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Abstract. In this study, we aim to investigate the neutrosophic soft semiregularization spaces associated with neutrosophic soft topological spaces. We introduce the concept of neutrosophic soft submaximal spaces and prove that corresponding to each neutrosophic soft topological space, there always exists a neutrosophic soft submaximal space which is an expansion of the given space. It is shown that neutrosophic soft submaximal and neutrosophic soft semiregular spaces are closely associated with those spaces which are minimal or maximal in accordance with certain types of properties which is called neutrosophic soft semiregular properties in this document. This has been an inspiration for us to deal with different characteristics for examination whether these are neutrosophic soft semiregular ones.
Keywords: Neutrosophic soft semi-regularization topology, neutrosophic soft ro-equivalence, neutrosophic soft submaximal space, neutrosophic soft nearly compact space, neutrosophic soft S-closed space
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INTRODUCTION

It is widely known that corresponding to each topological space, there always exists an associated semiregular space coarser than the space itself. Many scientists studied the semiregularization topology of this associated space thoroughly such as Bourbaki [7], Cameron [8], Mrsevic et al. [13] and many others. In [7], Bourbaki gave the definition of submaximal space and listed its properties. Furthermore, Cameron [6] studied the properties submaximal spaces.Bera and Mahapatra [4] defined neutrosophic soft relation. Smarandache[14] and Molodstov[12] initiated the theory of neutrosophic sets and the theory of soft sets in 2005 and 1999, respectively. These theories have always constituted research areas for scientists to make investigations as in [1, 3, 9, 15]. In 2013, Maji [11] presented the concept of neutrosophic soft set. Then, Bera presented neutrosophic soft topological spaces in [6]. And C.G. Aras, T.Y. Ozturk and S. Bayramov made a new approach to the concept of neutrosophic soft topological space in [2]. In this paper, our purpose is to extend these ideas to a neutrosophic soft topological space.

preliminaries

Definition 1 *[14] A neutrosophic set A on the universe set X is defined as:* $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$, *where* $T, I, F : X \to]^-0, 1^+[$ *and* $^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

Definition 2 *[12] Let X be an initial universe, E be a set of all parameters and* $P(X)$ *denote the power set of X. A pair* (F, E) *is called a* soft set *over X where F is a mapping given by* $F : E \longrightarrow P(X)$. *In other words, the soft set is a parameterized family of subsets of the set X. For* $e \in E$, $F(e)$ *may be considered as the set of e-elements of the soft set* (F, E) *or as the set of e-approximate elements of the soft set i.e.* $(F, E) = \{(e, F(e)) : e \in E, F : E \longrightarrow P(X)\}$.

Definition 3 *[10] Let X be an initial universe set and E be a set of parameters. Let* $P(X)$ *denote the set of all neutrosophic sets of X. Then a neutrosophic soft set* $\left(\widetilde{F}, E\right)$ *over X is a set defined by a set valued function* $\widetilde{F}$

representing a mapping $\widetilde{F}: E \to P(X)$ *where* $\widetilde{F}$ *is called the approximate function of the* neutrosophic soft set $\left(\widetilde{F}, E\right)$. *In other words, the neutrosophic soft set is a parametrized family of some elements of the set* $P(X)$ *and therefore it can be written as a set of ordered pairs:* $\left(\widetilde{F}, E\right) = \left\{\left(e, \left\langle x, T_{\widetilde{F}(e)}(x), I_{\widetilde{F}(e)}(x), F_{\widetilde{F}(e)}(x)\right\rangle : x \in X\right) : e \in E\right\}$ *where* $T_{\widetilde{F}(e)}(x), I_{\widetilde{F}(e)}(x), F_{\widetilde{F}(e)}(x) \in [0, 1]$ *are respectively called the truth-membership, indeterminacy-membership and falsity-membership function of* $\widetilde{F}(e)$. *Since the supremum of each* T, I, F *is* 1, *the inequality*
$0 \leq T_{\widetilde{F}(e)}(x) + I_{\widetilde{F}(e)}(x) F_{\widetilde{F}(e)}(x) \leq 3$ *is obvious.*

Definition 4 *[6] Let* $\left(\widetilde{F}, E\right)$ *be a neutrosophic soft set over the universe set X. The complement of* $\left(\widetilde{F}, E\right)$ *is denoted by* $\left(\widetilde{F}, E\right)^c$ *and is defined by:*

$$\left(\widetilde{F}, E\right)^c = \left\{\left(e, \left\langle x,\ F_{\widetilde{F}(e)}(x), 1 - I_{\widetilde{F}(e)}(x),\ T_{\widetilde{F}(e)}(x)\right\rangle : x \in X\right) : e \in E\right\}.$$

It is obvious that $\left[\left(\widetilde{F}, E\right)^c\right]^c = \left(\widetilde{F}, E\right)$.

Definition 5 *[11] Let* $\left(\widetilde{F}, E\right)$ *and* $\left(\widetilde{G}, E\right)$ *be two neutrosophic soft sets over the universe set X.* $\left(\widetilde{F}, E\right)$ *is said to be a neutrosophic soft subset of* $\left(\widetilde{G}, E\right)$ *if* $T_{\widetilde{F}(e)}(x) \leq T_{\widetilde{G}(e)}(x)$, $I_{\widetilde{F}(e)}(x) \leq I(x)$, $F_{\widetilde{F}(e)}(x) \leq F_{\widetilde{G}(e)}(x)$, $\forall e \in E$, $\forall x \in X$. *It is denoted by* $\left(\widetilde{F}, E\right) \subseteq \left(\widetilde{G}, E\right)$. $\left(\widetilde{F}, E\right)$ *is said to be neutrosophic soft equal to* $\left(\widetilde{G}, E\right)$ *if* $\left(\widetilde{F}, E\right) \subseteq \left(\widetilde{G}, E\right)$ *and* $\left(\widetilde{G}, E\right) \subseteq \left(\widetilde{F}, E\right)$. *It is denoted by* $\left(\widetilde{F}, E\right) = \left(\widetilde{G}, E\right)$.

Definition 6 *[2] Let* $NSS(X, E)$ *be the family of all neutrosophic soft sets over the universe set X and* $\tau \subset NSS(X, E)$. *Then* τ *is said to be a* neutrosophic soft topology *on X if:*

1. $0_{(X,E)}$ *and* $1_{(X,E)}$ *belong to* τ,
2. *the union of any number of neutrosophic soft sets in* τ *belongs to* τ,
3. *the intersection of a finite number of neutrosophic soft sets in* τ *belongs to* τ.

Then (X, τ, E) *is said to be a neutrosophic soft topological space over X. Each member of* τ *is said to be a neutrosophic soft open set [1].*

neutrosophic soft semiregularization

Definition 7 *[6] Let* (X, τ, E) *be a neutrosophic soft topological space and* $\left(\widetilde{F}, E\right) \in NSS(X, E)$ *be arbitrary. Then the* interior *of* $\left(\widetilde{F}, E\right)$ *is denoted by* $\left(\widetilde{F}, E\right)^\circ$ *and is defined as:*

$$\left(\widetilde{F}, E\right)^\circ = \bigcup\left\{\left(\widetilde{G}, E\right) : \left(\widetilde{G}, E\right) \subset \left(\widetilde{F}, E\right), \left(\widetilde{G}, E\right) \in \tau\right\}$$

i.e., it is the union of all open neutrosophic soft subsets of $\left(\widetilde{F}, E\right)$.

Definition 8 *[6] Let* (X, τ, E) *be a neutrosophic soft topological space and* $\left(\widetilde{F}, E\right) \in NSS(X, E)$ *be arbitrary. Then the* closure *of* $\left(\widetilde{F}, E\right)$ *is denoted by* $\overline{\left(\widetilde{F}, E\right)}$ *and is defined as:*

$$\overline{\left(\widetilde{F}, E\right)} = \bigcap\left\{\left(\widetilde{G}, E\right) : \left(\widetilde{G}, E\right) \subset \left(\widetilde{F}, E\right), \left(\widetilde{G}, E\right)^c \in \tau\right\}$$

i.e., it is the intersection of all closed neutrosophic soft super sets of $\left(\widetilde{F}, E\right)$.

Definition 9 *A neutrosophic soft set* $\left(\widetilde{F}, E\right)$ *in a neutrosophic soft topological space* (X, τ, E) *is called a neutrosophic regular open soft set if and only if* $\left(\widetilde{F}, E\right) = \left[\overline{\left(\widetilde{F}, E\right)}\right]^\circ$. *The complement of a neutrosophic soft regular open set is called a neutrosophic regular closed soft set.Let* (X, τ, E) *be a neutrosophic soft topological space. Consider the set of all neutrosophic soft regularly open sets in* (X, τ, E). *Then it is easy to see that it forms a base for some neutrosophic soft topology on X. We call this topology the neutrosophic soft semiregularization topology of* τ, *to be denoted by* τ_s.

Clearly $\tau_S \subseteq \tau$. (X, τ_S, E) *is called the neutrosophic soft semiregularization space. We can define a neutrosophic soft topology* (X, τ, E) *to be neutrosophic soft semiregular iff the neutrosophic soft regularly open sets in* (X, τ, E) *form a base for the neutrosophic soft topology* τ *on* X*. Thus according to the above definition,* (X, τ, E) *is neutrosophic soft semiregular iff* $\tau = \tau_S$.

Definition 10 *A neutrosophic soft point* $\left(\widetilde{F}, E\right)$ *is said to be neutrosophic soft quasi-coincident (neutrosophic soft q-coincident, for short) with* $\left(\widetilde{G}, E\right)$*, denoted by* $\left(\widetilde{F}, E\right) q \left(\widetilde{G}, E\right)$*, if and only if* $\left(\widetilde{F}, E\right) \nsubseteq \left(\widetilde{G}, E\right)^c$*. If* $\left(\widetilde{F}, E\right)$ *is not neutrosophic soft quasi-coincident with* $\left(\widetilde{G}, E\right)$*, we denote by* $\left(\widetilde{F}, E\right) q \left(\widetilde{G}, E\right)$.

Definition 11 *A neutrosophic soft point* $x^e_{(\alpha,\beta,\gamma)}$ *is said to be neutrosophic soft quasi-coincident (neutrosophic soft q-coincident, for short) with* $\left(\widetilde{F}, E\right)$*, denoted by* $x^e_{(\alpha,\beta,\gamma)} q \left(\widetilde{F}, E\right)$*, if and only if* $x^e_{(\alpha,\beta,\gamma)} \nsubseteq \left(\widetilde{F}, E\right)^c$*. If* $x^e_{(\alpha,\beta,\gamma)}$ *is not neutrosophic soft quasi-coincident with* $\left(\widetilde{F}, E\right)$*, we denote by* $x^e_{(\alpha,\beta,\gamma)} q \left(\widetilde{F}, E\right)$.

Theorem 1 *Let* (X, τ, E) *be a neutrosophic soft topological space. The following statements are equivalent:*

(a) (X, τ, E) *is neutrosophic soft semiregular;*

(b) for each neutrosophic soft open set $\left(\widetilde{U}, E\right)$ *and each neutrosophic soft point* $x^e_{(\alpha,\beta,\gamma)}$ *with* $x^e_{(\alpha,\beta,\gamma)} q \left(\widetilde{U}, E\right)$*, there exists a neutrosophic soft open set* $\left(\widetilde{V}, E\right)$ *such that* $x^e_{(\alpha,\beta,\gamma)} q \left(\widetilde{V}, E\right) \subseteq \left[\overline{\left(\widetilde{V}, E\right)}\right]^{\circ} \subseteq \left(\widetilde{U}, E\right)$*;*

(c) for each neutrosophic soft closed set $\left(\widetilde{A}, E\right)$ *and each neutrosophic soft point* $x^e_{(\alpha,\beta,\gamma)} \notin \left(\widetilde{A}, E\right)$*, there exists a neutrosophic soft regularly closed set* $\left(\widetilde{B}, E\right)$ *such that* $\left(\widetilde{A}, E\right) \subseteq \left(\widetilde{B}, E\right)$ *and* $x^e_{(\alpha,\beta,\gamma)} \notin \left(\widetilde{B}, E\right)$*;*

(d) for each neutrosophic soft set $\left(\widetilde{A}, E\right)$ *in* (X, τ, E) *and each neutrosophic soft open set* B *with* $\left(\widetilde{A}, E\right) q \left(\widetilde{B}, E\right)$*, there exists a neutrosophic soft regularly open set* $\left(\widetilde{U}, E\right)$ *such that* $\left(\widetilde{A}, E\right) q \left(\widetilde{B}, E\right) \subseteq \left(\widetilde{B}, E\right)$.

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