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Research article

Analysis of a basic SEIRA model with Atangana-Baleanu derivative

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Abstract: Since computer worms have very acute and negative effects on computer systems, they are considered as one of the malicious bodies that induce serious issues in these structures. This is why numerous efforts have been given for finding different ways to avert the unwanted occurrences which stem from computer worms' harmful behavior to this day. Our motivation is to make use of Atangana-Baleanu fractional derivative with Mittag-Leffler kernel which has latterly been brought into operation, and thus closely examine the basic SEIRA (susceptible-exposed-infectious-removed-antidotal) model associated with computer worms. To that end, we first prove the conditions that show the existence and uniqueness properties of the solutions for the fractional order model benefiting from fixed point theory. By using various values belonging to the fractional order, we also acquired different numerical simulations emphasizing that the aforementioned derivative is quite impactful.

Keywords: Atangana-Baleanu derivative; fixed point theory; existence; uniqueness; mathematical model
Mathematics Subject Classification: 34A08, 34A34, 47H10

1. Introduction

With the improvements in cyber world and the increasing use of internet, computer worms have become a critical problem. Worms are one of the malicious objects that attack through using the system vulnerability with the ability to copy itself from one machine to other machines. Worms have also infectivity, destructibility, invisibility and latent, therefore eliminating them is a serious problem. Because each malicious action gives rise to weakness in computer to gain access for several purposes, such as stealing password, credit card information, email address, deleting files or even anything on hard disk, causing unnecessary network traffic and so on, network experts consider worms as the extreme security risk on computers [1–3]. Morris, SQL Slammer and Code Red are considered as some of the well-known worms infecting a large number of computers and causing mass economic loss [3–5].

On account of the high similarity between computer worms propagation and spread of biological virus, the actions of malicious objects in the network environment can be investigated using classical epidemiological models [6–8]. Considering biological models such as SIR, SEIR, SIRS, there are a lot of models related to computer virus and worms in the literature [9–13].

Because of the hereditary and memory properties of fractional derivatives not owned by integer order derivatives, fractional operators have received increasing interest by several directions in the modeling of biological process, neural networks, engineering, physics, finance and many more [14–21]. Although there are many benefits of the classical fractional Riemann Liouville (RL) and Caputo derivatives to characterize real systems as more reliable, the singularity by virtue of their power kernels which leads to many significant computational hardships [22–24]. To eliminate these problems, at first, Caputo and Fabrizio [25] presented a new non-singular fractional derivative with exponential kernel which is called Caputo-Fabrizio (CF) derivative. Inspired by the definition of CF derivative, Atangana and Baleanu [26] have introduced two types of non-singular derivatives in terms of RL and Caputo named as Atangana-Baleanu (AB) derivatives with Mittag-Leffler kernel function. These operators have been interpreted as a filter regulator besides being a derivative [27]. Many applications of the new operators arise in various real-world problems. Atangana and Alkahtani [28] have obtained a detailed analysis for the existence and uniqueness of solutions of the Keller-Segel model involving CF derivative. Atangana and Koca [29] have investigated Baggs and Freedman model having exponential kernel. By means of Banach fixed point theory, Singh et al. [30] examined the epidemiological model for computer viruses equipped with CF derivative. Fractional partial differential equations containing AB derivative have been solved by Yavuz et al. [31]. Saad et al. [32] have been studied Naguma model with CF and AB derivative. The classical model of polluted lake system have been modified with the concept of fractional differentiation by Bildik et al. [33]. Uçar [34] has examined a smoking model as affected by determination and education related activities by means of CF and AB derivatives. Some other outstanding studies have been made in [35–42].

In the present work, regarding the great importance of AB fractional derivative we aim to promote the application of the AB derivative with Mittag-Leffler kernel to the basic SEIRA model and prove the detailed existence and uniqueness conditions of their solution through the fixed point theory. After that, we interpret the effect of this fractional derivative supporting some numerical simulations and also take into consideration the description memory and hereditary properties of fractional order models. Finally, the concluding remarks are discussed.

2. Basic definitions and preliminaries

In this section, we briefly give some basic definitions and properties that are useful in the next chapter.

Definition 2.1. *Let $g \in H^1(a,b)$, $a < b$ be a function and $\eta \in [0,1]$. The Atangana-Baleanu derivative in Caputo type of order η of g is given by [26]*

$$ {}_{a}^{ABC}D_t^{\eta}\left[g(t)\right] = \frac{F(\eta)}{1-\eta}\int_a^t g'(x)\,E_\eta\left[-\eta\frac{(t-x)^\eta}{1-\eta}\right]dx \tag{2.1}$$

where $F(\eta)$ is a normalization function with $F(0) = F(1) = 1$ and E_η is the Mittag-Leffler function.

Definition 2.2. *Let $g \in H^1(a,b)$, $a < b$ be a function and $\eta \in [0,1]$. The Atangana-Baleanu derivative in Riemann-Liouville type of order η of g is given by [26]:*

$$ {}_{a}^{ABR}D_t^{\eta}[g(t)] = \frac{F(\eta)}{1-\eta}\frac{d}{dt}\int_a^t g(x)E_{\eta}\left[-\eta\frac{(t-x)^{\eta}}{1-\eta}\right]dx. \tag{2.2} $$

Definition 2.3. *The fractional integral is defined by [26]:*

$$ {}_{a}^{AB}I_t^{\eta}[g(t)] = \frac{1-\eta}{F(\eta)}g(t) + \frac{\eta}{F(\eta)\Gamma(\eta)}\int_a^t g(\lambda)(t-\lambda)^{\eta-1}d\lambda. \tag{2.3} $$

Theorem 2.1. *For a continuous function g on $[a,b]$. The inequality given below holds on $[a,b]$ [26]:*

$$ \left\|{}_{0}^{ABR}D_t^{\eta}[g(t)]\right\| < \frac{F(\eta)}{1-\eta}\|g(t)\|, \tag{2.4} $$

where $\|g(t)\| = \max_{a\le t\le b}|g(t)|$.

Theorem 2.2. *[26] The Atangana-Baleanu derivative in Caputo and RL type satisfy Lipschitz condition:*

$$ \left\|{}_{0}^{ABC}D_t^{\eta}[g(t)] - {}_{0}^{ABC}D_t^{\eta}[h(t)]\right\| \le H\|g(t)-h(t)\| \tag{2.5} $$

and

$$ \left\|{}_{0}^{ABR}D_t^{\eta}[g(t)] - {}_{0}^{ABR}D_t^{\eta}[h(t)]\right\| \le H\|g(t)-h(t)\|. \tag{2.6} $$

3. Model description

Here, we focus on the basic SEIRA (susceptible-exposed-infectious-removed-antidotal) model described in [43]:

$$ \begin{aligned} \frac{dS(t)}{dt} &= \frac{-bS(t)I(t)}{N} - d_s S(t) + \mu A(t), \\ \frac{dE(t)}{dt} &= \frac{bS(t)I(t)}{N} - \tau E(t) - d_E E(t), \\ \frac{dI(t)}{dt} &= \tau E(t) - d_I I(t) - \theta I(t), \\ \frac{dR(t)}{dt} &= \theta I(t) - \varphi R(t), \\ \frac{dA(t)}{dt} &= d_S S(t) + d_E E(t) + d_I I(t) + \varphi R(t) - \mu A(t), \end{aligned} \tag{3.1} $$

with the initial conditions $S(0) = l_1$, $E(0) = l_2$, $I(0) = l_3$, $R(0) = l_4$, $A(0) = l_5$. This basic SEIRA model divides N computer hosts in total into five separate sections. $S(t)$ stands for the number of computer hosts susceptible to malicious object attack at time t; the number of computer hosts exposed to malicious object attack however are not yet actively infectious at time t is denoted by $E(t)$; the number of computer hosts actively infectious at time t is shown by $I(t)$; the number of computer hosts removed from the network owing to forced isolation as a consequence of system treatment effort or death from infection is described by $R(t)$; and finally $A(t)$ stands for the number of computer hosts restored (from the removed state) and equipped with modern anti-malicious software. The model parameters are as follows: The ratio of contact is b; the ratio of state transition from E to I is τ; the ratio of state transition from S to A is d_s; the ratio of state transition from E to A is d_E; the ratio of state transition from I to A is d_I; the ratio of state transition from A to S is μ; the ratio of death as a result of being infected by malicious objects is θ; and the ratio of restoration from R to A is φ.

Let us modify this model by replacing the integer order time derivative by the fractional time derivative:

$$\begin{aligned}
{}_{0}^{ABC}D_t^{\eta}(S(t)) &= \frac{-bS(t)I(t)}{N} - d_s S(t) + \mu A(t), \\
{}_{0}^{ABC}D_t^{\eta}(E(t)) &= \frac{bS(t)I(t)}{N} - \tau E(t) - d_E E(t), \\
{}_{0}^{ABC}D_t^{\eta}(I(t)) &= \tau E(t) - d_I I(t) - \theta I(t), \\
{}_{0}^{ABC}D_t^{\eta}(R(t)) &= \theta I(t) - \varphi R(t), \\
{}_{0}^{ABC}D_t^{\eta}(A(t)) &= d_S S(t) + d_E E(t) + d_I I(t) + \varphi R(t) - \mu A(t).
\end{aligned} \tag{3.2}$$

with the initial conditions $S(0) = l_1, E(0) = l_2, I(0) = l_3, R(0) = l_4, A(0) = l_5$, where ${}_{0}^{ABC}D_t^{\eta}$ is Atangana-Baleanu derivative in Caputo type and $\eta \in [0, 1]$.

4. Existence and uniqueness analysis

Providing the solution of nonlinear equations is known to be a hard subject in differential calculus. The fractional order model under consideration is nonlinear, it can be impossible to obtain the exact solution of such systems. To that end, we investigate the existence and uniqueness problems of the model given by (3.2). To achieve this, we will take advantage of fixed point theory.

Let $\mathcal{A} = C(N) \times C(N) \times C(N) \times C(N) \times C(N)$ and $C(N)$ be a Banach space of continuous $\mathbb{R} \to \mathbb{R}$ valued functions on the interval N with the norm

$$\|(S, E, I, R, A)\| = \|S\| + \|E\| + \|I\| + \|R\| + \|A\|,$$

where $\|S\| = sup\{|S(t)| : t \in N\}$, $\|E\| = sup\{|E(t)| : t \in N\}$, $\|I\| = sup\{|I(t)| : t \in N\}$, $\|R\| = sup\{|R(t)| : t \in N\}$, $\|A\| = sup\{|A(t)| : t \in N\}$. Now, we rearrange the model (3.2) in the following easy manner:

$$\begin{aligned}
{}_{0}^{ABC}D_t^{\eta}(S(t)) &= H_1(t, S), \\
{}_{0}^{ABC}D_t^{\eta}(E(t)) &= H_2(t, E), \\
{}_{0}^{ABC}D_t^{\eta}(I(t)) &= H_3(t, I), \\
{}_{0}^{ABC}D_t^{\eta}(R(t)) &= H_4(t, R), \\
{}_{0}^{ABC}D_t^{\eta}(A(t)) &= H_5(t, A).
\end{aligned} \tag{4.1}$$

Applying fractional integral to both sides of the Eq. (4.1) and by the fundamental theorem of fractional calculus, the above can be rewritten as:

$$\begin{aligned}
S(t)-S(0) &= \frac{1-\eta}{F(\eta)}H_1(t,S)+\frac{\eta}{F(\eta)\Gamma(\eta)}\int_0^t (t-\lambda)^{\eta-1}H_1(\lambda,S)\,d\lambda,\\
E(t)-E(0) &= \frac{1-\eta}{F(\eta)}H_2(t,E)+\frac{\eta}{F(\eta)\Gamma(\eta)}\int_0^t (t-\lambda)^{\eta-1}H_2(\lambda,E)\,d\lambda,\\
I(t)-I(0) &= \frac{1-\eta}{F(\eta)}H_3(t,I)+\frac{\eta}{F(\eta)\Gamma(\eta)}\int_0^t (t-\lambda)^{\eta-1}H_3(\lambda,I)\,d\lambda,\\
R(t)-R(0) &= \frac{1-\eta}{F(\eta)}H_4(t,R)+\frac{\eta}{F(\eta)\Gamma(\eta)}\int_0^t (t-\lambda)^{\eta-1}H_4(\lambda,R)\,d\lambda,\\
A(t)-A(0) &= \frac{1-\eta}{F(\eta)}H_5(t,A)+\frac{\eta}{F(\eta)\Gamma(\eta)}\int_0^t (t-\lambda)^{\eta-1}H_5(\lambda,A)\,d\lambda. \qquad (4.2)
\end{aligned}$$

Theorem 4.1. *If the following inequality holds*

$$0 \le \frac{b}{N}c + d_S < 1,$$

then the kernel H_1 satisfies the Lipschitz condition and contraction.

Proof. Let S and S_1 be two functions, then we have

$$\begin{aligned}
&\|H_1(t,S)-H_1(t,S_1)\|\\
=\;& \left\|-\frac{b}{N}S(t)I(t)-d_S S(t)+\frac{b}{N}S_1(t)I(t)+d_S S_1(t)\right\|\\
\le\;& \left(\frac{b}{N}\|I(t)\|+d_S\right)\|S(t)-S_1(t)\|\\
\le\;& \overline{\delta}_1\|S(t)-S_1(t)\|. \qquad (4.3)
\end{aligned}$$

where $\overline{\delta}_1 = \frac{b}{N}c + d_S$ and $\|S(t)\| \le a, \|E(t)\| \le b, \|I(t)\| \le c, \|R(t)\| \le d, \|A(t)\| \le e.$

$$\|H_1(t,S)-H_1(t,S_1)\| \le \overline{\delta}_1\|S(t)-S_1(t)\|. \qquad (4.4)$$

Hence, the Lipschitz condition satisfied for H_1 and $0 \le \frac{b}{N}c + d_S < 1$ implies H_1 is also contraction. □

Similarly, it can be shown that the Lipschitz condition and contraction fulfilled by the other kernels. Consider the system (4.2) in the following iterative formula:

$$S_n(t) = \frac{1-\eta}{F(\eta)}H_1(t,S_{n-1})+\frac{\eta}{F(\eta)\Gamma(\eta)}\int_0^t (t-\lambda)^{\eta-1}H_1(\lambda,S_{n-1})\,d\lambda,$$

$$
\begin{aligned}
E_n(t) &= \frac{1-\eta}{F(\eta)}H_2(t,E_{n-1})+\frac{\eta}{F(\eta)\Gamma(\eta)}\int_0^t (t-\lambda)^{\eta-1}H_2(\lambda,E_{n-1})\,d\lambda,\\
I_n(t) &= \frac{1-\eta}{F(\eta)}H_3(t,I_{n-1})+\frac{\eta}{F(\eta)\Gamma(\eta)}\int_0^t (t-\lambda)^{\eta-1}H_3(\lambda,I_{n-1})\,d\lambda,\\
R_n(t) &= \frac{1-\eta}{F(\eta)}H_4(t,R_{n-1})+\frac{\eta}{F(\eta)\Gamma(\eta)}\int_0^t (t-\lambda)^{\eta-1}H_4(\lambda,R_{n-1})\,d\lambda,\\
A_n(t) &= \frac{1-\eta}{F(\eta)}H_5(t,A_{n-1})+\frac{\eta}{F(\eta)\Gamma(\eta)}\int_0^t (t-\lambda)^{\eta-1}H_5(\lambda,A_{n-1})\,d\lambda, \qquad (4.5)
\end{aligned}
$$

where the initial conditions are

$$S_0(t)=S(0),E_0(t)=E(0),I_0(t)=I(0),R_0(t)=R(0),A_0(t)=A(0).$$

The difference between the successive terms take of the following expressions:

$$\varpi_{1n}(t)=S_n(t)-S_{n-1}(t)=\frac{1-\eta}{F(\eta)}\left[H_1(t,S_{n-1})-H_1(t,S_{n-2})\right]$$
$$+\frac{\eta}{F(\eta)\Gamma(\eta)}\int_0^t (t-\lambda)^{\eta-1}\left[H_1(\lambda,S_{n-1})-H_1(\lambda,S_{n-2})\right]d\lambda,$$

$$\varpi_{2n}(t)=E_n(t)-E_{n-1}(t)=\frac{1-\eta}{F(\eta)}\left[H_2(t,E_{n-1})-H_2(t,E_{n-2})\right]$$
$$+\frac{\eta}{F(\eta)\Gamma(\eta)}\int_0^t (t-\lambda)^{\eta-1}\left[H_2(\lambda,E_{n-1})-H_2(\lambda,E_{n-2})\right]d\lambda,$$

$$\varpi_{3n}(t)=I_n(t)-I_{n-1}(t)=\frac{1-\eta}{F(\eta)}\left[H_3(t,I_{n-1})-H_3(t,I_{n-2})\right]$$
$$+\frac{\eta}{F(\eta)\Gamma(\eta)}\int_0^t (t-\lambda)^{\eta-1}\left[H_3(\lambda,I_{n-1})-H_3(\lambda,I_{n-2})\right]d\lambda,$$

$$\varpi_{4n}(t)=R_n(t)-R_{n-1}(t)=\frac{1-\eta}{F(\eta)}\left[H_4(t,R_{n-1})-H_4(t,R_{n-2})\right]$$
$$+\frac{\eta}{F(\eta)\Gamma(\eta)}\int_0^t (t-\lambda)^{\eta-1}\left[H_4(\lambda,R_{n-1})-H_4(\lambda,R_{n-2})\right]d\lambda,$$

$$\varpi_{5n}(t) = A_n(t) - A_{n-1}(t) = \frac{1-\eta}{F(\eta)}\left[H_5(t, A_{n-1}) - H_5(t, A_{n-2})\right]$$

$$+\frac{\eta}{F(\eta)\Gamma(\eta)}\int_0^t (t-\lambda)^{\eta-1}\left[H_5(\lambda, A_{n-1}) - H_5(\lambda, A_{n-2})\right] d\lambda. \tag{4.6}$$

In the view of above calculations, it is clear that

$$\begin{aligned} S_n(t) &= \sum_{k=1}^{n} \varpi_{1k}(t), \\ E_n(t) &= \sum_{k=1}^{n} \varpi_{2k}(t), \\ I_n(t) &= \sum_{k=1}^{n} \varpi_{3k}(t), \\ R_n(t) &= \sum_{k=1}^{n} \varpi_{4k}(t), \\ A_n(t) &= \sum_{k=1}^{n} \varpi_{5k}(t). \end{aligned} \tag{4.7}$$

Implementing the norm to both sides of the Eq. (4.6) and then triangular identity, we get

$$\begin{aligned} &\|\varpi_{1n}(t)\| = \|S_n(t) - S_{n-1}(t)\| \\ &\leq \frac{1-\eta}{F(\eta)}\|[H_1(t, S_{n-1}) - H_1(t, S_{n-2})]\| \\ &+\frac{\eta}{F(\eta)\Gamma(\eta)}\left\|\int_0^t (t-\lambda)^{\eta-1}\left[H_1(\lambda, S_{n-1}) - H_1(\lambda, S_{n-2})\right] d\lambda\right\| \end{aligned} \tag{4.8}$$

Because the kernel H_1 fulfills the Lipschitz condition proved in Eq. (4.4), we find

$$\begin{aligned} \|\varpi_{1n}(t)\| &= \|S_n(t) - S_{n-1}(t)\| \\ &\leq \frac{1-\eta}{F(\eta)}\bar{\delta}_1 \|S_{n-1} - S_{n-2}\| + \frac{\eta}{F(\eta)\Gamma(\eta)}\bar{\delta}_1 \int_0^t (t-\lambda)^{\eta-1}\|S_{n-1} - S_{n-2}\| d\lambda \end{aligned} \tag{4.9}$$

and we have

$$\|\varpi_{1n}(t)\| \leq \frac{1-\eta}{F(\eta)}\bar{\delta}_1 \left\|\varpi_{1(n-1)}(t)\right\| + \frac{\eta}{F(\eta)\Gamma(\eta)}\bar{\delta}_1 \int_0^t (t-\lambda)^{\eta-1}\left\|\varpi_{1(n-1)}(\lambda)\right\| d\lambda \tag{4.10}$$

Analogously, for the rest equations of the model, we get the followings:

$$\begin{aligned}
\|\varpi_{2n}(t)\| &\leq \frac{1-\eta}{F(\eta)}\bar{\delta}_2\left\|\varpi_{2(n-1)}(t)\right\| + \frac{\eta}{F(\eta)\Gamma(\eta)}\bar{\delta}_2\int_0^t (t-\lambda)^{\eta-1}\left\|\varpi_{2(n-1)}(\lambda)\right\| d\lambda,\\
\|\varpi_{3n}(t)\| &\leq \frac{1-\eta}{F(\eta)}\bar{\delta}_3\left\|\varpi_{3(n-1)}(t)\right\| + \frac{\eta}{F(\eta)\Gamma(\eta)}\bar{\delta}_3\int_0^t (t-\lambda)^{\eta-1}\left\|\varpi_{3(n-1)}(\lambda)\right\| d\lambda,\\
\|\varpi_{4n}(t)\| &\leq \frac{1-\eta}{F(\eta)}\bar{\delta}_4\left\|\varpi_{4(n-1)}(t)\right\| + \frac{\eta}{F(\eta)\Gamma(\eta)}\bar{\delta}_4\int_0^t (t-\lambda)^{\eta-1}\left\|\varpi_{4(n-1)}(\lambda)\right\| d\lambda,\\
\|\varpi_{5n}(t)\| &\leq \frac{1-\eta}{F(\eta)}\bar{\delta}_5\left\|\varpi_{5(n-1)}(t)\right\| + \frac{\eta}{F(\eta)\Gamma(\eta)}\bar{\delta}_5\int_0^t (t-\lambda)^{\eta-1}\left\|\varpi_{5(n-1)}(\lambda)\right\| d\lambda. \qquad (4.11)
\end{aligned}$$

In the light of the results in hand, one can state the theorem given below.

Theorem 4.2. *The fractional model given as (3.2) has a solution, if we can find t_0 satisfying the equation*

$$\frac{1-\eta}{F(\eta)}\bar{\delta}_i + \frac{t_0^{\eta}}{F(\eta)\Gamma(\eta)}\bar{\delta}_i < 1 \; for \; i = 1,2,3,4,5. \qquad (4.12)$$

Proof. We know that $S(t)$, $E(t)$, $I(t)$, $R(t)$, $A(t)$ are bounded functions and carry out Lipschitz condition. Having regard the Eqs. (4.10) and (4.11), we get the succeeding relations:

$$\begin{aligned}
\|\varpi_{1n}(t)\| &\leq \|S_n(0)\|\left[\frac{1-\eta}{F(\eta)}\bar{\delta}_1 + \frac{t^{\eta}}{F(\eta)\Gamma(\eta)}\bar{\delta}_1\right]^n,\\
\|\varpi_{2n}(t)\| &\leq \|E_n(0)\|\left[\frac{1-\eta}{F(\eta)}\bar{\delta}_2 + \frac{t^{\eta}}{F(\eta)\Gamma(\eta)}\bar{\delta}_2\right]^n,\\
\|\varpi_{3n}(t)\| &\leq \|I_n(0)\|\left[\frac{1-\eta}{F(\eta)}\bar{\delta}_3 + \frac{t^{\eta}}{F(\eta)\Gamma(\eta)}\bar{\delta}_3\right]^n,\\
\|\varpi_{4n}(t)\| &\leq \|R_n(0)\|\left[\frac{1-\eta}{F(\eta)}\bar{\delta}_4 + \frac{t^{\eta}}{F(\eta)\Gamma(\eta)}\bar{\delta}_4\right]^n,\\
\|\varpi_{5n}(t)\| &\leq \|A_n(0)\|\left[\frac{1-\eta}{F(\eta)}\bar{\delta}_5 + \frac{t^{\eta}}{F(\eta)\Gamma(\eta)}\bar{\delta}_5\right]^n. \qquad (4.13)
\end{aligned}$$

Thus, the existence and continuity of the aforementioned solutions are proved. Now, our goal is to show that the above functions are solutions of Eq. (3.2), suppose that

$$\begin{aligned}
&S(t) - S(0) = S_n(t) - \overline{\upsilon}_{1n}(t),\\
&E(t) - E(0) = E_n(t) - \overline{\upsilon}_{2n}(t),\\
&I(t) - I(0) = I_n(t) - \overline{\upsilon}_{3n}(t),\\
&R(t) - R(0) = R_n(t) - \overline{\upsilon}_{4n}(t),\\
&A(t) - A(0) = A_n(t) - \overline{\upsilon}_{5n}(t). \qquad (4.14)
\end{aligned}$$

Next, we have

$$\|\overline{\upsilon}_{1n}(t)\| = \left\| \frac{1-\eta}{F(\eta)}[H_1(t,S) - H_1(t,S_{n-1})] + \frac{\eta}{F(\eta)\Gamma(\eta)} \int_0^t (t-\lambda)^{\eta-1} [H_1(\lambda,S) - H_1(\lambda,S_{n-1})]\, d\lambda \right\|$$
$$\leq \frac{1-\eta}{F(\eta)} \|H_1(t,S) - H_1(t,S_{n-1})\| + \frac{\eta}{F(\eta)\Gamma(\eta)} \int_0^t (t-\lambda)^{\eta-1} \|H_1(\lambda,S) - H_1(\lambda,S_{n-1})\, d\lambda\|$$
$$\leq \frac{1-\eta}{F(\eta)}\overline{\delta}_1 \|S - S_{n-1}\| + \frac{t^\eta}{F(\eta)\Gamma(\eta)}\overline{\delta}_1 \|S - S_{n-1}\|. \tag{4.15}$$

By continuing this method recursively, it yields at t_0

$$\|\overline{\upsilon}_{1n}(t)\| \leq \left(\frac{1-\eta}{F(\eta)} + \frac{t_0^\eta}{F(\eta)\Gamma(\eta)} \right)^{n+1} \overline{\delta}_1^{n+1} a. \tag{4.16}$$

As n tends to ∞ taking the limit both sides, we have $\|\overline{\upsilon}_{1n}(t)\| \to 0$. In an analogous way, it can be shown $\|\overline{\upsilon}_{2n}(t)\| \to 0$, $\|\overline{\upsilon}_{3n}(t)\| \to 0$, $\|\overline{\upsilon}_{4n}(t)\| \to 0$ and $\|\overline{\upsilon}_{5n}(t)\| \to 0$. □

It is another crucial subject to demonstrate the uniqueness of the solutions of the Eq. (3.2). Let $S_1(t)$, $E_1(t)$, $I_1(t)$, $R_1(t)$ and $A_1(t)$ be another solutions of the model (3.2), we find

$$S(t) - S_1(t) = \frac{1-\eta}{F(\eta)}[H_1(t,S) - H_1(t,S_1)] + \frac{\eta}{F(\eta)\Gamma(\eta)} \times \int_0^t (t-\lambda)^{\eta-1} [H_1(\lambda,S) - H_1(\lambda,S_1)]\, d\lambda \tag{4.17}$$

Taking the norm to the Eq. (4.17), and then since the kernel satisfies the Lipschitz condition, we obtain

$$\|S(t) - S_1(t)\| \leq \frac{1-\eta}{F(\eta)}\overline{\delta}_1 \|S(t) - S_1(t)\| + \frac{t^\eta}{F(\eta)\Gamma(\eta)}\overline{\delta}_1 \|S(t) - S_1(t)\| \tag{4.18}$$

This leads to

$$\|S(t) - S_1(t)\| \left(1 - \frac{1-\eta}{F(\eta)}\overline{\delta}_1 - \frac{t^\eta}{F(\eta)\Gamma(\eta)}\overline{\delta}_1 \right) \leq 0. \tag{4.19}$$

If the following inequality holds

$$\left(1 - \frac{1-\eta}{F(\eta)}\overline{\delta}_1 - \frac{t^{\eta}}{F(\eta)\Gamma(\eta)}\overline{\delta}_1\right) > 0, \tag{4.20}$$

then $\|S(t) - S_1(t)\| = 0$. So we have

$$S(t) = S_1(t).$$

Using the same attitude, we obtain the followings and complete the proof

$$E(t) = E_1(t), I(t) = I_1(t), R(t) = R_1(t), A(t) = A_1(t).$$

5. Numerical simulations and discussion

The purpose of this section is to observe what happens when fractional order η changes in the model (3.2). For this reason, several numerical simulations of this model will be given using the numerical technique which has been recently developed by Toufik and Atangana [44]. We use the initial conditions (70, 10, 10, 0, 10), respectively, and use the parameters $b = 0.5$, $\tau = 0.3$, $d_S = 0.0001$, $d_E = 0.0001$, $d_I = 0.0005$, $\mu = 0.0001$, $\theta = 0.005$, $\varphi = 0.1$ given in [43]. Figure 1 shows that the model scenario is exposed to the critical attitude where the system is attacked by infectious populations $I(t)$ for a long duration. In the Figure 2, it can be observed that as the fractional order η declines, the number of susceptible populations $S(t)$ rises whereas the number of infectious populations $I(t)$ reduces with $\eta = 0.5$ and 0.1. It can also clearly be seen that when a considerable increase and decrease occurs in $S(t)$ and $I(t)$ respectively, almost no change in observed $E(t)$, $R(t)$ and $A(t)$. On Figure 3, the behavior of the model components are displayed according to different values of the fractional order η. In addition, from Figure 3 it is clearly visible that as η goes up, the number of susceptible populations $S(t)$ decreases while the number of infectious populations $I(t)$ increases.

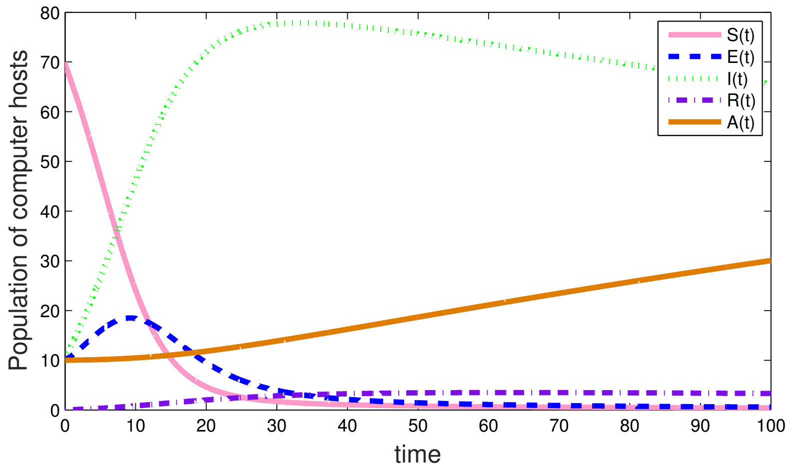

Figure 1. Numerical simulations for the Eq. (3.2) at $\eta = 0.9$.

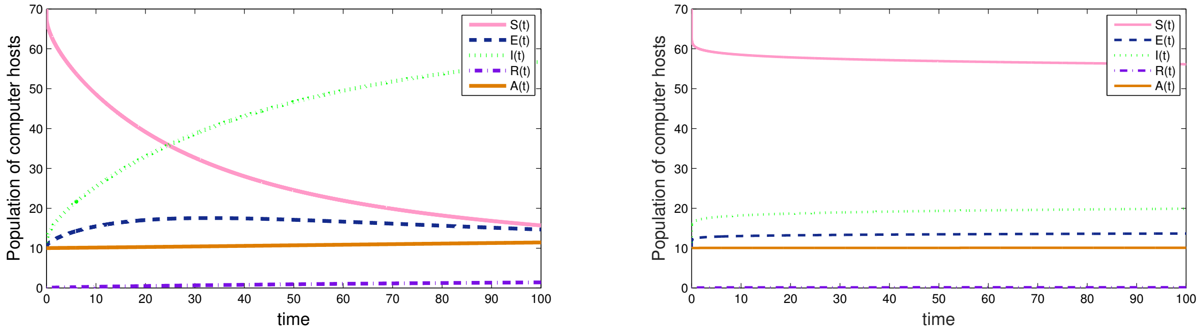

Figure 2. Numerical simulations for the Eq. (3.2) at $\eta = 0.5$ and $\eta = 0.1$, respectively.

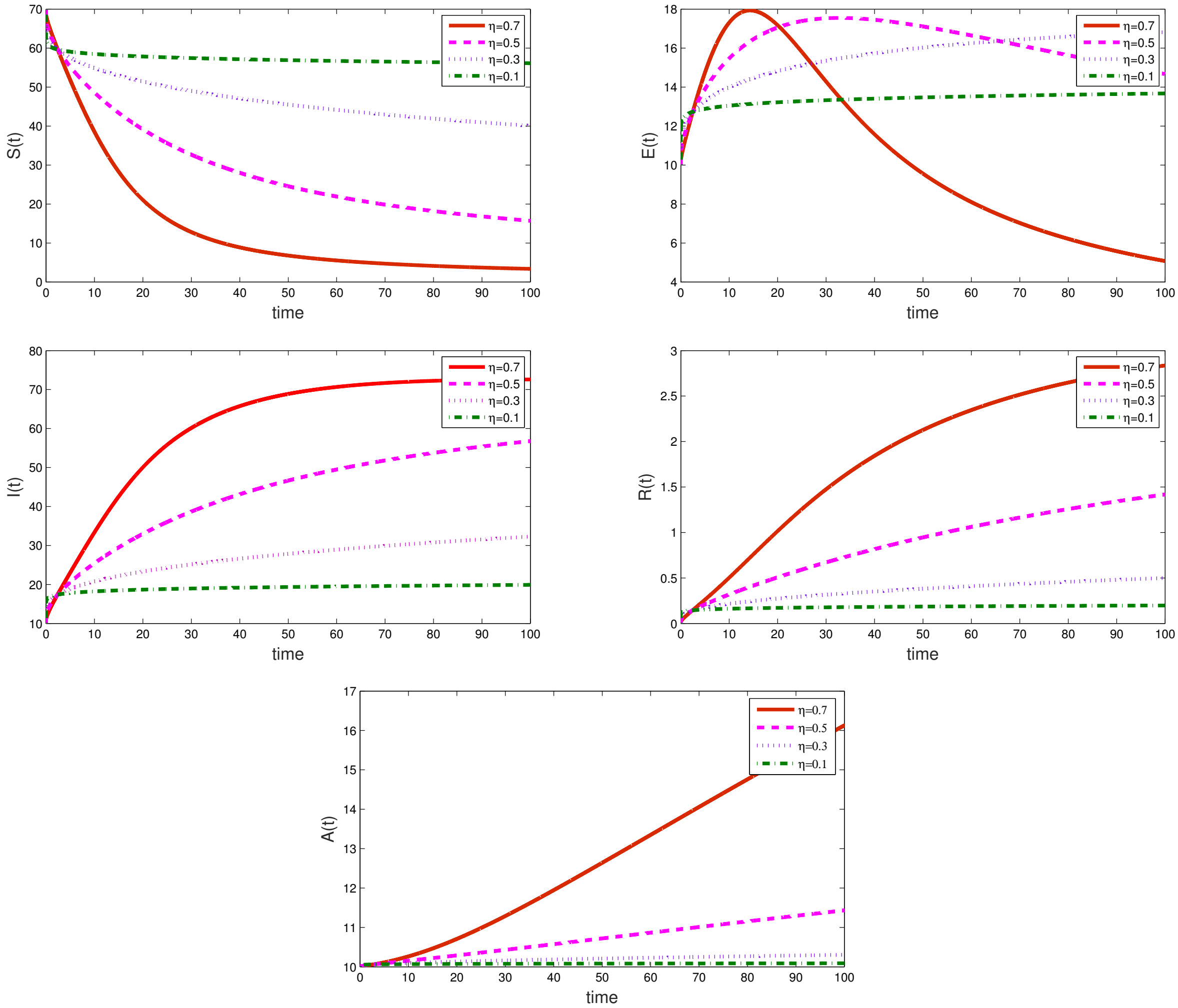

Figure 3. The behavior of the fractional basic SEIRA model components for distinct values of η.

6. Concluding remarks

Since malicious objects such as computer virus, worms etc. are major threats in the present days, a better understanding malicious objects is of critical significance for the computer security. In this study, the concept of ABC derivative serving a memory effect assists us for a comprehensive examination of the basic SEIRA model about computer worms. First, we remodel classical basic SEIRA model with the ABC fractional derivative. Second, the existence and uniqueness conditions for the fractional model are proved by means of the fixed point theory. Some numerical simulations are depicted with different values of η and briefly interpreted. Since in computer world, it is of utmost importance to decrease the number of infectious populations $I(t)$, we show in our fractional model that effectiveness of $I(t)$ declines as the fractional order η decreases, which is a great advantage of the AB derivative with hereditary properties. Consequently, in order to reveal the hidden properties of real-world phenomenas, we can conclude that the prospects of ABC fractional derivative ensure more suitable models of these phenomenas.

Conflict of interest

The author declares that no conflicts of interest in this paper.

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