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On $m^* - g-$Closed Sets and $m^* - R_0$ Spaces in a Hereditary $m-$Space (X, m, H)

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Abstract. Noiri and Popa [18] have defined the minimal local function and the minimal structure m_H^* which contains m in a hereditary minimal space (X, m, H). Moreover the concepts of $m - H_g-$closed sets and $(\Lambda, m_H^*)-$closed sets in a hereditary minimal space (X, m, H) are presented and investigated by Noiri and Popa in [18]. In this paper, we define the notions m^*-g-closed sets and $m^* - H_g-$closed sets in a hereditary minimal space (X, m, H) and explore some of their basic properties and few characterizations.
Keywords: $m^* - H_g - closed$, $m^* - g - closed$, $m^* - R_1$, $m - R_0$, $m^* - R_0$
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INTRODUCTION

The idea of minimal spaces was first discovered by Popa and Noiri [24]. A subfamily m of a nonempty set X is called a minimal structure if $\emptyset \in m$ and $X \in m$. Since then, many topologists have shown an increasing trend to study this notion. Insomuch that; A great deal of survey (e.g. see [1-7, 9, 11, 13-17, 19-23]) occurred as the by-products of this concept.

The notion of ideals in topological spaces was revealed by Kuratowski [12]. Janković and Hamlett [10] presented the notion of the local function in an ideal topological space (X, τ, I) and then generated the topology τ^*. And they examined in detail the properties of this topology.

Ozbakır and Yildirim [19] introduced and studied the concepts of m^*-closed sets and $m - I_g-$closed sets in an ideal minimal space. The $m-$local function and minimal $^*-$closures in an ideal minimal space (X, m, I) are presented and enquired.

A subfamily H of the power set of X is called a hereditary class [8] if B ⊂ A and A ∈ H implies B ∈ H.

T. Noiri and V. Popa [18] have defined a new set−operator on a minimal space by using the inherited class given Császár [8]. A minimal space (X, m) with a hereditary class H on X is called a hereditary minimal space (briefly hereditary $m-$ space) and is denoted by (X, m, H). Moreover, Noiri and Popa [18] have introduced $m - H_g-$closed sets and $(\Lambda, m_H^*)-$closed sets in a hereditary minimal space (X, m, I). And they have obtained decompositions of m_H^*-closed sets by using $m - H_g-$closed sets and $(\Lambda, m_H^*)-$closed sets.

In the first section of this paper, we recall the basic concepts that were required for the study. In the second section, we introduce the concepts of $m^* - g-$ closed sets and $m^* - H_g-$closed sets in a hereditary minimal space (X, m, H). And we study their properties and show that an $m^* - g-$closed set is weaker than an $m-$closed set and stronger than an $mg-$closed set. In the last section, by using $m^* - g-$closed sets we introduce some new types of separation axioms called $m^* - R_0$ and $m^* - R_1$ and investigate some of their characterizations.

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m^*-g-closed sets

Definition 1 *A subset A of a hereditary m−space (X, m, H) is said to be $m^* - Hg-$closed (resp. $m^* - g-$closed) set if $A^*_{mH} \subset U$ ($resp.\ mCl(A) \subset U$) whenever $A \subset U$ and U is $m^*_H -$ open. A subset A of X is said to be $m^* - H_g -$ open (resp. $m^* - g -$ open) if its complement is $m^* - H_g-$closed(resp. $m^* - g-$closed).*

Proposition 1 *Let (X, m, H) be a hereditary m−space. Then for a subset of X, the following implications hold:*

$$\begin{array}{ccc} m - closed & \Rightarrow & m^*_H - closed \\ \Downarrow & & \Downarrow \\ m^* - g - closed & \Rightarrow & m^* - H_g - closed \\ \Downarrow & & \Downarrow \\ mg - closed & \Rightarrow & m - H_g - closed \end{array}$$

m^*-R_0 spaces

In this section we study some new types of separation axiom in a hereditary $m-$space (X, m, H) by using m^*-g-closed sets.

Definition 2 *An m−space (X, m) is said to be $m - R_0$ [7] if for each m−open set U and each $x \in U$, $mCl(\{x\}) \subseteq U$.*

The notion of $m^* - R_0$ spaces is defined as follows:

Definition 3 *A hereditary m−space (X, m, H) said to be $m^* - R_0$ if for every m^*_H-open set U and each $x \in U$, $mCl(\{x\}) \subseteq U$.*

Remark 1 *Since m−open sets are m^*_H-open, every $m^* - R_0$ space is $m - R_0$.*

We define a separation axiom called $m^* - R_1$ which is stronger than $m^* - R_0$.

Definition 4 *A hereditary m−space (X, m, H) is said to be $m^* - R_1$ if for every $x, y \in X$ with $mCl(\{x\}) \neq mCl^*_H(\{y\})$, there exist two disjoint m^*_H-open sets U and V such that $mCl(\{x\}) \subseteq U$ and $mCl^*_H(\{y\}) \subseteq V$.*

Theorem 1 *We obtain the implications for a hereditary m−space (X, m, H).*

$$\begin{array}{c} m^* - R_1 \ space \\ \Downarrow \\ m^* - R_0 \ space \\ \Downarrow \\ m - R_0 \ space \end{array}$$

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