

# A Type of Continuous Multifunction in Topological Spaces

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**Abstract** The aim of our study is to define and examine the notions of new classes of multifunctions and to obtain some properties of these multifunctions in fuzzy topological spaces.

**Keywords** Multifunctions, Fuzzy Sets, Fuzzy Continuity

## 1. Introduction

Up to the present, many studies have been made on closed sets and set concepts reproduced from this set. The concept of  $g$ -closed sets was introduced by Levine[3]. On the other hand, Yuksel and Beceren[6] have defined the notion of  $\beta^*$ -set and established a decomposition of continuity. Açıkgöz[1] introduced and studied the notions of  $\beta^*g$ -closed sets which are between the class of closed sets and  $g$ -closed sets. Fuzzy logic is firstly emerged with definition of the fuzzy set concept by Zadeh in 1965[7]. Also, fuzzy topological spaces are studied using the concept of fuzzy set. Chang introduced the concept of fuzzy topological space in 1968[2]. Then, many concepts of fuzzy topology are organized according to general topology. In this context, many of the concept of continuity of the general topology are transferred to fuzzy topology by different researchers. The idea of fuzzy multifunctions was first put forward by Papageorgio in 1985[5] and in 1991, Mukherjee ve Malakar studied semi-continuity, almost continuity and weak continuity for fuzzy multifunctions by using the concept of quasi-coincident[4]. The aim of this study is to expand  $\beta^*g$ -continuous functions to multifunctions and to examine some of the features of these functions.

## 2. Preliminaries

Let  $(X, \tau)$  be a topological space and  $A \subset X$ . If  $A = U \cap V$ , for  $U$  is open set and  $\text{int}(V) = \text{cl}(\text{int}(V))$ , then  $A$  is  $\beta^*$ -set[6]. If  $\text{cl}(A) \subset U$  for  $A \subset U$  and  $U \beta^*$ -set,  $A$  is  $\beta^*g$ -closed set[1]. The class of all  $\beta^*g$ -open ( $\beta^*g$ -closed) subsets of  $(X, \tau)$  is denoted by  $\beta^*g\mathcal{O}(X)$  ( $\beta^*g\mathcal{C}(X)$ ). For a subset  $A$  of  $(X, \tau)$ , we define the

$\beta^*g$ -closure and the  $\beta^*g$ -interior of  $A$  respectively, as follows:  $\beta^*g\text{-cl}(A) = \bigcap \{F: A \subset F \text{ and } F, \beta^*g\text{-closed}\}$  [1],  $\beta^*g\text{-int}(A) = \bigcup \{G: G \subset A \text{ ve } G, \beta^*g\text{-open}\}$ . Let  $X$  is nonempty set and  $I = [0, 1]$  closed interval. Every element of  $I^X$  is a fuzzy set of  $X$  for all  $\alpha: X \rightarrow I$  functions. that  $I^X$  is functions sets[7]. Fuzzy sets are denoted by  $\alpha, \beta, \gamma$ . We use  $\leq, \vee, \wedge, '$  notions for fuzzy sets instead of inclusion, union, intersection and complementation, respectively.

## 3. Fuzzy Continuous Multifunctions

Let  $x \in X$  and  $F: (X, \tau) \rightarrow (Y, \tau_Y)$  be a fuzzy multifunction.

### 3.1. Fuzzy Semi $\beta^*g$ -Continuous Multifunctions

#### 3.1.1. Definition

a)  $F$  is defined to be upper semi  $\beta^*g$ -continuous at  $x \in X$  if for every  $\mu \in \tau_Y$  such that  $x \in F^+(\mu)$  there exists a  $\beta^*g$ -open neighborhood  $U$  of  $x$  such that  $U \subset F^+(\mu)$ .

b)  $F$  is defined to be lower semi  $\beta^*g$ -continuous at  $x \in X$  if for every  $\mu \in \tau_Y$  such that  $x \in F^-(\mu)$  there exists a  $\beta^*g$ -open neighborhood  $U$  of  $x$  such that  $U \subset F^-(\mu)$ .

c)  $F$  is defined to be (upper) lower semi  $\beta^*g$ -continuous in  $X$  if it is (upper) lower semi  $\beta^*g$ -continuous for every  $x \in X$ .

#### 3.1.2. Theorem

$F: (X, \tau) \rightarrow (Y, \tau_Y)$  is fuzzy lower semi  $\beta^*g$ -continuous if and only if  $F^-(\mu)$  is  $\beta^*g$ -open in  $X$  for every  $\mu \in \tau_Y$ .

#### Proof

$\Rightarrow$ : Let  $F$  be fuzzy lower semi  $\beta^*g$ -continuous and  $x \in F^-(\mu)$  for  $\mu \in \tau_Y$ . Then, there exists a  $\beta^*g$ -open set  $U$  such that  $x \in U \subset F^-(\mu)$ . Hence,  $x \in \beta^*g\text{-int}(F^-(\mu))$  and  $F^-(\mu) = \beta^*g\text{-int}(F^-(\mu))$ . Consequently,  $F^-(\mu)$  is  $\beta^*g$ -open in  $X$ .

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$\Leftarrow$ : Let  $F^-(\mu)$  be  $\beta^*g$ -open in  $X$  for every  $\mu \in \tau_Y$ ,  $x \in X$  and  $x \in F^-(\mu)$ . If we take  $F^-(\mu) = U$ , then  $x \in U \subset F^-(\mu)$ . Thus,  $F$  is fuzzy lower semi  $\beta^*g$ -continuous at  $x \in X$ . Hence  $F$  is fuzzy lower semi  $\beta^*g$ -continuous in  $X$ .

### 3.1.3. Theorem

$F: (X, \tau) \rightarrow (Y, \tau_Y)$  is fuzzy upper semi  $\beta^*g$ -continuous if and only if  $F^+(\mu)$  is  $\beta^*g$ -open in  $X$  for every  $\mu \in \tau_Y$ .

#### Proof

$\Rightarrow$ : Let  $F$  be fuzzy upper semi  $\beta^*g$ -continuous and  $x \in F^+(\mu)$  for  $\mu \in \tau_Y$ . Then, there exists a  $\beta^*g$ -open set  $U$  such that  $x \in U \subset F^+(\mu)$ . Hence,  $x \in \beta^*g\text{-int}(F^+(\mu))$  and  $F^+(\mu) = \beta^*g\text{-int}(F^+(\mu))$ . Consequently,  $F^+(\mu)$  is  $\beta^*g$ -open in  $X$ .

$\Leftarrow$ : Let  $F^+(\mu)$  be  $\beta^*g$ -open in  $X$  for every  $\mu \in \tau_Y$ ,  $x \in X$  and  $x \in F^+(\mu)$ . If we take  $F^+(\mu) = U$ , then  $x \in U \subset F^+(\mu)$ . Thus,  $F$  is fuzzy upper semi  $\beta^*g$ -continuous at  $x \in X$ . Hence  $F$  is fuzzy upper semi  $\beta^*g$ -continuous in  $X$ .

## 3.2. Fuzzy Almost $\beta^*g$ -Continuous Multifunctions

### 3.2.1. Definition

Let  $x \in X$  and  $F: (X, \tau) \rightarrow (Y, \tau_Y)$  be a fuzzy multifunction.

a)  $F$  is defined to be upper almost  $\beta^*g$ -continuous at  $x \in X$  if for every  $\mu \in \tau_Y$  such that  $x \in F^+(\mu)$  there exists a  $\beta^*g$ -open neighborhood  $U$  of  $x$  such that  $U \subset F^+(\beta\text{-int}(f\text{-cl}(\mu)))$ .

b)  $F$  is defined to be lower almost  $\beta^*g$ -continuous at  $x \in X$  if for every  $\mu \in \tau_Y$  such that  $x \in F^-(\mu)$  there exists a  $\beta^*g$ -open neighborhood  $U$  of  $x$  such that  $U \subset F^-(\beta\text{-int}(f\text{-cl}(\mu)))$ .

c)  $F$  is defined to be (upper) lower almost  $\beta^*g$ -continuous in  $X$  if it is (upper) lower almost  $\beta^*g$ -continuous for every  $x \in X$ .

### 3.2.2. Theorem

The following are equivalent for a fuzzy multifunction  $F: (X, \tau) \rightarrow (Y, \tau_Y)$ :

- (1)  $F$  is fuzzy lower almost  $\beta^*g$ -continuous.
- (2)  $F^-(\mu) \subset \beta^*g\text{-int}(F^-(\beta\text{-int}(f\text{-cl}(\mu))))$  for every  $\mu \in \tau_Y$ .
- (3)  $F^-(\mu)$  is  $\beta^*g$ -open in  $X$  for every fuzzy regular open set  $\mu$  in  $Y$ .
- (4)  $F^-(\mu) \subset \beta^*g\text{-int}(F^-(\beta\text{-int}(f\text{-cl}(\mu))))$  for every  $\mu \in \tau_Y$ .
- (5)  $F^-(\mu)$  is  $\beta^*g$ -open in  $X$  for every fuzzy regular open set  $\mu$  in  $Y$ .
- (6)  $F^-(\beta\text{-int}(f\text{-cl}(\mu)))$  is  $\beta^*g$ -open in  $X$  for every  $\mu \in \tau_Y$ .
- (7)  $\beta^*g\text{-cl}(F^+(\beta\text{-cl}(f\text{-int}(\beta)))) \subset F^+(\beta)$  for every fuzzy closed set  $\beta$  in  $Y$ .

(8)  $F^+(\beta)$  is  $\beta^*g$ -closed in  $X$  for every  $\beta$  fuzzy regular closed set in  $Y$ .

#### Proof

(1) $\Rightarrow$ (2) Let  $\mu \in \tau_Y$  and  $x \in F^-(\mu)$ . By (1), there exists a  $\beta^*g$ -open neighborhood  $U$  of  $x$  such that  $x \in U \subset F^-(\beta\text{-int}(f\text{-cl}(\mu)))$ . Then,  $x \in \beta^*g\text{-int}(F^-(\beta\text{-int}(f\text{-cl}(\mu))))$ . Hence,  $F^-(\mu) \subset \beta^*g\text{-int}(F^-(\beta\text{-int}(f\text{-cl}(\mu))))$ .

(2) $\Rightarrow$ (3) Let  $\mu$  be fuzzy regular open set, that is,  $\mu = \beta\text{-int}(f\text{-cl}(\mu))$ . By (2),  $F^-(\mu) \subset \beta^*g\text{-int}(F^-(\beta\text{-int}(f\text{-cl}(\mu)))) \subset \beta^*g\text{-int}(F^-(\mu))$  for  $\mu \in \tau_Y$ . Thus,  $F^-(\mu) = \beta^*g\text{-int}(F^-(\mu))$ . Then,  $F^-(\mu)$  is  $\beta^*g$ -open in  $X$ .

(3) $\Rightarrow$ (4) Let  $\mu$  be fuzzy open set in  $Y$ . Then,  $\beta\text{-int}(f\text{-cl}(\mu))$  is fuzzy regular open in  $Y$ . By (3),  $F^-(\beta\text{-int}(f\text{-cl}(\mu)))$  is  $\beta^*g$ -open set in  $X$ .

(4) $\Rightarrow$ (1) Let  $\mu$  be fuzzy open set in  $Y$  such that  $x \in F^-(\mu)$ . By (4), if we take  $F^-(\beta\text{-int}(f\text{-cl}(\mu))) = U$ ,  $U$  is  $\beta^*g$ -open in  $X$ . Since  $\mu \leq \beta\text{-int}(f\text{-cl}(\mu))$  and  $x \in F^-(\mu)$ ,  $x \in F^-(\beta\text{-int}(f\text{-cl}(\mu)))$ . If we take  $x \in F^-(\beta\text{-int}(f\text{-cl}(\mu)))$ , then  $x \in F^-(\mu)$ . Hence,  $F$  is fuzzy lower almost  $\beta^*g$ -continuous.

(2) $\Rightarrow$ (5) Let  $\beta$  be a fuzzy closed set in  $Y$ . By (2),  $F^-(1 - \beta) \subset \beta^*g\text{-int}(F^-(\beta\text{-int}(f\text{-cl}(1 - \beta)))) = \beta^*g\text{-int}(F^-(\beta\text{-int}(1 - \beta\text{-int}(\beta)))) = \beta^*g\text{-int}(F^-(1 - \beta\text{-cl}(f\text{-int}(\beta))))$ .

By using  $F^-(1 - \beta) = X - F^+(\beta)$  equation,  $X - F^+(\beta) \subset \beta^*g\text{-int}(F^-(1 - (\beta\text{-cl}(f\text{-int}(\beta)))) = \beta^*g\text{-int}(X - F^+(\beta\text{-cl}(f\text{-int}(\beta))))$ . Thus,  $X - F^+(\beta) \subset X - (\beta^*g\text{-cl}(F^+(\beta\text{-cl}(f\text{-int}(\beta))))$  is obtained. Consequently,  $\beta^*g\text{-cl}(F^+(\beta\text{-cl}(f\text{-int}(\beta)))) \subset F^+(\beta)$  is found.

(5) $\Rightarrow$ (6) Let  $\beta$  be a fuzzy regular closed set in  $Y$ . Then,  $\beta$  is a fuzzy closed set in  $Y$ . By (5),  $\beta^*g\text{-cl}(F^+(\beta\text{-cl}(f\text{-int}(\beta)))) \subset F^+(\beta)$  and since  $\beta$  is fuzzy regular closed,  $\beta \leq \beta\text{-cl}(f\text{-int}(\beta))$  is obtained. Hence,  $\beta\text{-cl}(F^+(\beta)) \subset F^+(\beta)$  is found. Thus,  $F^+(\beta)$  is  $\beta^*g$ -closed in  $X$ .

(6) $\Rightarrow$ (3) Let  $\mu$  be a fuzzy regular open set in  $Y$ . Then,  $1 - \mu$  is fuzzy regular closed in  $Y$ . By (6),  $F^+(1 - \mu)$  is  $\beta^*g$ -closed in  $X$ . From the equation of  $F^+(1 - \mu) = X - F^-(\mu)$ ,  $F^-(\mu)$  is  $\beta^*g$ -open in  $X$ .

### 3.2.3. Theorem

The following are equivalent for a fuzzy multifunction  $F: (X, \tau) \rightarrow (Y, \tau_Y)$ :

- (1)  $F$  is fuzzy upper almost  $\beta^*g$ -continuous.
- (2)  $F^+(\mu) \subset \beta^*g\text{-int}(F^+(\beta\text{-int}(f\text{-cl}(\mu))))$  for every  $\mu \in \tau_Y$ .
- (3)  $F^+(\mu)$  is  $\beta^*g$ -open in  $X$  for every fuzzy regular open set  $\mu$  in  $Y$ .
- (4)  $F^+(\beta\text{-int}(f\text{-cl}(\mu)))$  is  $\beta^*g$ -open in  $X$  for every  $\mu \in \tau_Y$ .
- (5)  $\beta^*g\text{-cl}(F^-(\beta\text{-cl}(f\text{-int}(\beta)))) \subset F^-(\beta)$  for every fuzzy closed set  $\beta$  in  $Y$ .

(6)  $F^-(\beta)$  is  $\beta^*g$ -closed in  $X$  for every  $\beta$  fuzzy regular closed set in  $Y$ .

**Proof**

The proof is similar to Theorem 3.2.2

3.2.4. Theorem

Let  $F: (X, \tau) \rightarrow (Y, \tau_Y)$  be a fuzzy multifunction.  $F$  is fuzzy lower almost  $\beta^*g$ -continuous if and only if for every  $\mu$  fuzzy semi open set in  $Y$   $\beta^*g-cl((F^+(\mu)) \subset F^+(f-cl(\mu)))$  relation is exist.

**Proof**

$\Rightarrow$ : Let  $F$  be fuzzy loweralmost  $\beta^*g$ -continuous and  $\mu$  be a fuzzy semi open set in  $Y$ . Then,  $\mu \leq f-cl(f-int(\mu))$  and if we take  $\beta = f-cl(f-int(\mu))$ ,  $\beta$  is fuzzy regular closed in  $Y$ . By Theorem 3.2.2 (6),  $F^+(\beta)$  is  $\beta^*g$ -closed in  $X$ . Thus

$$\beta^*g-cl((F^+(\mu)) \subset \beta^*g-cl((F^+(\beta)) = F^+(\beta))$$

$F^+(\beta) \subset F^+(f-cl(\mu))$  ise  $\beta^*g-cl((F^+(\mu)) \subset F^+(f-cl(\mu))$  is obtained.

$\Leftarrow$ : Every fuzzy regular closed set is fuzzy semi open. Let  $\beta$  be fuzzy regular closed in  $Y$ . Then,  $\beta^*g-cl((F^+(\mu)) = F^+(\beta))$  is obtained and  $F^+(\beta)$  is  $\beta^*g$ -closed in  $X$ . By Theorem 3.2.2,  $F$  is fuzzy loweralmost  $\beta^*g$ -continuous.

3.2.5. Theorem

Let  $F: (X, \tau) \rightarrow (Y, \tau_Y)$  be a fuzzy multifunction.  $F$  is fuzzy upper almost  $\beta^*g$ -continuous if and only if for every  $\mu$  fuzzy semi open set in  $Y$   $\beta^*g-cl((F^-(\mu)) \subset F^-(f-cl(\mu)))$  relation is exist.

**Proof**

$\Rightarrow$ : Let  $F$  be fuzzy upper almost  $\beta^*g$ -continuous and  $\mu$  be a fuzzy semi open set in  $Y$ . Then,  $\mu \leq f-cl(f-int(\mu))$  and if we take  $\beta = f-cl(f-int(\mu))$ ,  $\beta$  is fuzzy regular closed in  $Y$ . By Theorem 3.2.3 (6),  $F^-(\beta)$  is  $\beta^*g$ -closed in  $X$ . Thus

$$\beta^*g-cl((F^-(\mu)) \subset \beta^*g-cl((F^-(\beta)) = F^-(\beta))$$

$F^-(\beta) \subset F^-(f-cl(\mu))$  ise  $\beta^*g-cl((F^-(\mu)) \subset F^-(f-cl(\mu))$  is obtained.

$\Leftarrow$ : Every fuzzy regular closed set is fuzzy semi open. Let  $\beta$  be fuzzy regular closed in  $Y$ . Then,  $\beta^*g-cl((F^-(\mu)) = F^-(\beta))$  is obtained and  $F^-(\beta)$  is  $\beta^*g$ -closed in  $X$ . By Theorem 3.2.2,  $F$  is fuzzy lower almost  $\beta^*g$ -continuous.

3.3. Fuzzy Weakly  $\beta^*g$ -Continuous Multifunctions

3.3.1. Definition

Let  $F: (X, \tau) \rightarrow (Y, \tau_Y)$  be a fuzzy multifunction.

a) If there exist a  $\beta^*g$ -open neighborhood of  $xU$  such that  $U \subset F^-(f-cl(\mu))$  for every  $\mu \in \tau_Y$  such that  $x \in F^-(\mu)$ ,  $F$  is defined to be fuzzy lower weakly  $\beta^*g$ -continuous at  $x$ .

b) If there exist a  $\beta^*g$ -open neighborhood of  $xU$  such

that  $U \subset F^+(f-cl(\mu))$  for every  $\mu \in \tau_Y$  such that  $x \in F^+(\mu)$ ,  $F$  is defined to be fuzzy lower weakly  $\beta^*g$ -continuous at  $x$ .

c) If  $F$  is fuzzy lower (upper) weakly  $\beta^*g$ -continuous multifunction at every  $x \in X$ , then  $F$  is fuzzy lower(upper) weakly  $\beta^*g$ -continuous in  $X$ .

3.3.2. Theorem

$F: (X, \tau) \rightarrow (Y, \tau_Y)$  is fuzzy lower weakly  $\beta^*g$ -continuous multifunction if and only if  $F^-(\mu) \subset \beta^*g-int(F^-(f-cl(\mu)))$  for every  $\mu$  fuzzy open set in  $Y$ .

**Proof**

$\Rightarrow$ : Let  $\mu \in \tau_Y$  such that  $x \in F^-(\mu)$ . From hypothesis, there exist  $\beta^*g$ -open set  $U$  such that  $U \subset F^-(f-cl(\mu))$ . Since  $U$  is  $\beta^*g$ -open in  $X$  such that  $U \subset \beta^*g-int(F^-(f-cl(\mu)))$  and  $x \in \beta^*g-int(F^-(f-cl(\mu)))$ . Hence,  $F^-(\mu) \subset \beta^*g-int(F^-(f-cl(\mu)))$ .

$\Leftarrow$ : Let  $F^-(\mu) \subset \beta^*g-int(F^-(f-cl(\mu)))$  for every fuzzy set  $\mu$  in  $Y$ . Let we take  $\beta^*g-int(F^-(f-cl(\mu))) = U$  and fuzzy open set  $\mu$  in  $Y$  such that  $x \in F^-(\mu)$ .  $U$  is  $\beta^*g$ -open neighborhood of  $x$  and  $x \in U \subset F^-(f-cl(\mu))$ . Hence  $F$  is fuzzy lowerweakly  $\beta^*g$ -continuous.

3.3.3. Theorem

$F: (X, \tau) \rightarrow (Y, \tau_Y)$  is fuzzy upper weakly  $\beta^*g$ -continuous multifunction if and only if  $F^+(\mu) \subset \beta^*g-int(F^+(f-cl(\mu)))$  for every  $\mu$  fuzzy open set in  $Y$ .

**Proof**

$\Rightarrow$ : Let  $\mu \in \tau_Y$  such that  $x \in F^+(\mu)$ . From hypothesis, there exist  $\beta^*g$ -open set  $U$  such that  $U \subset F^+(f-cl(\mu))$ . Since  $U$  is  $\beta^*g$ -open in  $X$  such that  $U \subset \beta^*g-int(F^+(f-cl(\mu)))$  and  $x \in \beta^*g-int(F^+(f-cl(\mu)))$ . Hence,  $F^+(\mu) \subset \beta^*g-int(F^+(f-cl(\mu)))$ .

$\Leftarrow$ : Let  $F^+(\mu) \subset \beta^*g-int(F^+(f-cl(\mu)))$  for every fuzzy set  $\mu$  in  $Y$ . Let we take  $\beta^*g-int(F^+(f-cl(\mu))) = U$  and fuzzy open set  $\mu$  in  $Y$  such that  $x \in F^+(\mu)$ .  $U$  is  $\beta^*g$ -open neighborhood of  $x$  and  $x \in U \subset F^+(f-cl(\mu))$ . Hence  $F$  is fuzzy lower weakly  $\beta^*g$ -continuous.

3.4. The Connections between Fuzzy Semi  $\beta^*g$ -Continuous, Fuzzy Almost  $\beta^*g$ -Continuous and Fuzzy Weakly  $\beta^*g$ -Continuous Multifunctions

3.4.1. Theorem

Let  $F: (X, \tau) \rightarrow (Y, \tau_Y)$  be a fuzzy multifunction. If  $F$  is fuzzy lower (upper) semi  $\beta^*g$ -continuous, then  $F$  is fuzzy lower (upper) almost  $\beta^*g$ -continuous.

**Proof**

This proof is for fuzzy upper continuous multifunctions. Let  $\mu \in \tau_Y$  such that  $x \in F^+(\mu)$ . Since  $F$  is fuzzy lower (upper) semi  $\beta^*g$ -continuous, there is an open set in  $X$  such that  $U \subset F^+(\mu)$ . The relation  $\mu \leq f-cl(\mu)$  is exist and  $f-int(\mu) \leq f-int(f-cl(\mu))$ . Because  $\mu$  is fuzzy open set,  $\mu \leq f-int(f-cl(\mu))$ . Hence  $F^+(\mu) \leq F^+(f-int(f-cl(\mu)))$ . Then  $\beta^*g-int(F^+(\mu)) \leq \beta^*g-int(F^+(f-int(f-cl(\mu))))$ .

$\text{int}(F^+(f - \text{int}(f - \text{cl}(\mu)))$ ). We can reach the relation  $F^+(\mu) \leq \beta^*g - \text{int}(F^+(f - \text{int}(f - \text{cl}(\mu)))$ . Then  $F$  is fuzzy lower (upper) almost  $\beta^*g$ -continuous.

The proof is similar for fuzzy lower continuous multifunctions.

#### 3.4.2. Theorem

Let  $F: (X, \tau) \rightarrow (Y, \tau_Y)$  lower (upper) almost  $\beta^*g$ -continuous,  $F$  is fuzzy lower (upper) weakly  $\beta^*g$ -continuous.

#### Proof

This proof is for fuzzy upper continuous multifunctions. Let  $F^+(\mu) \subset \beta^*g - \text{int}(F^+(f - \text{int}(f - \text{cl}(\mu))))$  for every  $\mu \in \tau_Y$ . Since the relation  $f - \text{int}(f - \text{cl}(\mu)) \leq f - \text{cl}(\mu)$  is exist and  $F$  upper almost  $\beta^*g$ -continuous  $\beta^*g - \text{int}(F^+(f - \text{int}(f - \text{cl}(\mu)))) \subset \beta^*g - \text{int}(F^+(f - \text{cl}(\mu)))$ . Then,  $F$  is fuzzy upper weakly  $\beta^*g$ -continuous.

The proof is similar for fuzzy lower continuous multifunctions.

#### 3.4.4. Theorem

Let  $F: (X, \tau) \rightarrow (Y, \tau_Y)$  be a fuzzy multifunction. If  $Y$  is fuzzy regular topological space and  $F$  is fuzzy lower weakly  $\beta^*g$ -continuous,  $F$  is fuzzy lower semi  $\beta^*g$ -continuous.

#### Proof

Let  $\mu$  be a fuzzy open set in  $Y$  such that  $F(x) q \mu$ . There exist  $y \in Y$  such that  $(F(x))(y) \neq 0$ . Fuzzy point  $y_\lambda$  and fuzzy set  $\mu$  are quasi-coincident for  $\lambda = (F(x))(y)$ . That is,  $y_\lambda q \mu$ . Since  $Y$  is fuzzy regular topological space, there exist a  $\beta$  fuzzy open set in  $Y$  such that  $y_\lambda q \beta \leq f - \text{cl}(\beta) \leq \mu$ . Because of fuzzy lower weakly continuity of  $F$ , there exist  $\beta^*g$ -open neighborhood  $U$  of  $x$  such that

$F(z) q f - \text{cl}(\beta)$  for each  $z \in U$ . Hence,  $F(z) q \mu$  for each  $z \in U$ . Thus,  $U \subset F^-(\mu)$  is obtained. Therefore,  $F$  is fuzzy lower semi  $\beta^*g$ -continuous.

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