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Stellar structure and rates of stellar precession

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ABSTRACT

We investigate the detectability and behaviour of spin–axis precession, applying classical formulae to twelve close binary systems (α CrB, AR Cas, FT Ori, QX Car, AI Hya, AG Per, V526 Sgr, V346 Cen, YZ Cas, MY Cyg, β Aur, RZ Cha), for which recent data have been published (Berkowitz, 2024). Using EZ-Web ZAMS models together with an extended version of the RADAU code, we computed internal structure parameters k_2, gyration radii β, and mechanical ellipticities ε_m from which corresponding theoretical precession ratios (P/U) can be derived. Comparison with long-term photometric inclination changes $|\Delta i|$ reveals a clear physical trend: systems with high precessional rates and relatively large masses (e.g., QX Car, AR Cas, AI Hya) exhibit the largest inclination variations, whereas older and more evolved systems (such as FT Ori and α CrB) show significantly weaker signatures. In all cases, the observed $|\Delta i|$ values lie systematically below the corresponding ZAMS predictions, consistent with increasing central concentration, reduced mechanical ellipticity, and progressive tidal damping as the stars evolve. Overall, the results show that spin–axis precession is not only detectable in young massive binaries, but its amplitude directly traces their internal structure, underscoring precession's potential as a powerful diagnostic for probing stellar interiors and guiding future evolutionary modelling.

Introduction

Precession and nutation

Spin–axis precession in close binary stars arises from torques generated by rotational and tidally induced quadrupole and higher order distortions. Classical tidal theory, developed by and formalized extensively by Kopal (1959), shows that these distortions depend sensitively on the internal density distribution through structural constants such as the apsidal-motion coefficient k_2. When the stellar spin axis is misaligned with the orbital angular-momentum vector, the resulting torque causes the spin axis to precess. The physical origin was clarified by Alexander (1973, 1976), who demonstrated that rotational–tidal coupling naturally produces precession on timescales governed by the star's internal mass distribution and tidal synchronization processes. The additional wobbling of the spin axis in the direction perpendicular to its precessional motion, taken as a generalization of the precession, is the nutation. We concentrate on regular precession in this article.

Apart from Kopal (1959) monograph (see also the update in *Kopal's Dynamics of Close Binary Systems, Springer, 1978*), significant background to precession in astrophysical contexts can be found in Brouwer (1946), Goldstein (1950), Savonije & Witte (2002), and Fitzpatrick (2012). Precession effects have been confirmed in a few well-sttudied cases, or binaries with more evolutionary advance, as presented by Hambleton et al. (2018).

Recent developments

Recent observational work has revived interest in this phenomenon. Berkowitz (2024) reported extremely small but measurable long-term variations in apparent orbital inclination ($|\Delta i|$) in several detached binaries, which he claimed were consistent with expectations from precession theory. The present paper looks into this point at an introductory level so that the need for further data collection and analysis can be checked, as well as clarifying the type of quantitative results that can enhance theoretical understanding.

The relevant theoretical ratio of the orbital period (P) to the precession period (U) was given by Budding & Demircan (2022) for the primary star, as:

$$|P/U| = 3/2\, q/(1+q)(n/\omega_z)\varepsilon_m \cos\theta \quad (1)$$

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where q is the secondary-to-primary mass ratio, n and ω_z are the orbital and spin angular velocities, and θ is the spin–orbit obliquity. The quantity ε_m represents the mechanical ellipticity of the primary star and can be approximated (Kopal, 1959) as:

εm=2κk2 q r13/3B (2)

where k_2 is the quadrupole structural constant, $\beta = I/(MR^2)$ is the gyration factor, in which I denotes the principal moment of inertia about the spin axis of the undistorted star and M and R represent its mass and mean radius, respectively. The quantity $r_1 = R_1 / a$ is the fractional radius of the primary component in terms of the mean separation of the two mass centres a. The parameter $\kappa = (\omega_z/n)^2$ measures the degree of departure from spin–orbit synchronism, since tidal interactions generally drive ω_z toward n over Myr timescales (Hut, 1981; Zahn, 1977).

Modern stellar models allow accurate computation of the internal structure quantities required for evaluating Eqns. (1)–(2). The RADAU program, used for calculating k_2 and higher-order structural constants for Eggleton-type stellar models (Inlek and Budding, 2023), can also be easily modified to derive corresponding values of the gyration factor β. Using these (Paxton, 2004) internal structure outputs, we compute precession time-scales for a range of stellar models and compare the theoretical expectations with several observationally interesting binary systems.

The computation of structural constants such as k_2 is based on the theoretical framework in which Radau's differential equation (Eqn (3), below) links the internal density distribution of a star to the alternative structural parameter η_j. Once η_j is obtained from the numerical solution of Radau's equation, the corresponding distortion coefficient Δ_j and apsidal motion constant k_j follows directly. These quantities provide a consistent description of the star's tidal and rotational response and form the essential inputs for precession calculations. Radau's equation derives from the theory of figures of equilibrium of rotating and gravitating fluid masses, pioneered by the 18th-century mathematician. Radau's equation rearranges the equilibrium conditions expressed by Clairaut into a more convenient form for deriving relevant parameters, such as the moment of inertia

Radau's differential equation for the stellar density distribution is (Kopal, 1959):

$$a d\eta/da + 6\rho(\eta+1)/\overline{\rho} + \eta(\eta-1) - j(j+1) = 0 \quad (3)$$

Here, a is the mean radial coordinate, varying from 0 at the stellar centre to a_0 at the surface. The density at radius a is ρ, while $\overline{\rho}$ denotes the mean internal density interior to that radius. In this form, Radau's equation directly connects the star's internal density profile to the resulting structure constants. For numerical stellar models, Radau's equation is rewritten in discrete form as:

$$\Delta\eta_{j,i} = (\Delta a/a)_i f(\rho, \overline{\rho}, a, \eta_j)_{i-1} \quad (4a)$$

$$\eta_{j,i} = \eta_{j,i-1} + \Delta\eta_{j,i} \quad (4b)$$

where the index $i = 1 \ldots N$ labels the radial mesh points from the core to the surface (typically $N \sim 100$ to 200). The function f arises from rearranging Eqn (3) so that all the terms, apart from the first, are shifted to the right. The starting value $\eta_{j,0}$ is evaluated at the innermost mesh point, where $\overline{\rho} = \rho_0$. In most astrophysical applications, including the apsidal motion and precession calculations considered here, the quadrupole term $j = 2$ is of primary interest, as it dominates the tidal effect.

Once η_j is obtained, the distortion parameter is computed from Clairaut's relation:

$$\Delta_j = (2j+1)/(j+\eta_j) \quad (5)$$

where η_j is the structure-dependent variable that encodes the star's internal density distribution. The well-known apsidal motion constants are then obtained from the relation (5) through

$$k_j = \frac{\Delta_j - 1}{2} \quad (6)$$

From these expressions, it is evident that Δ_j exceeds unity for real stars, but reduce to exactly 1 in the point-mass (Roche) approximation.

Although precession can in principle occur in many binaries, its detectability must depend strongly on evolutionary stage. As stars age, increasing central condensation reduces both k_2 and ε_m, leading to weaker torques and therefore smaller observable effects such as $|\Delta i|$. Precession should therefore be more conspicuous in relatively massive systems, where rotational and tidal distortions remain significant. For this reason, the present study concentrates on five young binaries, selected because their ages (log age in yr ≈ 6–8) and structural characteristics make them the most promising candidates for measurable precessional signatures.

Results

The internal structure parameters k_2, β, and ε_m for a range of Zero-Age Main Sequence (ZAMS) stellar masses were computed using EZ-Web (MadStar), the modern web-based implementation of Eggleton's STARS stellar evolution code. EZ-Web provides ZAMS stellar models with radial profiles of density, pressure, temperature, radius, and enclosed mass m(r). These profiles were extracted directly for each stellar mass and supplied as input to the modified RADAU routine. EZ-Web is particularly well suited for this analysis because its internal physics—OPAL opacities, standard equation of state, and mixing-length convection treatment—is fully compatible with the structural-constant calculations required by the Radau formalism. The radial grids and internal density distributions generated by EZ-Web are ideal for evaluating the tidal and rotational deformation parameters that govern spin–axis precession.

The RADAU program (Inlek and Budding 2023), originally designed to compute the apsidal-motion constant k_2, was extended in this study to also compute the gyration radius factor β and the mechanical ellipticity ε_m. All three parameters were evaluated consistently from the same EZ-Web radial mesh. Thus, for each stellar mass, the RADAU integration yields a self-consistent set of structural constants that can be inserted directly into Eq. (1) to calculate the theoretical precession ratio P/U.

For consistent comparison across all ZAMS models, the spin–orbit obliquity was set to $\theta = 30^\circ$, and the rotation/orbital frequency ratio $(\omega_z/n)$was set to unity The resulting values of k_2, β, ε_m, and the theoretical precession rate P/U are given in Table 1.

These results demonstrate the strong dependence of spin–axis precession on stellar internal structure. A notable transition occurs near 1.5 $M_\odot$, marking the change from convective to radiative envelopes. This transition directly affects the apsidal-motion constant k_2, the gyration factor β, and consequently the mechanical ellipticity ε_m. As stellar mass increases, the larger quadrupole distortion leads to higher ε_m values and stronger predicted precession (larger P/U). Lower-mass stars display reduced deformation and correspondingly weaker precession. These ZAMS predictions form the theoretical foundation for our initial comparison with observed inclination variations in real binary systems, as discussed in Section 3.

Table 1
Values of the parameters k2, β, εm, and the resulting ratio P/U for ZAMS models.

ZAMS Mass (Solar units)	k_2	β	ε_m	P/U
1	0.0229	0.0925	0.0101	$1.93.10^{-3}$
2	0.0041	0.0472	0.0035	$6.68.10^{-4}$
5	0.0083	0.0617	0.0055	$1.05.10^{-3}$
10	0.0133	0.0734	0.0074	$1.41.10^{-3}$
20	0.0183	0.0827	0.0090	$1.72.10^{-3}$

Comparison with observations

Spin–axis precession in close binary systems is expected to manifest observationally as slow, long-term variations in the apparent orbital inclination *i*. Such variations arise when the stellar spin axis precesses relative to the orbital plane, producing small but detectable changes in eclipse geometry in multi-epoch light-curve analyses (Kopal, 1959; Berkowitz, 2024). Although the rates involved are extremely small, several well-studied detached binaries exhibit inclination variations that may plausibly be attributed to precessional motion.

Table 2 lists twelve representative young and relatively massive systems for which long-term photometric or spectroscopic monitoring provides measurements of inclination variation, here expressed through $|\Delta i|$ following Berkowitz (2024). The primary masses of these systems were used to interpolate the ZAMS structure parameters k_2, β, and ε_m from the EZ-Web models in Table 1. These structural constants were then inserted into Eq. (1) to compute the theoretical ratio *P/U*. For consistency across the sample, we adopted a single illustrative set of parameters ($\kappa = 1.0$, $q = 0.7$, $r_1 = 0.2$), noting that these values are not system-specific but allow a relative comparison of theoretical precession strengths.

We may note that Berkowitz' measure for the rate of precession specifies Δi as the apparent change of inclination per orbit (in the same units). In terms of more familiar parameters, Δi should be generally of the same order as the period of the orbit, *P*, divided by the time it takes for the spin axis to complete one revolution about the orbital axis, *U*. The value of Δi depends on just where in the precesional cycle, the measurements are taken, while Berkowitz suggests an average was found from a number of separate evaluations. Given the relative scarcity of observational data on precession, Berkowitz' results offer a broad appraisal to check if a deeper level of investigation could be warranted.

A pattern emerges from Table 2 in which the largest observed inclination variations are found in the more massive or structurally distorted systems. In particular, V346 Cen exhibits the largest $|\Delta i|$ value (2.72×10^{-4}), followed by V526 Sgr (2.61×10^{-4}) and QX Car (1.72×10^{-4}), consistent with their relatively high stellar masses and strong quadrupole distortions. Significant variations are also observed in AI Hya (1.52×10^{-4}) and AR Cas (1.22×10^{-4}). In contrast, older and less distorted systems such as FT Ori and especially α CrB display much smaller $|\Delta i|$ values (8.46×10^{-5} and 8.23×10^{-5}, respectively). Although the sample is limited, the ordering broadly reflects the expectation that young systems with stronger quadrupole distortions—typically associated with higher mass or less-evolved internal structures—tend to produce larger precessional inclination changes.

Fig. 1 compares the observed inclination variation $|\Delta i|$ with the predicted precession ratio P/U. Despite considerable scatter, a positive correlation is evident: systems with larger predicted P/U values generally exhibit larger $|\Delta i|$. Most stars fall within an order-of magnitude band around a roughly proportional relation $|\Delta i| \propto (P/U)$. This level of agreement is reasonable given the uncertainties inherent in measuring variations of order 10^{-5}–10^{-4} deg per orbit and the simplifying assumptions in the theoretical modelling (e.g. ZAMS structure, synchronism, assumed obliquity). Linear regression between the observed inclination variation $|\Delta i|$ and the theoretical precession ratio P/U yields a positive correlation with Pearson coefficient $R = 0.63$ ($R^2 = 0.40$). This indicates a moderate but physically meaningful trend, consistent with the expectation that systems with larger theoretical precession rates exhibit larger inclination variations, despite the limited sample size.

Table 2
Data on representative sample of stars showing possible precessional effects.

Name	prim. mass ($M_\odot$)	*P/U*, ZAMS	$\lvert\Delta i\rvert$, obs	log Age(yr)
α CrB	2.57	7.40×10^{-4}	8.23×10^{-5}	8.64
AR Cas	5.9	1.11×10^{-3}	1.22×10^{-4}	7.7
FT Ori	2.2	6.93×10^{-4}	8.46×10^{-5}	8.94
QX Car	9.27	1.36×10^{-3}	1.72×10^{-4}	6.8
AI Hya	2.15	6.87×10^{-4}	1.52×10^{-4}	9.0
AG Per	5.35	1.08×10^{-3}	2.37×10^{-5}	6.6
V526 Sgr	2.27	7.02×10^{-4}	2.61×10^{-4}	8.6
V346 Cen	12.0	1.47×10^{-3}	2.72×10^{-4}	6
YZ Cas	2.31	7.07×10^{-4}	3.91×10^{-5}	8.4
MY Cyg	1.81	9.08×10^{-4}	2.3×10^{-5}	9.1
β Aur	2.38	7.16×10^{-4}	7.90×10^{-6}	8.6
RZ Cha	1.49	1.31×10^{-3}	7.33×10^{-5}	9.3
	$\kappa = 1.0$	$q = 0.7$	$r_1 = 0.2$	

Fig. 2 examines the relationship between *P/U* and stellar age. The data show a clear trend for systems of greater age to exhibit lower theoretical *P/U* values. The correlation shown in Fig. 2 applies to the typical adopted parameters (k, q and r1) mentioned above. The more general situation pertaining to the individual stars would be complicated by multiple competing effects. Eq. (1) contains the geometric factor cos θ, which would increase *P/U* if tidal dissipation reduces the obliquity θ over time. In contrast, structural evolution—particularly increasing central condensation—reduces β and ε_m, thereby decreasing *P/U*. The observed behaviour likely reflects a mixture of these influences, together with uncertainties in system ages and observationally inferred $|\Delta i|$. For these reasons, the apparent decline in Fig. 2 should be regarded as suggestive for real situations, rather than definitive.

Other factors also apply to the present comparison. The observational baselines for most systems remain far shorter than the expected precessional periods, limiting the precision of $|\Delta i|$ measurements. The small amplitudes involved approach the limits of detectability for ground-based photometry. On the theoretical side, the use of ZAMS structural parameters for systems spanning a wide range of ages introduces additional uncertainty, as do the assumptions of solid-body rotation and neglect of magnetic braking or differential rotation.

A linear fit between the theoretical precession ratio P/U and stellar age gives a strong negative correlation ($R = -0.996$, $R^2 = 0.99$) for this five-star sample. Given the small and deliberately restricted sample of young systems, this result should be regarded as indicative rather than definitive, highlighting the combined effects of structural evolution and tidal damping on precessional strength.

Despite these limitations, the overall agreement between predicted *P/U* values and observed $|\Delta i|$ indicates that inclination variations in these systems are broadly consistent with spin–axis precession. The established connection between precession and apsidal-motion constraints (Claret, 2023) further underscores the importance of internal stellar structure in shaping long-term dynamical evolution. Young, relatively massive binaries appear to offer the most favourable conditions for detecting precession, whereas older systems exhibit weaker signatures owing to diminishing mechanical ellipticity and progressive spin–orbit alignment.

Discussion and conclusions

Spin–axis precession in close binaries offers a sensitive diagnostic of stellar internal structure, manifested observationally as long-term, low-amplitude variations in orbital inclination. The systems analysed here overlap with those studied by Berkowitz (2024), who reported $|\Delta i|$ values in the range 10^{-6}–10^{-4} degrees per orbit—consistent with expectations for moderately distorted stellar components. Using EZ-Web project (2026) models together with our extended RADAU routines, we computed structural constants (k_2, β, ε_m) and theoretical precession ratios *P/U*. A broad trend is found among the twelve systems selected from observational findings: Several systems in the sample exhibit relatively large inclination variations, with the most pronounced values found in V346 Cen and V526 Sgr, followed by QX Car, AI Hya, and AR Cas. In contrast, FT Ori and especially α CrB display noticeably smaller $|\Delta i|$ values. This ordering broadly reflects differences in their predicted *P/U* values, which depend on stellar mass and internal structure. This mass–age dependence matches expectations from tidal alignment and internal structural evolution (Hut, 1981).

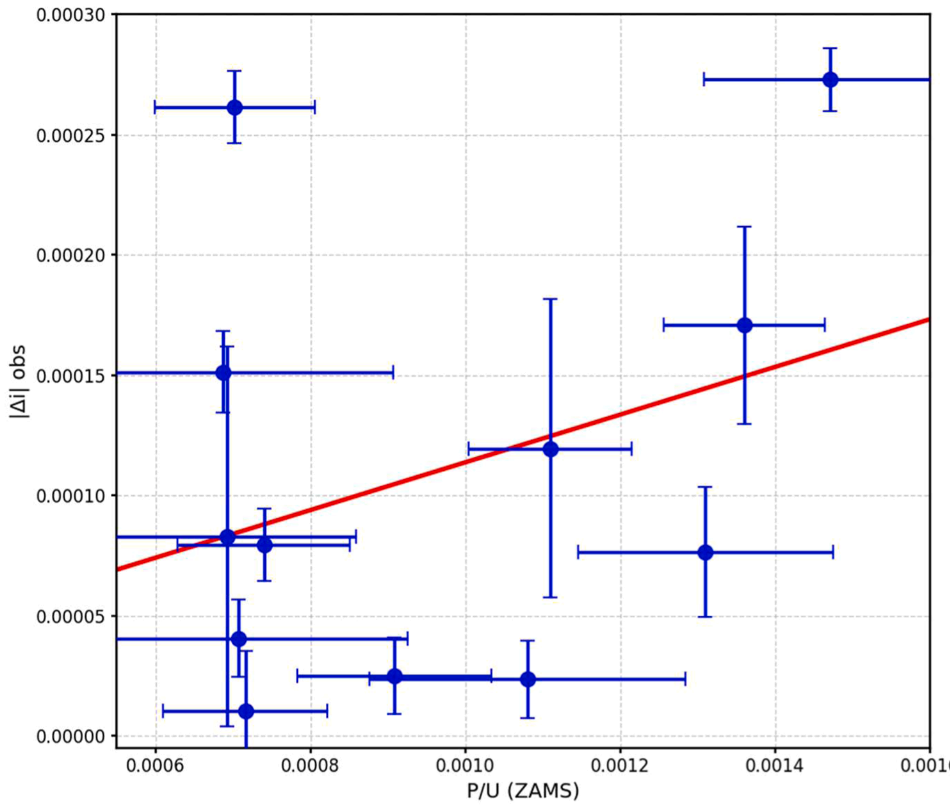

Fig. 1. Observed inclination variation |Δi| versus theoretical precession rate P/U for selected close binary systems from Berkowitz (2024).

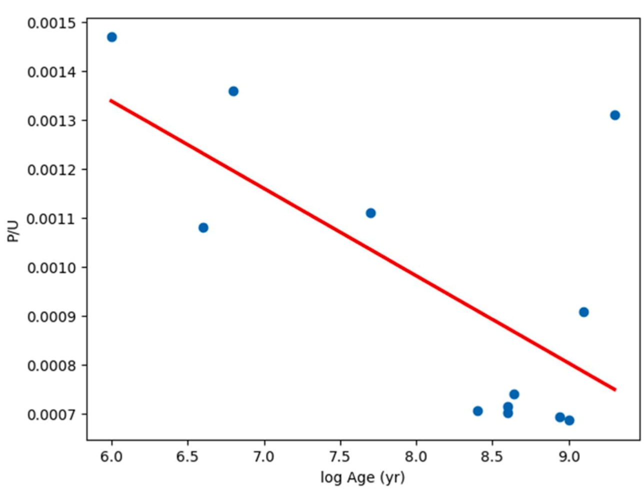

Fig. 2. Trend of precessional motion with age, formed by plotting columns 3 against 5 from Table 2.

Theoretical P/U estimates generally exceed reported $|\Delta i|$ values by factors of 5–10, which is reasonable given that ZAMS models overestimate mechanical ellipticity for evolved stars, most observations cover incomplete precession cycles, and $|\Delta i|$ uncertainties remain significant (Berkowitz, 2024). Fig. 1 shows a generally positive correlation between P/U and the observed $|\Delta i|$ values. In this set of young, relatively massive binaries, systems with stronger theoretical precession (larger P/U) also display larger inclination variations, confirming that the ZAMS-based structural parameters predict the observed precession signature. Fig. 2 indicates that P/U decreases systematically with increasing stellar age. This behaviour reflects two opposing effects: structural evolution reduces β and ε_m, lowering P/U, while tidal dissipation gradually reduces the spin–orbit obliquity θ, which acts in the opposite direction. The combined outcome is a general decline of P/U with age.

The results show that precession should be most prominent in young massive stars, where quadrupole distortion remains strongest. The small $|\Delta i|$ values in older systems suggest gradual suppression of precession due to increasing central condensation and decreasing ε_m, consistent with apsidal motion studies (Claret, 2024) and tidal theory (Fuller et al., 2025). Part of the residual variation in $|\Delta i|$ may be attributed to differential rotation within the stellar envelope (Lurie et al., 2017) or to magnetic-field–regulated tidal dissipation, which alters the internal angular-momentum distribution over time (Wei, 2022).

Overall, the combined observational and theoretical evidence supports precession as a physically plausible mechanism for producing measurable inclination variations in young and moderately massive binaries. In contrast, more evolved systems display significantly weaker signatures of precession, consistent with the expected reduction of the apsidal motion constant and increasing central concentration as stars age. This trend aligns with the findings of Claret et al. (2021), who demonstrated that internal structural evolution leads to a progressive decline in the stellar parameters that govern precessional and apsidal motion behaviour.

In summary, our analysis indicates that (1) observed inclination variations are consistent with precession but are damped relative to ZAMS predictions, (2) precession amplitude decreases with age due to structural evolution rather than geometric alignment alone, and (3) system-specific evolutionary modelling will be essential for refining these conclusions and improving the future use of precession as a diagnostic of stellar interiors.

Looking ahead, the next stage of research will extend this work to evolved stars, where structural changes become more pronounced and precessional behaviour may differ significantly from the ZAMS-based trends identified here. By constructing full-age MESA evolutionary tracks and recalculating k_2, β, and ε_m at each evolutionary phase, we aim to evaluate how precession evolves beyond the Main Sequence. Such studies, when photometric resolution will be significantly higher, will provide a deeper, more comprehensive understanding of precession in binary stars and further clarify its diagnostic value for probing internal stellar structure.

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No data was used for the research described in the article.

CRediT authorship contribution statement

G. Inlek: Writing – original draft, Validation, Software, Methodology, Investigation, Conceptualization. **E. Budding:** Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that she have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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