivative with Mittag-Leffler kernel named as Atangana-Baleanu (AB) derivative. It is evident from the researches which has been carried out pursuant to these advancements in the recent years that these fractional derivatives provide the scientists a good chance to describe diverse problems. For example, benefiting from the Caputo-Fabrizio derivative, Atangana et al. [8] examined Baggs and Freedman model, Ullah et al. [41] investigated the dynamics of tuberculosis infection, Singh et al. [35] studied existence and uniqueness theorems about giving up smoking dynamics. Furthermore, by concentrating AB derivative, Atangana and Koca [7] analyzed a nonlinear chaotic system, Gomez-Aguilar [19] examined a nonlinear alcoholism model under the influence of Twitter, Singh et al. [33] gave a new analysis of fractional fish farm model, Veeresha et al. [42] invesitgated fractional extended Fisher–Kolmogorov equation. Other remarkable studies can be found in [26], [34], [1], [5], [25], [40], [10], [18], [44], [3], [4], [2], [21], [32], [39].

Therefore, inspired by the applicability of non-singular derivatives, we aim to further investigate a smoking model in the view of fractional concept. The rest of the paper is constructed as follows: We present some preliminaries related to CF and AB derivatives in the next section. The mathematical formulation of smoking model with determination and education is presented in Section 3. In Section 4, by means of fixed point theory, it is proved existence and uniqueness of solutions for our fractional smoking models equipped with exponential kernel and Mittag Leffler kernel sense. In Section 5, some numerical results are put into place and these outcomes shortly commented. At the end of study, we see the conclusions.

2. **Some preliminaries.**

In this part, we give basic definitions related to the Caputo-Fabrizio and Atangana-Baleanu fractional derivatives.

Definition 2.1. [15] Let $a < b$, $g \in H^1(a,b)$ and $\sigma \in [0,1]$, the Caputo-Fabrizio derivative is given by

$$D_t^\sigma [g(t)] = \frac{M(\sigma)}{1-\sigma} \int_a^t g'(x) \exp\left[-\sigma \frac{t-x}{1-\sigma}\right] dx, \tag{1}$$

where $M(\sigma)$ is a normalization function satisfying $M(0) = M(1) = 1$. If $g \notin H^1(a,b)$, this derivative can be rearranged as below:

$$D_t^\sigma [g(t)] = \frac{\sigma M(\sigma)}{1-\sigma} \int_a^t (g(t) - g(x)) \exp\left[-\sigma \frac{t-x}{1-\sigma}\right] dx. \tag{2}$$

Remark 1. If $\eta = \frac{1-\sigma}{\sigma} \in [0,\infty)$, $\sigma = \frac{1}{1+\eta} \in [0,1]$, then Eq. (2) is of the form:

$$D_t^{\sigma}[g(t)] = \frac{N(\eta)}{\eta} \int_a^t g'(x) \exp\left[-\frac{t-x}{\eta}\right] dx,$$

with $N(0) = N(\infty) = 1$. Additionally,

$$\lim_{\eta \to 0} \frac{1}{\eta} \exp\left[-\frac{t-x}{\eta}\right] = \delta(x-t).$$

The related integral of the new derivative was proposed by Nieto and Losada [2], [27].

Definition 2.2. Let $0 < \sigma < 1$ and g be a function. The fractional integral of order σ is defined by [2], [27]:

$$I_t^{\sigma}[g(t)] = \frac{2(1-\sigma)}{(2-\sigma)M(\sigma)} g(t) + \frac{2\sigma}{(2-\sigma)M(\sigma)} \int_0^t g(s)\, ds, \quad t \geq 0. \tag{3}$$

Moreover, the below result holds

$$\frac{2(1-\sigma)}{(2-\sigma)M(\sigma)} + \frac{2\sigma}{(2-\sigma)M(\sigma)} = 1,$$

then $M(\sigma) = \frac{2}{2-\sigma}$ for $0 < \sigma < 1$.

Using the above results, another form of the new Caputo derivative of order $0 < \sigma < 1$ given as [27]:

$$D_t^{\sigma}[g(t)] = \frac{1}{1-\sigma} \int_a^t g'(x) \exp\left[-\sigma \frac{t-x}{1-\sigma}\right] dx. \tag{4}$$

Definition 2.3. Let $\sigma \in [0,1]$, $a < b$ and $g \in H^1(a,b)$ be a function. The Atangana-Baleanu derivative in Caputo sense of order σ of g is defined as [6]:

$${}_a^{ABC}D_t^{\sigma}[g(t)] = \frac{F(\sigma)}{1-\sigma} \int_a^t g'(x) E_{\sigma}\left[-\sigma \frac{(t-x)^{\sigma}}{1-\sigma}\right] dx$$

where $F(\eta)$ is a normalization function such as $F(\sigma) = 1 - \sigma + \frac{\sigma}{\Gamma(\sigma)}$ with $F(0) = F(1) = 1$.

Definition 2.4. Let $\sigma \in [0,1]$, $a < b$ and $g \in H^1(a,b)$ be a function. The Atangana-Baleanu derivative in Riemann-Liouville sense of order σ of g is represented as [6]:

$${}_a^{ABR}D_t^{\sigma}[g(t)] = \frac{F(\sigma)}{1-\sigma} \frac{d}{dt} \int_a^t g(x) E_{\sigma}\left[-\sigma \frac{(t-x)^{\sigma}}{1-\sigma}\right] dx.$$

Definition 2.5. The fractional integral relative to the fractional derivative is given as [6]:

$${}_a^{AB}I_t^{\sigma}[g(t)] = \frac{1-\sigma}{F(\sigma)} g(t) + \frac{\sigma}{F(\sigma)\Gamma(\sigma)} \int_a^t g(\lambda)(t-\lambda)^{\sigma-1} d\lambda.$$

3. **Mathematical model formulation.** In this part, we deal with the model which is able to describe smoking model with determination and education in the following expression [43]: In their study, it is presumed that the aggregate number of the collection under consideration, which is denoted by N, remains fixed for the duration of modeling. The overall population is separated into three subcategories, where P signifies potential smokers; S denotes smokers and R signifies removed. The class inclined to consume tobacco products constitutes the potential smokers category, while the class that regularly consume cigarettes forms the smokers category. The class comprising both quitters and the people who have enough knowledge and education to stay away from the smoking habit for a lifetime is named removed, and denoted by R. Thus $N = P + S + Q$. The other model parameters are: Both the ratio of influx into category P and each category's ratio of death due to natural causes are indicated by a_1. The transference ratio of the smoking behavior is signified by a_2, while the ratio of departure from the smokers category is designated by a_3. The determination ratio is denoted by a_4, and the ratio involving potential smokers who shift to category R because of education a_5. The so-called mathematical model given in [43]:

$$\begin{aligned} \frac{dP(t)}{dt} &= a_1 N - a_2 \frac{PS}{N} + a_3 (1 - a_4) S - a_1 P - a_5 P, \\ \frac{dS(t)}{dt} &= a_2 \frac{PS}{N} - a_1 S - a_3 S, \\ \frac{dR(t)}{dt} &= a_3 a_4 S - a_1 R + a_5 P. \end{aligned} \tag{5}$$

with the initial conditions $P(0) \geq 0$, $S(0) \geq 0$, $R(0) \geq 0$. Because of N is constant, if we use the transformation $p = \frac{P}{N}$, $s = \frac{S}{N}$, $r = \frac{R}{N}$ in the Eq. (5), we find

$$\begin{aligned} \frac{dp(t)}{dt} &= a_1 - a_2 ps + a_3 (1 - a_4) s - a_1 p - a_5 p \\ \frac{ds(t)}{dt} &= a_2 ps - a_1 s - a_3 s \\ \frac{dr(t)}{dt} &= a_3 a_4 s - a_1 r + a_5 p. \end{aligned} \tag{6}$$

as the above same initial conditions. To extend and promote this model, we redefine the model (6) by substituting the integer order time derivative by the fractional time derivative:

$$\begin{aligned} {}_0^{CF}D_t^{\sigma}(p(t)) &= a_1 - a_2 p(t) s(t) + a_3 (1 - a_4) s(t) - a_1 p(t) - a_5 p(t), \\ {}_0^{CF}D_t^{\sigma}(s(t)) &= a_2 p(t) s(t) - a_1 s(t) - a_3 s(t), \\ {}_0^{CF}D_t^{\sigma}(r(t)) &= a_3 a_4 s(t) - a_1 r(t) + a_5 p(t). \end{aligned} \tag{7}$$

Similarly, the fractional model under Atangana-Baleanu derivative

$$\begin{aligned} {}_0^{ABC}D_t^{\sigma}(p(t)) &= a_1 - a_2 p(t) s(t) + a_3 (1 - a_4) s(t) - a_1 p(t) - a_5 p(t), \\ {}_0^{ABC}D_t^{\sigma}(s(t)) &= a_2 p(t) s(t) - a_1 s(t) - a_3 s(t), \\ {}_0^{ABC}D_t^{\sigma}(r(t)) &= a_3 a_4 s(t) - a_1 r(t) + a_5 p(t). \end{aligned} \tag{8}$$

The related initial conditions to the above models are

$$p(0) \geq 0, s(0) \geq 0, r(0) \geq 0.$$

4. **Existence and uniqueness analysis.** In the subfield of differential calculus, it is an arduous matter to achieve the solution of nonlinear equations. Since we are also dealing with a nonlinear fractional order model, it might not be possible to reach an exact solution for this kind of systems. Hence we dedicate this section to elaborate on the existence and uniqueness of the solution concerning the aforementioned models (7) and (8) in view of the fixed point theory.

Let $\mathcal{P} = C(N) \times C(N) \times C(N)$ and $C(N)$ be a Banach space of continuous $\mathbb{R} \to \mathbb{R}$ valued functions on the interval N having the norm:

$$\|(p,s,r)\| = \|p\| + \|s\| + \|r\|,$$

where $\|p\| = sup\{|p(t)| : t \in N\}$, $\|s\| = sup\{|s(t)| : t \in N\}$, $\|r\| = sup\{|r(t)| : t \in N\}$.

4.1. **Existence and uniqueness of solutions for smoking model with Caputo-Fabrizio derivative.** Applying fractional integral in [27] to the Eq. (7), we have

$$\begin{aligned} p(t) - p(0) &= {}_0^{CF}I_t^{\sigma}\left[a_1 - a_2 p(t) s(t) + a_3(1-a_4) s(t) - a_1 p(t) - a_5 p(t)\right], \\ s(t) - s(0) &= {}_0^{CF}I_t^{\sigma}\left[a_2 p(t) s(t) - a_1 s(t) - a_3 s(t)\right], \\ r(t) - r(0) &= {}_0^{CF}I_t^{\sigma}\left[a_3 a_4 s(t) - a_1 r(t) + a_5 p(t)\right]. \end{aligned} \tag{9}$$

Utilizing the notation introduced by Losada and Nieto [27], we find

$$p(t) - p(0) = \frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}\left[a_1 - a_2 p(t) s(t) + a_3(1-a_4) s(t) - a_1 p(t) - a_5 p(t)\right]$$
$$+ \frac{2\sigma}{(2-\sigma)M(\sigma)}\int_0^t \left[a_1 - a_2 p(\lambda) s(\lambda) + a_3(1-a_4) s(\lambda) - a_1 p(\lambda) - a_5 p(\lambda)\right] d\lambda$$

$$s(t) - s(0) = \frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}\left[a_2 p(t) s(t) - a_1 s(t) - a_3 s(t)\right]$$
$$+ \frac{2\sigma}{(2-\sigma)M(\sigma)}\int_0^t \left[a_2 p(\lambda) s(\lambda) - a_1 s(\lambda) - a_3 s(\lambda)\right] d\lambda,$$

$$r(t) - r(0) = \frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}\left[a_3 a_4 s(t) - a_1 r(t) + a_5 p(t)\right]$$
$$+ \frac{2\sigma}{(2-\sigma)M(\sigma)}\int_0^t \left[a_3 a_4 s(\lambda) - a_1 r(\lambda) + a_5 p(\lambda)\right] d\lambda. \tag{10}$$

For the sake of clearness, we identify the kernels as below:

$$\begin{aligned} G_1(t,p) &= a_1 - a_2 p(t) s(t) + a_3(1-a_4) s(t) - a_1 p(t) - a_5 p(t), \\ G_2(t,s) &= a_2 p(t) s(t) - a_1 s(t) - a_3 s(t), \\ G_3(t,r) &= a_3 a_4 s(t) - a_1 r(t) + a_5 p(t). \end{aligned} \tag{11}$$

Theorem 4.1. *If the below inequality holds*

$$0 \leq a_2 \varepsilon_2 + a_1 + a_5 < 1$$

then the kernel G_1 satisfies Lipschitz condition and contraction.

Proof. Let p and p_1 be two functions, and then we have

$$\begin{aligned}\left\|G_1(t,p)-G_1(t,p_1)\right\| &= \left\|-a_2p(t)s(t)-a_1p(t)-a_5p(t)+a_2p_1(t)s(t)\right.\\ &\quad \left.+a_1p_1(t)+a_5p_1(t)\right\|\\ &\le \left[a_2\left\|s(t)\right\|+a_1+a_5\right]\left\|p(t)-p_1(t)\right\|.\end{aligned}$$

where $\|p(t)\| \le \varepsilon_1$, $\|s(t)\| \le \varepsilon_2$, $\|r(t)\| \le \varepsilon_3$ are bounded functions and taking $\overline{\psi}_1 = a_2\varepsilon_2 + a_1 + a_5$, we have

$$\left\|G_1(t,p)-G_1(t,p_1)\right\| \le \overline{\psi}_1\left\|p(t)-p_1(t)\right\|. \tag{12}$$

Hence, the Lipschitz condition is proved for the kernel G_1 and $0 \le a_2\varepsilon_2+a_1+a_5 < 1$ gives G_1 is a contraction. □

Similarly, it can be shown that the Lipschitz condition and contraction performed by the kernels G_2 and G_3.

By using the aforementioned kernels, Eq. (10) becomes

$$p(t)=p(0)+\frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}G_1(t,p)+\frac{2\sigma}{(2-\sigma)M(\sigma)}\int_0^t G_1(\lambda,p)\,d\lambda,$$

$$s(t)=s(0)+\frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}G_2(t,s)+\frac{2\sigma}{(2-\sigma)M(\sigma)}\int_0^t G_2(\lambda,s)\,d\lambda,$$

$$r(t)=r(0)+\frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}G_3(t,r)+\frac{2\sigma}{(2-\sigma)M(\sigma)}\int_0^t G_3(\lambda,r)\,d\lambda. \tag{13}$$

We focus on the below recursive formula:

$$p_n(t)=\frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}G_1(t,p_{n-1})+\frac{2\sigma}{(2-\sigma)M(\sigma)}\int_0^t G_1(\lambda,p_{n-1})\,d\lambda,$$

$$s_n(t)=\frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}G_2(t,s_{n-1})+\frac{2\sigma}{(2-\sigma)M(\sigma)}\int_0^t G_2(\lambda,s_{n-1})\,d\lambda,$$

$$r_n(t)=\frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}G_3(t,r_{n-1})+\frac{2\sigma}{(2-\sigma)M(\sigma)}\int_0^t G_3(\lambda,r_{n-1})\,d\lambda. \tag{14}$$

with

$$p_0(t)=p(0),\ s_0(t)=s(0),\ r_0(t)=r(0) \tag{15}$$

as the initial conditions.

The difference between successive terms is of the form:

$$\begin{aligned}\phi_{1n}^*(t) &= p_n(t)-p_{n-1}(t)=\frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}\left[G_1(t,p_{n-1})-G_1(t,p_{n-2})\right]\\ &\quad+\frac{2\sigma}{(2-\sigma)M(\sigma)}\int_0^t\left[G_1(\lambda,p_{n-1})-G_1(\lambda,p_{n-2})\right]d\lambda,\end{aligned}$$

$$\begin{aligned}\phi_{2n}^{*}(t) &= s_n(t) - s_{n-1}(t) = \frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}\left[G_2(t,s_{n-1}) - G_2(t,s_{n-2})\right] \\ &\quad + \frac{2\sigma}{(2-\sigma)M(\sigma)}\int_0^t \left[G_2(\lambda,s_{n-1}) - G_2(\lambda,s_{n-2})\right]d\lambda,\end{aligned}$$

$$\begin{aligned}\phi_{3n}^{*}(t) &= r_n(t) - r_{n-1}(t) = \frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}\left[G_3(t,r_{n-1}) - G_3(t,r_{n-2})\right] \\ &\quad + \frac{2\sigma}{(2-\sigma)M(\sigma)}\int_0^t \left[G_3(\lambda,r_{n-1}) - G_3(\lambda,r_{n-2})\right]d\lambda. \qquad (16)\end{aligned}$$

In the line with the above calculations, it is evident that

$$\begin{aligned}p_n(t) &= \sum_{k=0}^{n}\phi_{1k}^{*}(t), \\ s_n(t) &= \sum_{k=0}^{n}\phi_{2k}^{*}(t), \\ r_n(t) &= \sum_{k=0}^{n}\phi_{3k}^{*}(t). \qquad (17)\end{aligned}$$

Performing the norm to both sides of the Eq. (16) and using triangular identity, we assess

$$\begin{aligned}\left\|\phi_{1n}^{*}(t)\right\| &= \left\|p_n(t) - p_{n-1}(t)\right\| \\ &\leq \left\|\frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}\left[G_1(t,p_{n-1}) - G_1(t,p_{n-2})\right]\right\| \\ &\quad + \frac{2\sigma}{(2-\sigma)M(\sigma)}\int_0^t \left\|\left[G_1(\lambda,p_{n-1}) - G_1(\lambda,p_{n-2})\right]d\lambda\right\| \qquad (18)\end{aligned}$$

Since the kernel G_1 carries out the Lipschitz condition, we find

$$\begin{aligned}\left\|\phi_{1n}^{*}(t)\right\| &= \left\|p_n(t) - p_{n-1}(t)\right\| \\ &\leq \frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}\overline{\psi}_1\left\|p_{n-1} - p_{n-2}\right\| \\ &\quad + \frac{2\sigma}{(2-\sigma)M(\sigma)}\overline{\psi}_1\int_0^t \left\|p_{n-1} - p_{n-2}\right\|d\lambda \qquad (19)\end{aligned}$$

and then

$$\begin{aligned}\left\|\phi_{1n}^{*}(t)\right\| &\leq \frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}\overline{\psi}_1\left\|\phi_{1(n-1)}^{*}(t)\right\| \\ &\quad + \frac{2\sigma}{(2-\sigma)M(\sigma)}\overline{\psi}_1\int_0^t \left\|\phi_{1(n-1)}^{*}(\lambda)\right\|d\lambda. \qquad (20)\end{aligned}$$

Continuing the same attitude, we gain

$$\begin{aligned}\left\|\phi_{2n}^{*}(t)\right\| &\leq \frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}\overline{\psi}_{2}\left\|\phi_{2(n-1)}^{*}(t)\right\| \\ &+\frac{2\sigma}{(2-\sigma)M(\sigma)}\overline{\psi}_{2}\int_{0}^{t}\left\|\phi_{2(n-1)}^{*}(\lambda)\right\|d\lambda, \\ \left\|\phi_{3n}^{*}(t)\right\| &\leq \frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}\overline{\psi}_{3}\left\|\phi_{3(n-1)}^{*}(t)\right\| \\ &+\frac{2\sigma}{(2-\sigma)M(\sigma)}\overline{\psi}_{3}\int_{0}^{t}\left\|\phi_{3(n-1)}^{*}(\lambda)\right\|d\lambda. \end{aligned} \tag{21}$$

Considering the achieved findings, we state the theorem as below:

Theorem 4.2. *If we can find t_0 such that*

$$\frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}\overline{\psi}_{i}+\frac{2\sigma}{(2-\sigma)M(\sigma)}\overline{\psi}_{i}t_{0}<1 \text{ for } i=1,2,3.$$

then the fractional model (7) has a solution.

Proof. Benefiting from the Eq. (20) and Eq. (21), then taking into consideration the fact that the functions $p(t)$, $s(t)$, $r(t)$ are bounded and the kernels fulfil Lipschitz condition, we obtain the succeeding relations as below:

$$\begin{aligned}\left\|\phi_{1n}^{*}(t)\right\| &\leq \left\|p_{n}(0)\right\|\left[\frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}\overline{\psi}_{1}+\frac{2\sigma}{(2-\sigma)M(\sigma)}\overline{\psi}_{1}t\right]^{n}, \\ \left\|\phi_{2n}^{*}(t)\right\| &\leq \left\|s_{n}(0)\right\|\left[\frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}\overline{\psi}_{2}+\frac{2\sigma}{(2-\sigma)M(\sigma)}\overline{\psi}_{2}t\right]^{n}, \\ \left\|\phi_{3n}^{*}(t)\right\| &\leq \left\|r_{n}(0)\right\|\left[\frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}\overline{\psi}_{3}+\frac{2\sigma}{(2-\sigma)M(\sigma)}\overline{\psi}_{3}t\right]^{n}. \end{aligned} \tag{22}$$

So, these solutions exist and are continuous. In order to show that the above functions are solution of the Eq. (7), we suppose

$$\begin{aligned} p(t)-p(0) &= p_{n}(t)-\zeta_{1n}(t), \\ s(t)-s(0) &= s_{n}(t)-\zeta_{2n}(t), \\ r(t)-r(0) &= r_{n}(t)-\zeta_{3n}(t). \end{aligned} \tag{23}$$

Therefore, we find

$$\begin{aligned}\left\|\zeta_{1n}\right\| &= \left\|\frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}\left[G_{1}(t,p)-G_{1}(t,p_{n-1})\right]\right. \\ &\left.+\frac{2\sigma}{(2-\sigma)M(\sigma)}\int_{0}^{t}\left[G_{1}(\lambda,p)-G_{1}(\lambda,p_{n-1})\right]d\lambda\right\|\end{aligned}$$

$$\begin{aligned}
&\leq \frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}\left\|G_1(t,p)-G_1(t,p_{n-1})\right\| \\
&\quad+\frac{2\sigma}{(2-\sigma)M(\sigma)}\int_0^t \left\|G_1(\lambda,p)-G_1(\lambda,p_{n-1})\,d\lambda\right\| \\
&\leq \frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}\overline{\psi}_1\left\|p-p_{n-1}\right\|+\frac{2\sigma}{(2-\sigma)M(\sigma)}\overline{\psi}_1 t\left\|p-p_{n-1}\right\|. \qquad (24)
\end{aligned}$$

By repeating this process, it gives at t_0

$$\left\|\zeta_{1n}(t)\right\| \leq \left(\frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}+\frac{2\sigma}{(2-\sigma)M(\sigma)}t_0\right)^{n+1}\overline{\psi}_1^{n+1}a. \qquad (25)$$

As n approaches to infinity, taking the limit Eq. (25), we have

$$\left\|\zeta_{1n}(t)\right\| \to 0.$$

Similarly, we have

$$\left\|\zeta_{2n}(t)\right\| \to 0 \text{ and } \left\|\zeta_{3n}(t)\right\| \to 0.$$

□

It is a critical matter to achieve the uniqueness for the solutions of the model (7). Let $p_1(t)$, $s_1(t)$ and $r_1(t)$ be another solutions then

$$\begin{aligned}
p(t)-p_1(t) &= \frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}\left[G_1(t,p)-G_1(t,p_1)\right] \\
&\quad+\frac{2\sigma}{(2-\sigma)M(\sigma)}\int_0^t \left[G_1(\lambda,p)-G_1(\lambda,p_1)\right]d\lambda. \qquad (26)
\end{aligned}$$

Regarding the fact that the kernel provides Lipschitz condition and taking the norm Eq. (26), we have

$$\begin{aligned}
\left\|p(t)-p_1(t)\right\| &\leq \frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}\overline{\psi}_1\left\|p(t)-p_1(t)\right\| \\
&\quad+\frac{2\sigma}{(2-\sigma)M(\sigma)}\overline{\psi}_1 t\left\|p(t)-p_1(t)\right\| \qquad (27)
\end{aligned}$$

It yields

$$\left\|p(t)-p_1(t)\right\|\left(1-\frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}\overline{\psi}_1-\frac{2\sigma}{(2-\sigma)M(\sigma)}\overline{\psi}_1 t\right) \leq 0. \qquad (28)$$

If the inequality $\left(1-\frac{2(1-\sigma)}{(2-\sigma)M(\sigma)}\overline{\psi}_1-\frac{2\sigma}{(2-\sigma)M(\sigma)}\overline{\psi}_1 t\right) \geq 0$ exists, then $\left\|p(t)-p_1(t)\right\|=0$. So we obtain

$$p(t)=p_1(t).$$

In an analogous way, we have

$$s(t)=s_1(t) \text{ and } r(t)=r_1(t), \qquad (29)$$

which completes the uniqueness of the solutions for the model (7).

4.2. **Existence and uniqueness of solutions for smoking model with Atangana-Baleanu derivative.** Implementing the fractional integral to both sides of the Eq. (8), the model can be written as follows:

$$p(t) - p(0) = \frac{1-\sigma}{F(\sigma)}\left[a_1 - a_2 p(t) s(t) + a_3(1-a_4)s(t) - a_1 p(t) - a_5 p(t)\right]$$
$$+\frac{\sigma}{F(\sigma)\Gamma(\sigma)}\int_0^t (t-\lambda)^{\sigma-1}\left[a_1 - a_2 p(\lambda) s(\lambda) + a_3(1-a_4)s(\lambda) - a_1 p(\lambda) - a_5 p(\lambda)\right] d\lambda,$$
$$s(t) - s(0) = \frac{1-\sigma}{F(\sigma)}\left[a_2 p(t) s(t) - a_1 s(t) - a_3 s(t)\right]$$
$$+\frac{\sigma}{F(\sigma)\Gamma(\sigma)}\int_0^t (t-\lambda)^{\sigma-1}\left[a_2 p(\lambda) s(\lambda) - a_1 s(\lambda) - a_3 s(\lambda)\right] d\lambda,$$

$$r(t) - r(0) = \frac{1-\sigma}{F(\sigma)}\left[a_3 a_4 s(t) - a_1 r(t) + a_5 p(t)\right]$$
$$+\frac{\sigma}{F(\sigma)\Gamma(\sigma)}\int_0^t (t-\lambda)^{\sigma-1}\left[a_3 a_4 s(\lambda) - a_1 r(\lambda) + a_5 p(\lambda)\right] d\lambda. \tag{30}$$

For simplification, we assign the below kernels:

$$\begin{aligned}
\overline{G}_1(t,p) &= a_1 - a_2 p(t) s(t) + a_3(1-a_4)s(t) - a_1 p(t) - a_5 p(t), \\
\overline{G}_2(t,s) &= a_2 p(t) s(t) - a_1 s(t) - a_3 s(t), \\
\overline{G}_3(t,r) &= a_3 a_4 s(t) - a_1 r(t) + a_5 p(t).
\end{aligned} \tag{31}$$

Theorem 4.3. *If the following inequality holds*

$$0 \leq a_2\overline{\varepsilon}_2 + a_1 + a_5 < 1$$

then the kernel $\overline{G}_1$ *satisfies Lipschitz condition and contraction.*

Proof. Let p and p_1 be two functions, and then we have

$$\left\|\overline{G}_1(t,p) - \overline{G}_1(t,p_1)\right\| \leq \left[a_2\|s(t)\| + a_1 + a_5\right]\|p(t) - p_1(t)\|,$$

where $\|p(t)\| \leq \overline{\varepsilon}_1$, $\|s(t)\| \leq \overline{\varepsilon}_2$, $\|r(t)\| \leq \overline{\varepsilon}_3$ are bounded functions and taking $\widetilde{\psi}_1 = a_2\overline{\varepsilon}_2 + a_1 + a_5$, we have

$$\left\|\overline{G}_1(t,p) - \overline{G}_1(t,p_1)\right\| \leq \widetilde{\psi}_1\|p(t) - p_1(t)\|.$$

So, the kernel $\overline{G}_1$ satisfies the Lipschitz condition and $0 \leq a_2\overline{\varepsilon}_2 + a_1 + a_5 < 1$ gives $\overline{G}_1$ is contraction. □

Similarly, it can be shown that $\overline{G}_2$ and $\overline{G}_3$ fulfil the Lipschitz condition and contraction.

Regarding the above kernels, Eq. (30) becomes

$$\begin{aligned}
p(t) &= p(0) + \frac{1-\sigma}{F(\sigma)}\overline{G}_1(t,p) + \frac{\sigma}{F(\sigma)\Gamma(\sigma)}\int_0^t (t-\lambda)^{\sigma-1}\overline{G}_1(\lambda,p)\,d\lambda, \\
s(t) &= s(0) + \frac{1-\sigma}{F(\sigma)}\overline{G}_2(t,s) + \frac{\sigma}{F(\sigma)\Gamma(\sigma)}\int_0^t (t-\lambda)^{\sigma-1}\overline{G}_2(\lambda,s)\,d\lambda,
\end{aligned}$$

$$r(t) = r(0) + \frac{1-\sigma}{F(\sigma)}\overline{G}_3(t,r) + \frac{\sigma}{F(\sigma)\Gamma(\sigma)}\int_0^t (t-\lambda)^{\sigma-1}\overline{G}_3(\lambda,r)\,d\lambda. \quad (32)$$

Here, we give the below recursive formula:

$$\begin{aligned}
p_n(t) &= \frac{1-\sigma}{F(\sigma)}\overline{G}_1(t,p_{n-1}) + \frac{\sigma}{F(\sigma)\Gamma(\sigma)}\int_0^t (t-\lambda)^{\sigma-1}\overline{G}_1(\lambda,p_{n-1})\,d\lambda, \\
s_n(t) &= \frac{1-\sigma}{F(\sigma)}\overline{G}_2(t,s_{n-1}) + \frac{\sigma}{F(\sigma)\Gamma(\sigma)}\int_0^t (t-\lambda)^{\sigma-1}\overline{G}_2(\lambda,s_{n-1})\,d\lambda, \\
r_n(t) &= \frac{1-\sigma}{F(\sigma)}\overline{G}_3(t,r_{n-1}) + \frac{\sigma}{F(\sigma)\Gamma(\sigma)}\int_0^t (t-\lambda)^{\sigma-1}\overline{G}_3(\lambda,r_{n-1})\,d\lambda. \quad (33)
\end{aligned}$$

with

$$p_0(t) = p(0),\ s_0(t) = s(0),\ r_0(t) = r(0)$$

as the initial conditions. The difference between the successive terms is of the following expression:

$$\begin{aligned}
\overline{\phi}^*_{1n}(t) &= p_n(t) - p_{n-1}(t) = \frac{1-\sigma}{F(\sigma)}\left[\overline{G}_1(t,p_{n-1}) - \overline{G}_1(t,p_{n-2})\right] \\
&\quad + \frac{\sigma}{F(\sigma)\Gamma(\sigma)}\int_0^t (t-\lambda)^{\sigma-1}\left[\overline{G}_1(\lambda,p_{n-1}) - \overline{G}_1(\lambda,p_{n-2})\right]d\lambda, \\
\overline{\phi}^*_{2n}(t) &= s_n(t) - s_{n-1}(t) = \frac{1-\sigma}{F(\sigma)}\left[\overline{G}_2(t,s_{n-1}) - \overline{G}_2(t,s_{n-2})\right] \\
&\quad + \frac{\sigma}{F(\sigma)\Gamma(\sigma)}\int_0^t (t-\lambda)^{\sigma-1}\left[\overline{G}_2(\lambda,s_{n-1}) - \overline{G}_2(\lambda,s_{n-2})\right]d\lambda, \\
\overline{\phi}^*_{3n}(t) &= r_n(t) - r_{n-1}(t) = \frac{1-\sigma}{F(\sigma)}\left[\overline{G}_3(t,r_{n-1}) - \overline{G}_3(t,r_{n-2})\right] \\
&\quad + \frac{\sigma}{F(\sigma)\Gamma(\sigma)}\int_0^t (t-\lambda)^{\sigma-1}\left[\overline{G}_3(\lambda,r_{n-1}) - \overline{G}_3(\lambda,r_{n-2})\right]d\lambda. \quad (34)
\end{aligned}$$

It is evident that

$$\begin{aligned}
p_n(t) &= \sum_{k=0}^{n}\overline{\phi}^*_{1k}(t), \\
s_n(t) &= \sum_{k=0}^{n}\overline{\phi}^*_{2k}(t), \\
r_n(t) &= \sum_{k=0}^{n}\overline{\phi}^*_{3k}(t). \quad (35)
\end{aligned}$$

Taking the norm to both sides of the Eq. (34) and using the triangular inequality

$$\begin{aligned}\left\|\overline{\phi}_{1n}^{*}(t)\right\| &= \|p_n(t)-p_{n-1}(t)\| \\ &\leq \frac{1-\sigma}{F(\sigma)}\left\|\overline{G}_1(t,p_{n-1})-\overline{G}_1(t,p_{n-2})\right\| \\ &+ \frac{\sigma}{F(\sigma)\Gamma(\sigma)}\left\|\int_0^t (t-\lambda)^{\sigma-1}\left[\overline{G}_1(\lambda,p_{n-1})-\overline{G}_1(\lambda,p_{n-2})\right]d\lambda\right\| \quad (36)\end{aligned}$$

Since the kernel $\overline{G}_1$ satisfies Lipschitz condition, we get

$$\begin{aligned}\left\|\overline{\phi}_{1n}^{*}(t)\right\| &= \|p_n(t)-p_{n-1}(t)\| \\ &\leq \frac{1-\sigma}{F(\sigma)}\widetilde{\psi}_1\|p_{n-1}-p_{n-2}\| \\ &+ \frac{\sigma}{F(\sigma)\Gamma(\sigma)}\widetilde{\psi}_1\int_0^t (t-\lambda)^{\sigma-1}\|p_{n-1}-p_{n-2}\|\,d\lambda \quad (37)\end{aligned}$$

and

$$\begin{aligned}\left\|\overline{\phi}_{1n}^{*}(t)\right\| &\leq \frac{1-\sigma}{F(\sigma)}\widetilde{\psi}_1\left\|\overline{\phi}_{1(n-1)}^{*}(t)\right\| \\ &+ \frac{\sigma}{F(\sigma)\Gamma(\sigma)}\widetilde{\psi}_1\int_0^t (t-\lambda)^{\sigma-1}\left\|\overline{\phi}_{1(n-1)}^{*}(\lambda)\right\|d\lambda. \quad (38)\end{aligned}$$

By the similar way, we gain

$$\begin{aligned}\left\|\overline{\phi}_{2n}^{*}(t)\right\| &\leq \frac{1-\sigma}{F(\sigma)}\widetilde{\psi}_2\left\|\overline{\phi}_{2(n-1)}^{*}(t)\right\| \\ &+ \frac{\sigma}{F(\sigma)\Gamma(\sigma)}\widetilde{\psi}_2\int_0^t (t-\lambda)^{\sigma-1}\left\|\overline{\phi}_{2(n-1)}^{*}(\lambda)\right\|d\lambda,\end{aligned}$$

$$\begin{aligned}\left\|\overline{\phi}_{3n}^{*}(t)\right\| &\leq \frac{1-\sigma}{F(\sigma)}\widetilde{\psi}_3\left\|\overline{\phi}_{3(n-1)}^{*}(t)\right\| \\ &+ \frac{\sigma}{F(\sigma)\Gamma(\sigma)}\widetilde{\psi}_3\int_0^t (t-\lambda)^{\sigma-1}\left\|\overline{\phi}_{3(n-1)}^{*}(\lambda)\right\|d\lambda. \quad (39)\end{aligned}$$

Within the framework of the results in hand, we state a new theorem.

Theorem 4.4. *If we can find t_0 such that*

$$\frac{1-\sigma}{F(\sigma)}\widetilde{\psi}_i+\frac{t_0^{\sigma}}{F(\sigma)\Gamma(\sigma)}\widetilde{\psi}_i<1 \text{ for } i=1,2,3$$

then the fractional model (8) has a solution.

Proof. Considering the functions $p(t)$, $s(t)$, $r(t)$ are bounded and using the Eqs. (38) and (39), we obtain the following relations:

$$\begin{aligned}
\left\|\overline{\phi}_{1n}^{*}(t)\right\| &\leq \|p(0)\|\left[\frac{1-\sigma}{F(\sigma)}\widetilde{\psi}_1+\frac{t_0^{\sigma}}{F(\sigma)\Gamma(\sigma)}\widetilde{\psi}_1\right]^n, \\
\left\|\overline{\phi}_{2n}^{*}(t)\right\| &\leq \|s(0)\|\left[\frac{1-\sigma}{F(\sigma)}\widetilde{\psi}_2+\frac{t_0^{\sigma}}{F(\sigma)\Gamma(\sigma)}\widetilde{\psi}_2\right]^n, \\
\left\|\overline{\phi}_{3n}^{*}(t)\right\| &\leq \|r(0)\|\left[\frac{1-\sigma}{F(\sigma)}\widetilde{\psi}_3+\frac{t_0^{\sigma}}{F(\sigma)\Gamma(\sigma)}\widetilde{\psi}_3\right]^n. \qquad (40)
\end{aligned}$$

Thus, the solutions exist and are continuous. In order to prove that the above functions are solution of the model (8), suppose that

$$\begin{aligned}
p(t)-p(0) &= p_n(t)-\overline{\zeta}_{1n}(t), \\
s(t)-s(0) &= s_n(t)-\overline{\zeta}_{2n}(t), \\
r(t)-r(0) &= r_n(t)-\overline{\zeta}_{3n}(t). \qquad (41)
\end{aligned}$$

Next, we obtain

$$\begin{aligned}
\left\|\overline{\zeta}_{1n}\right\| &= \left\|\frac{1-\sigma}{F(\sigma)}\left[\overline{G}_1(t,p)-\overline{G}_1(t,p_{n-1})\right]\right. \\
&\quad \left.+\frac{\sigma}{F(\sigma)\Gamma(\sigma)}\int_0^t (t-\lambda)^{\sigma-1}\left[\overline{G}_1(\lambda,p)-\overline{G}_1(\lambda,p_{n-1})\right]d\lambda\right\| \\
&\leq \frac{1-\sigma}{F(\sigma)}\left\|\overline{G}_1(t,p)-\overline{G}_1(t,p_{n-1})\right\| \\
&\quad +\frac{\sigma}{F(\sigma)\Gamma(\sigma)}\int_0^t (t-\lambda)^{\sigma-1}\left\|\overline{G}_1(\lambda,p)-\overline{G}_1(\lambda,p_{n-1})\,d\lambda\right\| \\
&\leq \frac{1-\sigma}{F(\sigma)}\widetilde{\psi}_1\|p-p_{n-1}\|+\frac{t^{\sigma}}{F(\sigma)\Gamma(\sigma)}\widetilde{\psi}_1\|p-p_{n-1}\|. \qquad (42)
\end{aligned}$$

Continuing this process recursively, it gives at t_0

$$\left\|\overline{\zeta}_{1n}(t)\right\| \leq \left(\frac{1-\sigma}{F(\sigma)}+\frac{t_0^{\sigma}}{F(\sigma)\Gamma(\sigma)}\right)^{n+1}\widetilde{\psi}_1^{n+1}M. \qquad (43)$$

As n approaches to infinity, taking the limit to both sides of the Eq. (43), we have

$$\left\|\overline{\zeta}_{1n}(t)\right\| \to 0.$$

Analogously, we get

$$\left\|\overline{\zeta}_{2n}(t)\right\| \to 0 \text{ and } \left\|\overline{\zeta}_{3n}(t)\right\| \to 0.$$

□

Now, we show the uniqueness for the solutions of the model (8), which is another significant matter.

Let $p_1(t)$, $s_1(t)$ and $r_1(t)$ be another solutions. Then, we obtain

$$\begin{aligned}
p(t)-p_1(t) &= \frac{1-\sigma}{F(\sigma)}\left[\overline{G}_1(t,p)-\overline{G}_1(t,p_1)\right] \\
&\quad +\frac{\sigma}{F(\sigma)\Gamma(\sigma)}\int_0^t (t-\lambda)^{\sigma-1}\left[\overline{G}_1(\lambda,p)-\overline{G}_1(\lambda,p_1)\right]d\lambda. \qquad (44)
\end{aligned}$$

We know that the kernel carries out Lipschitz condition and implementing the norm Eq. (44), we have

$$\begin{aligned}\left\|p\left(t\right)-p_{1}\left(t\right)\right\| &\leq \frac{1-\sigma}{F\left(\sigma\right)}\widetilde{\psi}_{1}\left\|p\left(t\right)-p_{1}\left(t\right)\right\| \\ &\quad +\frac{t^{\sigma}}{F\left(\sigma\right)\Gamma\left(\sigma\right)}\widetilde{\psi}_{1}\left\|p\left(t\right)-p_{1}\left(t\right)\right\|. \end{aligned} \tag{45}$$

It gives

$$\left\|p\left(t\right)-p_{1}\left(t\right)\right\|\left(1-\frac{1-\sigma}{F\left(\sigma\right)}\widetilde{\psi}_{1}-\frac{t^{\sigma}}{F\left(\sigma\right)\Gamma\left(\sigma\right)}\widetilde{\psi}_{1}\right)\leq 0. \tag{46}$$

If the inequality $\left(1-\frac{1-\sigma}{F(\sigma)}\widetilde{\psi}_{1}-\frac{t^{\sigma}}{F(\sigma)\Gamma(\sigma)}\widetilde{\psi}_{1}\right)\geq 0$ exists, then $\left\|p\left(t\right)-p_{1}\left(t\right)\right\|=0$. So, we find

$$p\left(t\right)=p_{1}\left(t\right).$$

Clearly, we can show .

$$s\left(t\right)=s_{1}\left(t\right) \text{ and } r\left(t\right)=r_{1}\left(t\right),$$

which gives the uniqueness for the solutions of the model (8).

5. **Numerical results.** Smoking is a complex habit affected by a distinctive and overlapping mixture of biologic, psychosocial, environmental, and educational elements. These elements can act as risk or defensive elements. Risk elements are characterized by rises in frequency and density, although they raise the possibility of starting smoking and continuous use. In contrast, defensive elements reduce the possibility of starting smoking and lower the probability of switching experimental to habitual use. Moreover, the possibility of smoking of educated and determined people are lower, and if they make a decision for smoking they smoke less cigarettes each day. Considering these facts, we set forth various numerical examples providing evidence to our theoretical outcomes concerning the relevant models (7) and (8) by employing the method which is shown in [9] and [37] seriatim. To this end, as given in [43], it is presumed that the initial conditions would be $p(0)=0.6$, $s(0)=0.3$, $r(0)=0.1$, and the parameters $a_1=0.02$, $a_2=0.4$, $a_3=0.05$, $a_4=0.2$, $a_5=0.06$ are selected. According to both CF and AB derivatives, the influence of education is evident from Figs. 1-2, which demonstrate that not only the number of smokers diminishes but also the removed population escalates with the widespread presence of education. First, it is determined that $a_5=0.6$, and after that, our conclusions $a_4=0.2$-0.6 are displayed without altering the rest of the parameters for observing the manner in which the smokers populations of our fractional models are affected by the change in determination and fractional order σ. In parallel, Figs. 3-4 show us that the number of smokers decreases when determination rises. Moreover, from Figs. 5-6, the effect of alteration in education and fractional order σ is represented by fixing a_4 as equal to 0.6. It is consequently perceived from both of these fractional derivatives that the smokers population shows a decrease when education is raised from $a_5=0.02$ to 0.06.

6. **Concluding remarks and future works.** In recent times, Caputo with collaboration Fabrizio have introduced a new fractional order derivative with exponential kernel. Then, Atangana and Baleanu have defined AB derivative comprising Mittag-Leffler kernel. In order to see further applications of these fractional derivatives and better explore smoking matter, we present the fractional smoking model

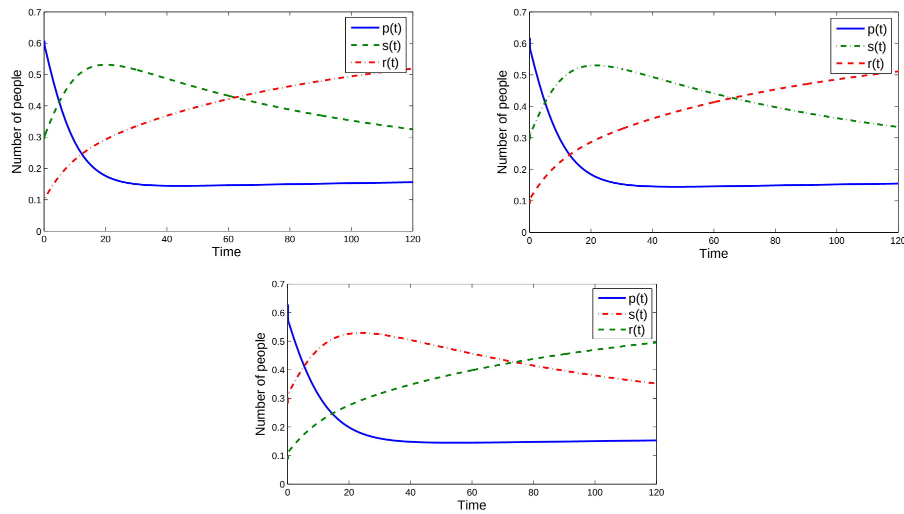

FIGURE 1. Numerical simulations for the model (7) at $\sigma = 0.93$, $\sigma = 0.75$ and $\sigma = 0.6$, respectively.

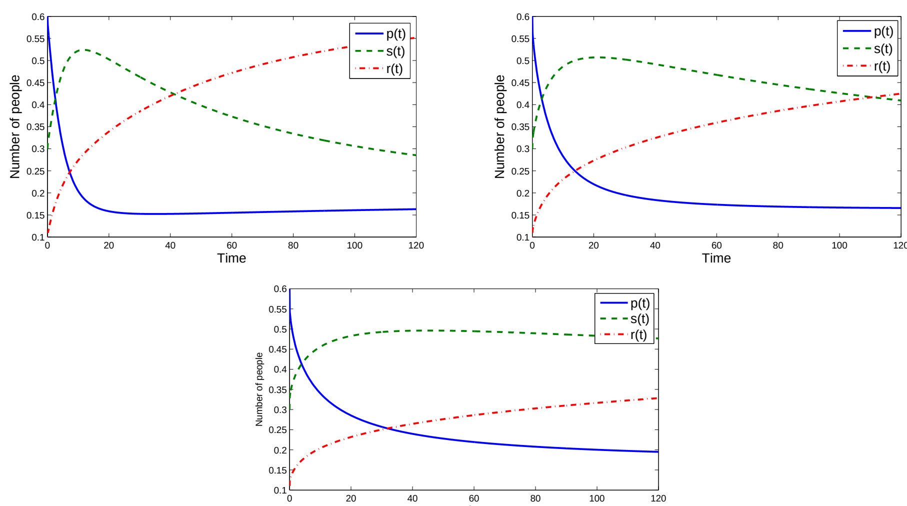

FIGURE 2. Numerical simulations for the model (8) at $\sigma = 0.93$, $\sigma = 0.75$ and $\sigma = 0.6$ respectively.

with determination and education linked with the model [43] for the first time by the concept of CF and AB derivatives. To the best of our knowledge, this model has never been modelled with these derivatives in the literature. In the line with fixed point method, we aim to give the conditions for the existence and uniqueness solutions of the models (7) and (8). The numerical simulations for these fractional models have been performed in order to understand the effectiveness of the fractional order σ as well as determination parameter a_4 and education parameter a_5. These simulations indicate that increase in determination and education leads to

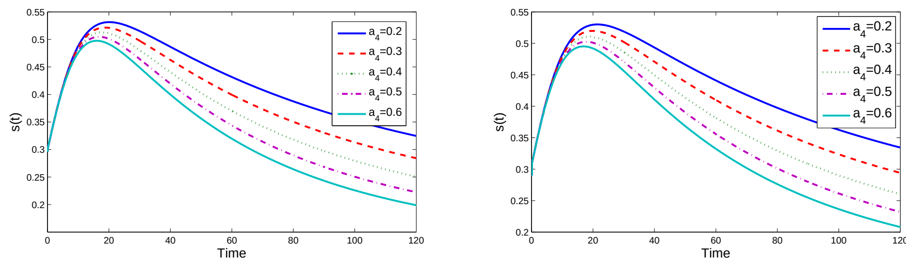

FIGURE 3. The effect of the parameters a_4 on the smokers population s of the model (7) for the fractional order $\sigma = 0.95$ and $\sigma = 0.75$, respectively.

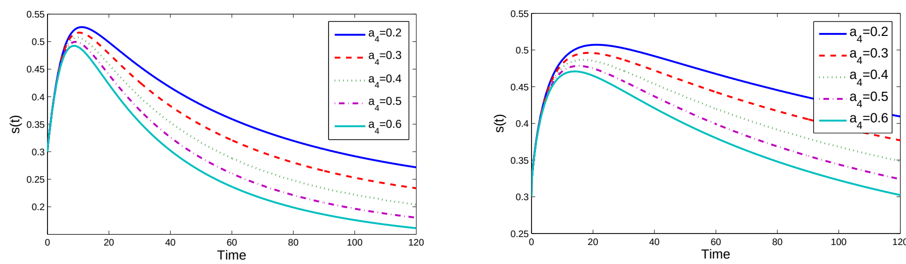

FIGURE 4. The effect of the parameters a_4 on the smokers population s of the model (8) for the fractional order $\sigma = 0.95$ and $\sigma = 0.75$, respectively.

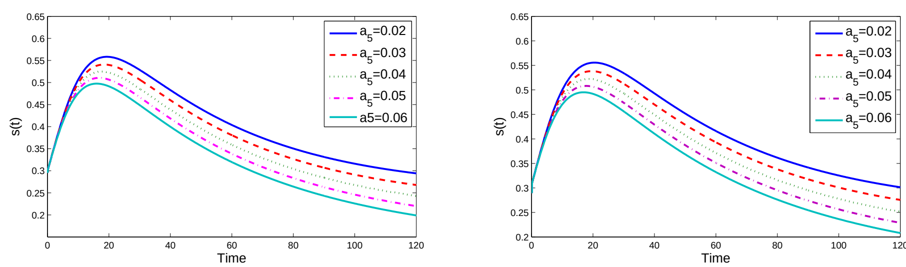

FIGURE 5. The effect of the parameters a_5 on the smokers population s of the model (7) for the fractional order $\sigma = 0.95$ and $\sigma = 0.75$, respectively.

decrease the smokers according to the different fractional orders. We anticipate that the current study will be more useful in the description of smoking matter thinking of determination and education. Maybe, in the future, the gained elements may lead us to do more research on this subject. For example, the mathematical model can be updated by considering various dynamical structures and can be examined with different types of derivatives.

Acknowledgment. This research is supported by Balikesir University Research Grant No. Bap 2019/099.

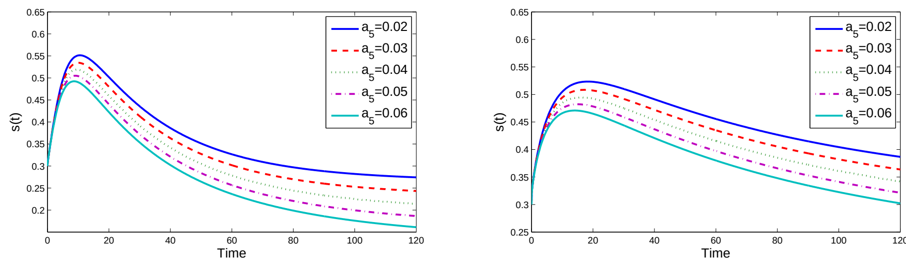

FIGURE 6. The effect of the parameters a_5 on the smokers population s of the model (8) for the fractional order $\sigma = 0.95$ and $\sigma = 0.75$, respectively.

REFERENCES

[1] B. S. T. Alkahtani, A. Atangana and I. Koca, Huge analysis of Hepatitis C model within the scope of fractional calculus, *J. Nonlinear Sci. Appl.*, **9** (2016), 6195–6203.

[2] A. Atangana, *Fractional Operators with Constant and Variable Order with Application to Geo-Hydrology*, Academic Press, London, 2018.

[3] A. Atangana, On the new fractional derivative and application to nonlinear Fisher's reaction-diffusion equation, *Appl. Math. Comput.*, **273** (2016), 948–956.

[4] A. Atangana and B. S. T. Alkahtani, Analysis of the Keller-Segel model with a fractional derivative without singular kernel, *Entropy*, **17** (2015), 4439–4453.

[5] A. Atangana and B. S. T. Alkahtani, New model of groundwater flowing within a confine aquifer: application of Caputo-Fabrizio derivative, *Arabian Journal of Geosciences*, **9** (2016), 6pp.

[6] A. Atangana and D. Baleanu, New fractional derivatives with nonlocal and non-singular kernel: Theory and application to heat transfer model, *Thermal Science*, **20** (2016), 763–769.

[7] A. Atangana and I. Koca, Chaos in a simple nonlinear system with Atangana-Baleanu derivatives with fractional order, *Chaos Solitons Fractals*, **89** (2016), 447–454.

[8] A. Atangana and I. Koca, On the new fractional derivative and application to nonlinear Baggs and Freedman model, *J. Nonlinear Sci. Appl.*, **9** (2016), 2467–2480.

[9] A. Atangana and K. M. Owolabi, New numerical approach for fractional differential equations, *Math. Model. Nat. Phenom.*, **13** (2018), Paper No. 3, 21 pp.

[10] D. Baleanu and A. Fernandez, On some new properties of fractional derivatives with Mittag-Leffler kernel, *Commun. Nonlinear Sci. Numer. Simul.*, **59** (2018), 444–462.

[11] D. Baleanu, Z. B. Guvenc and J. A. T. Machado, *New Trends in Nanotechnology and Fractional Calculus Applications*, Springer, Dordrecht, 2010.

[12] H. M. Baskonus, T. Mekkaoui, Z. Hammouch and H. Bulut, Active control of a chaotic fractional order economic system, *Entropy*, **17** (2015), 5771–5783.

[13] T. J. Brinker, S. S. Balderjahn, W. Seeger and D. A. Groneberg, Education Against Tobacco (EAT): A quasi-experimental prospective evaluation of a programme for preventing smoking in secondary schools delivered by medical students: A study protocol, *BMJ Open*, **4** (2014), 1–7.

[14] C. Bullen, Impact of tobacco smoking and smoking cessation on cardiovascular risk and disease, *Expert Review of Cardiovascular Therapy*, **6** (2008), 883–895.

[15] M. Caputo and M. Fabrizio, A new definition of fractional derivative without singular kernel, *Progress in Fractional Differentiation and Applications*, **1** (2015), 73–85.

[16] F. Evirgen, Analyze the optimal solutions of optimization problems by means of fractional gradient based system using VIM, *Int. J. Optim. Control. Theor. Appl. IJOCTA*, **6** (2016), 75–83.

[17] F. Evirgen and N. Özdemir, A fractional order dynamical trajectory approach for optimization problem with HPM, *Fractional Dynamics and Control*, Springer, New York, (2012), 145–155.

[18] A. Fernandez, D. Baleanu and H. M. Srivastava, Series representations for fractional-calculus operators involving generalised Mittag-Leffler functions, *Commun. Nonlinear Sci. Numer. Simul.*, **67** (2019), 517–527.

[19] J. F. Gómez-Aguilar, Analytical and numerical solutions of a nonlinear alcoholism model via variable-order fractional differential equations, *Phys. A*, **494** (2018), 52–75.

[20] O. K. Ham, Stages and processes of smoking cessation among adolescents, *Western Journal of Nursing Research*, **29** (2007), 301–315.

[21] F. Haq, K. Shah, G. ur Rahman and M. Shahzad, Numerical solution of fractional order smoking model via laplace Adomian decomposition method, *Alexandria Engineering Journal*, **57** (2018), 1061–1069.

[22] K. O. Haustein and D. Groneberg, *Tobacco or Health?: Physiological and Social Damages Caused by Tobacco Smoking*, Springer-Verlag Berlin, 2010.

[23] M. Khalid, F. S. Khan and A. Iqbal, Perturbation-iteration algorithm to solve fractional giving up smoking mathematical model, *International Journal of Computer Applications*, **142** (2016), 1–6.

[24] A. A. Kilbas, H. M. Srivastava and J. J. Trujillo, *Theory and Applications of Fractional Differential Equations*, Elsevier, 2006.

[25] İ. Koca, Analysis of rubella disease model with non-local and non-singular fractional derivatives, *Int. J. Optim. Control. Theor. Appl. IJOCTA*, **8** (2018), 17–25.

[26] S. Kumar, A. Ahmadian, R. Kumar, D. Kumar, J. Singh, D. Baleanu and M. Salimi, An efficient numerical method for fractional SIR epidemic model of infectious disease by using Bernstein wavelets, *Mathematics*, **8** (2020), 558.

[27] J. Losada and J. J. Nieto, Properties of a new fractional derivative without singular kernel, *Progress in Fractional Differentiation and Applications*, **1** (2015), 87-92.

[28] J. T. Machado and A. M. Lopes, Artistic painting: A fractional calculus perspective, *Applied Mathematical Modelling*, **65** (2019), 614–626.

[29] N. Özdemir, O. P. Agrawal, B. B. İskender and D. Karadeniz, Fractional optimal control of a 2-dimensional distributed system using eigenfunctions, *Nonlinear Dynamics*, **55** (2009), 251–260.

[30] N. Özdemir, D. Karadeniz and B. B. İskender, Fractional optimal control problem of a distributed system in cylindrical coordinates, *Phys. Lett. A*, **373** (2009), 221-226.

[31] N. Özdemir and M. Yavuz, Numerical solution of fractional Black-Scholes equation by using the multivariate Padé approximation, *Acta Physica Polonica A*, **132** (2017), 1050–1053.

[32] J. Singh, Analysis of fractional blood alcohol model with composite fractional derivative, *Chaos Solitons Fractals*, **140** (2020), 110127, 6 pp.

[33] J. Singh, D. Kumar and D. Baleanu, A new analysis of fractional fish farm model associated with Mittag-Leffler-type kernel, *Int. J. Biomath.*, **13** (2020), 2050010, 17 pp.

[34] J. Singh, D. Kumar, Z. Hammouch and A. Atangana, A fractional epidemiological model for computer viruses pertaining to a new fractional derivative, *Appl. Math. Comput.*, **316** (2018), 504–515.

[35] J. Singh, D. Kumar, M. A. Qurashi and D. Baleanu, A new fractional model for giving up smoking dynamics, *Adv. Difference Equ.*, (2017), Paper No. 88, 16 pp.

[36] N. H. Sweilam, A. M. Nagy and A. A. El-Sayed, Second kind shifted Chebyshev polynomials for solving space fractional order diffusion equation, *Chaos Solitons Fractals*, **73** (2015), 141–147.

[37] M. Toufik and A. Atangana, New numerical approaximation of fractional derivative with non-local and non-singular kernel: Application to chaotic models, *Eur. Phys. J. Plus*, **132** (2017), Article number: 444.

[38] E. Uçar, N. Özdemir and E. Altun, Fractional order model of immune cells influenced by cancer cells, *Math. Model. Nat. Phenom.*, **14** (2019), Paper No. 308, 12 pp.

[39] S. Ucar, N. Ozdemir, I. Koca and E. Altun, Novel analysis of the fractional glucose-insulin regulatory system with non-singular kernel derivative, *European Physical Journal Plus*, **135** (2020), 414.

[40] S. Uçar, E. Uçar, N. Özdemir and Z. Hammouch, Mathematical analysis and numerical simulation for a smoking model with Atangana-Baleanu derivative, *Chaos Solitons Fractals*, **118** (2019), 300–306.

[41] S. Ullah, M. A. Khan, M. Farooq, Z. Hammouch and D. Baleanu, A fractional model for the dynamics of tuberculosis infection using Caputo-Fabrizio derivative, *Discrete Contin. Dyn. Syst. Ser. S*, **13** (2020), 975–993.

[42] P. Veeresha, D. G. Prakasha, J. Singh, I. Khan and D. Kumar, Analytical approach for fractional extended Fisher-Kolmogorov equation with Mittag-Leffler kernel, *Adv. Difference Equ.*, (2020), Paper No. 174, 17 pp.

[43] A. Yadav, P. K. Srivastava and A. Kumar, Mathematical model for smoking: Effect of determination and education, *Int. J. Biomath.*, **8** (2015), 1550001, 14 pp.

[44] M. Yavuz and E. Bonyah, New approaches to the fractional dynamics of schistosomiasis disease model, *Phys. A*, **525** (2019), 373–393.

Received April 2019; 1st revision May 2019; 2nd revision January 2021.

E-mail address: sumeyraucar@balikesir.edu.tr