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Neutrosophic Soft $\delta-$Topology and Neutrosophic Soft Compactness

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Abstract. We introduce the concepts of neutrosophic soft $\delta-$interior, neutrosophic soft quasicoincidence, neutrosophic soft q-neighborhood, neutrosophic regular open soft set, neutrosophic soft $\delta-$closure, neutrosophic soft $\theta-$closure, neutrosophic semi open soft set and show that the set of all neutrosophic soft $\delta-$open sets is also a neutrosophic soft topology, which is called the neutrosophic soft $\delta-$topology. We obtain equivalent forms of neutrosophic soft $\delta-$continuity. Moreover, the notions of neutrosophic soft $\delta-$compactness and neutrosophic soft locally $\delta-$compactness are defined and their basic properties under neutrosophic soft $\delta-$continuous mappings are investigated.
Keywords: Neutrosophic soft quasi-coincidence, neutrosophic regular open soft set, neutrosophic $\delta-$closed soft, neutrosophic semi open soft, neutrosophic soft $\delta-$topology
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INTRODUCTION

In 2005, the concept of a neutrosophic set was introduced by Smarandache as a generalization of classical sets, fuzzy set theory [20], intuitionistic fuzzy set theory [3], etc. By using this theory of neutrosophic set, many researches was made by mathematicians in sub branches of mathematics [7,16]. There are many inherent difficulties in classical methods for the inadequacy of the theories of parametrization tools. So, classical methods are insufficient in dealing with several practical problems in some other disciplines such as economics, engineering, environment, social science, medical science, etc. In 1999, Molodtsov pointed out the inherent difficulties of these theories [14]. A different approach was initiated by Molodtsov for modeling uncertainties. This approach was applied in some other directions such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration and so on. The theory of soft topological spaces was introduced by Shabir and Naz [17] for the first time in 2011. Soft topological spaces were defined over an initial universe with a fixed set of parameters and showed that a soft topological space gave a parameterized family of topological spaces. In [1,2,5,6,9,10,12] , some scientists made researches and did theoretical studies in soft topological spaces. In 2013, Maji [13] defined the concept of neutrosophic soft sets for the first time. Then, Deli and Broumi [11] modified this concept. In 2017, Bera presented neutrosophic soft topological spaces in [8].

Definition 1 *[18] A neutrosophic set A on the universe set X is defined as:* $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$, *where* $T, I, F : X \to]^-0, 1^+[$ *and* $^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

Definition 2 *[11] Let X be an initial universe set and E be a set of parameters. Let $P(X)$ denote the set of all neutrosophic sets of X. Then a neutrosophic soft set $\left(\widetilde{F}, E\right)$ over X is a set defined by a set valued function $\widetilde{F}$ representing a mapping $\widetilde{F} : E \to P(X)$ where $\widetilde{F}$ is called the approximate function of the* neutrosophic soft set $\left(\widetilde{F}, E\right)$. *In other words, the neutrosophic soft set is a parametrized family of some elements of the set $P(X)$ and therefore it can be written as a set of ordered pairs:*

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$$\left(\widetilde{F},E\right)=\left\{\left(e,\left\langle x,T_{\widetilde{F}(e)}(x),I_{\widetilde{F}(e)}(x),F_{\widetilde{F}(e)}(x)\right\rangle : x\in X\right):e\in E\right\},$$

where $T_{\widetilde{F}(e)}(x), I_{\widetilde{F}(e)}(x), F_{\widetilde{F}(e)}(x) \in [0,1]$ are respectively called the truth-membership, indeterminacy-membership and falsity-membership function of $\widetilde{F}(e)$. Since the supremum of each T, I, F is 1, the inequality

$$0\leq T_{\widetilde{F}(e)}(x)+I_{\widetilde{F}(e)}(x)F_{\widetilde{F}(e)}(x)\leq 3$$

is obvious.

Definition 3 *[4] Let $NSS(X,E)$ be the family of all neutrosophic soft sets over the universe set X and $\tau\subset NSS(X,E)$. Then τ is said to be a neutrosophic soft topology on X if:*

1) $0_{(X,E)}$ and $1_{(X,E)}$ belong to τ;

2) the union of any number of neutrosophic soft sets in τ belongs to τ;

3) the intersection of a finite number of neutrosophic soft sets in τ belongs to τ. Then (X,τ,E) is said to be a neutrosophic soft topological space over X. Each member of τ is said to be a neutrosophic soft open set [4].

Some definitions

Definition 4 *[8] Let (X,τ,E) be a neutrosophic soft topological space and $\left(\widetilde{F},E\right)\in NSS(X,E)$ be arbitrary. Then the* interior *of $\left(\widetilde{F},E\right)$ is denoted by $\left(\widetilde{F},E\right)^{\circ}$ and is defined as:*

$$\left(\widetilde{F},E\right)^{\circ}=\bigcup\left\{\left(\widetilde{G},E\right):\left(\widetilde{G},E\right)\subset\left(\widetilde{F},E\right),\left(\widetilde{G},E\right)\in\tau\right\}$$

i.e., it is the union of all open neutrosophic soft subsets of $\left(\widetilde{F},E\right)$. Also, the closure *of $\left(\widetilde{F},E\right)$ is denoted by $\overline{\left(\widetilde{F},E\right)}$ and is defined as:*

$$\overline{\left(\widetilde{F},E\right)}=\bigcap\left\{\left(\widetilde{G},E\right):\left(\widetilde{G},E\right)\subset\left(\widetilde{F},E\right),\left(\widetilde{G},E\right)^{c}\in\tau\right\}$$

i.e., it is the intersection of all closed neutrosophic soft super sets of $\left(\widetilde{F},E\right)$.

Definition 5 *Let $\left(\widetilde{F},E\right)$ be a neutrosophic soft set in a neutrosophic soft topological space (X,τ,E). Then,*

$$NS\,cl_{\delta}\left(\widetilde{F},E\right)=\bigcap\left\{\left(\widetilde{G},E\right)\in NSS\,(X,E):\left(\widetilde{F},E\right)\subset\left(\widetilde{G},E\right),\left(\widetilde{G},E\right)=\overline{\left[\left(\widetilde{G},E\right)^{\circ}\right]}\right\}$$

Definition 6 *A neutrosophic soft set $\left(\widetilde{F},E\right)$ is said to be neutrosophic $\delta-$closed soft if and only if $\left(\widetilde{F},E\right)=NS\,cl_{\delta}\left(\widetilde{F},E\right)$ and the complement of a neutrosophic $\delta-$closed soft set is called a neutrosophic $\delta-$open soft set.*

Theorem 1 *The finite union of neutrosophic $\delta-$closed soft sets is also neutrosophic soft $\delta-$closed. That is, if $\left(\widetilde{F},E\right)=NS\,cl_{\delta}\left(\widetilde{F},E\right)$ and $\left(\widetilde{G},E\right)=NS\,cl_{\delta}\left(\widetilde{G},E\right)$, then $\left(\widetilde{F},E\right)\cup\left(\widetilde{G},E\right)=NS\,cl_{\delta}\left[\left(\widetilde{F},E\right)\cup\left(\widetilde{G},E\right)\right]$.*

Corollary 1 *For any neutrosophic soft set $\left(\widetilde{F},E\right)$ in a neutrosophic in a neutrosophic soft topological space (X,τ,E), $NS\,cl_{\delta}\left(\widetilde{F},E\right)$ is a neutrosophic $\delta-$closed soft set. That is, $NS\,cl_{\delta}\left(NS\,cl_{\delta}\left(\widetilde{F},E\right)\right)=NS\,cl_{\delta}\left(\widetilde{F},E\right)$.*

Clearly, $NS\,cl_{\delta}\left(0_{(X,E)}\right)=0_{(X,E)}$. And for any neutrosophic soft subsets $\left(\widetilde{F},E\right)$ and $\left(\widetilde{G},E\right)$, if $\left(\widetilde{F},E\right)\subseteq\left(\widetilde{G},E\right)$ then $NS\,cl_{\delta}\left(\widetilde{F},E\right)\subseteq NS\,cl_{\delta}\left(\widetilde{G},E\right)$.

Therefore, by Theorem 7 and Corollary 8, the neutrosophic soft $\delta-$closure operation on a neutrosophic soft topological space (X,τ,E) satisfies the Kuratowski Closure Axioms. So, there exists one and only one topology on X. We will define the topology as follows.

Definition 7 *The set of all neutrosophic $\delta-$open soft sets of (X,τ,E) is also a neutrosophic soft topology on X. We denote it by τ_{δ} and it is called a neutrosophic soft $\delta-$topology on X. An ordered pair (X,τ_{δ}) is called a neutrosophic soft $\delta-$topological space.*

Conclusion

Therefore, some properties of the notions of neutrosophic $\delta-$open soft sets, neutrosophic $\delta-$closed soft sets, neutrosophic soft $\delta-$interior, neutrosophic soft $\delta-$closure, neutrosophic soft $\delta-$interior point, neutrosophic soft $\delta-$cluster point and neutrosophic soft $\delta-$topology are introduced. Also, the notions of neutrosophic soft $\delta-$compactness and neutrosophic soft locally $\delta-$compactness are introduced. Furthermore, the properties of neutrosophic soft $\delta-$ compactness and neutrosophic soft locally $\delta-$compactness are analized under the neutrosophic soft $\delta-$continuous mappings. Additionally, a new approach is made to the concept of quasi-coincidence in neutrosophic soft topology. Since topological structures on neutrosophic soft sets have been introduced by many scientists, we generalize the $\delta-$topological properties to the neutrosophic soft sets which may be useful in some other disciplines. For the existence of compact connections between soft sets and information systems [15, 19], the results obtained from the studies on neutrosophic soft topological space can be used to develop these connections. We hope that many researchers will benefit from the findings in this document to further their studies on neutrosophic soft topology to carry out a general framework for their applications in practical life.

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