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A review on swing-up methods in inverted pendulums

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Abstract

Inverted pendulum systems are considered a fundamental criterion in comparing and validating control methods due to their nonlinear, unstable, and underactuated nature. In this study, swing-up methods that raise the inverted pendulum from a hanging position to a vertical position are comprehensively examined in light of the literature. Linear and rotary inverted pendulum systems are presented along with their mathematical models, and inverted pendulum swing-up methods found in the literature are presented along with the studies conducted,, the methods used to keep the inverted pendulum in equilibrium are listed, and inverted pendulum swing-up methods found in the literature are presented along with the studies conducted. The methods used are evaluated starting from their first proposed studies and including their current applications. In this context, the new roles of classical energy-based methods in soft robotics applications are discussed; the Hedge Algebra approach, which optimizes the computational load in fuzzy logic-based systems, is examined. In addition, the success of Deep Reinforcement Learning algorithms, which offer model-independent control, especially in overcoming "Sim-to-Real" challenges, is discussed through current publications. This review provides a broad perspective, ranging from traditional control approaches to AI-driven learning systems, offering a current and comprehensive roadmap for researchers.

Keywords: *Linear inverted pendulum, rotary inverted pendulum, pendulum swing-up, classical control, reinforcement learning*

Ters sarkaçta yükseltme yöntemleri incelemesi

Öz

Ters sarkaç sistemleri, doğrusal olmayan, kararsız ve eksik tahrikli doğası nedeniyle kontrol yöntemlerinin kıyaslanması ve doğrulanmasında temel bir kıstas olarak kabul

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edilmektedir. Bu çalışmada, ters sarkacın aşağı sarkık konumdan dikey konuma getirilmesini sağlayan yükseltme (swing-up) yöntemleri literatür ışığında kapsamlı bir şekilde incelenmiştir. Lineer ve dönen ters sarkaç sistemleri matematik modelleriyle birlikte sunulmuş, literatürde yer alan ters sarkaç yükseltme yöntemleri yapılan çalışmalarla birlikte verilmiş, ters sarkacın dengede tutulması için kullanılan yöntemler listelenmiş ve literatürde yer alan ters sarkaç yükseltme yöntemleri yapılan çalışmalarla birlikte verilmiştir. Kullanılan yöntemler ilk ortaya konduğu çalışmalardan başlayıp güncel uygulamalarıyla da değerlendirilmiştir. Bu bağlamda, klasik enerji tabanlı yöntemlerin yumuşak robotik (soft robotics) uygulamalarındaki yeni rolleri ele alınmış; bulanık mantık tabanlı sistemlerde hesaplama yükünü optimize eden Hedge Cebiri (Hedge Algebras) yaklaşımı incelenmiştir. Ayrıca, modelden bağımsız kontrol imkânı sunan Derin Pekiştirmeli Öğrenme (Deep Reinforcement Learning) algoritmalarının, özellikle "Simülasyondan Gerçeğe Transfer" (Sim-to-Real) zorluklarını aşmadaki başarısı güncel yayınlar üzerinden tartışılmıştır. Bu derleme çalışması, geleneksel kontrol yaklaşımlarından yapay zekâ odaklı öğrenen sistemlere kadar geniş bir perspektif sunarak, araştırmacılar için güncel ve bütünsel bir yol haritası niteliği taşımaktadır.

Anahtar kelimeler: *Doğrusal ters sarkaç, döner ters sarkaç, sarkaç yükseltme, klasik kontrol, pekiştirmeli öğrenme*

1. Introduction

The inverted pendulum system is one of the intuitive classical control problems. Due to its nonlinear [1], unstable [2] and underactuated [3] nature, the inverted pendulum mechanism is often used as a benchmark and test system for control algorithms. With a wide range of applications from rocket launch pads [4] to the balance mechanisms of humanoid robots, these systems offer an ideal platform for testing the performance, robustness and real-time applicability of control algorithms. An inverted pendulum system consists of a rigid rod and a car to which the rod is attached and on which it can rotate freely [5]. Besides the linear inverted pendulum system [6], various topological configurations of inverted pendulums are prevalent in the literature. The main examples of these are the rotary inverted pendulum [7], the double linear inverted pendulum [8], the double rotary inverted pendulum [9], the triple linear inverted pendulum [10], the pendubot [11] and soft inverted pendulum [12].

The two of the most studied topics in inverted pendulum systems are: firstly, maintaining the balance of the pendulum so that it remains in an upward direction despite potential disturbances - such as external mechanical impulses, air currents, or sensor noise - and the system's inherent instability, starting from non-zero initial conditions; and secondly, raising the pendulum from its downward vertical equilibrium position to a range where this control can be performed. Traditional control approaches generally rely on linearized mathematical models of the system around its upright equilibrium point. While methods like Proportional-Integral-Derivative (PID) and Linear Quadratic Regulator (LQR) successfully perform the balancing task when the pendulum reaches the vertical position (stable equilibrium point), the problem of raising the pendulum from its hanging position (swing-up) requires more complex strategies due to the highly nonlinear nature of the system. In this context, examples of methods used for balance control include PID [13], [14], [15], Sliding Mode [16], [17], LQR [18], [19], Fuzzy Logic [20], [21], Neural Networks [22], Grey Prediction [23], Virtual Model Control [24] and ANFIS (Adaptive

Neuro Fuzzy Inference System) [25], [26]. Energy-based [27], [28], [29], [30], Fuzzy Logic [1], [31], [32], [33] and Sliding Mode [34], [35] control methods are among the main ones used to perform swing-up control. However, many of these methods require a precise system model or may create limitations in terms of computational overhead in real-time applications. In particular, model uncertainties, external disturbances, and differences between the simulation environment and the physical world -such as unmodeled stiction in the joints, encoder quantization noise, and actuator dead-zones- are key factors that limit the performance of classical methods.

Recent advances in artificial intelligence and computational intelligence have opened new horizons for overcoming these limitations. This study aims to examine the current literature on the problem of raising and balancing an inverted pendulum from a perspective ranging from classical methods to modern "learning-based" approaches. In this context, one of the main contributions of the study is its perspective on the synthesis of classical and modern methods. In addition to fundamental approaches such as energy-based control, the literature examines how emerging Hedge Algebra (HA)-based controllers reduce computational load and increase robustness compared to classical fuzzy logic methods [36].

Another key contribution is its perspective on the transition to learning systems. The success of algorithms such as Reinforcement Learning (RL), Deep Deterministic Policy Gradient (DDPG), and Proximal Policy Optimization (PPO), which offer model-independent control, in addressing the upgrade problem is examined.

The final main contribution topic addresses the problems of transferring simulation to reality (Sim-to-Real). The problem of transferring the simulation policy to real hardware, which is the biggest challenge of RL-based methods, has been discussed in light of current solutions such as "Two-Stage Learning Protocol" and "Detail Reward Functions" [37], [38].

This review aims not only to provide researchers with a list of existing methods, but also to highlight the evolving direction of control theory towards data-driven and intelligent systems. Furthermore, the main types of inverted pendulums used in the literature (linear and rotary inverted pendulums) will be presented along with their mathematical models, and the most commonly used swing-up methods for these pendulums will be examined and compared.

The remainder of this manuscript is organized as follows: Section 2 introduces the linear and rotary inverted pendulum systems along with their mathematical modelling. Section 3 provides a brief literature review on foundational stabilization and balance control methods. Section 4 delivers a comprehensive and detailed examination of all major swing-up methods found in the literature, ranging from classical energy-based techniques to computational intelligence and deep reinforcement learning approaches. Section 5 presents the results and a thorough discussion of the comparative evaluations, including sim-to-real challenges. Finally, Section 6 outlines future research directions and concludes the paper.

2. Types of mechanical pendulums

2.1. Linear inverted pendulum

The linear inverted pendulum consists of a rod with one degree of freedom that is attached via a shaft to a carriage mounted on a linear slide and can rotate freely around the Y-axis (Figure 1).

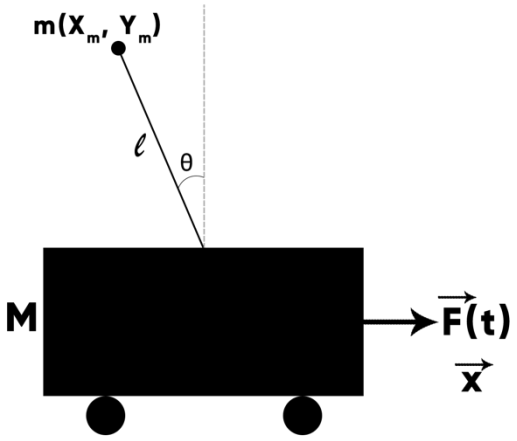


Figure 1: Model of inverted pendulum

One commonly used method for obtaining the mathematical model of an inverted pendulum is the Euler-Lagrangian equations. In this method, the difference between the kinetic and potential energy of the system is found to obtain the Lagrangian term. Then, the derivatives of this term with respect to the velocity variables are taken to obtain the characteristic equations of the system.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \tag{1}$$

Kinematic Equations

$$X_m = X - \ell \sin\theta \tag{2}$$

$$Y_m = \ell \cos\theta \tag{3}$$

$$\dot{X}_m = \dot{X} - \ell\dot{\theta} \cos\theta \tag{4}$$

$$\dot{Y}_m = -\ell\dot{\theta} \sin\theta \tag{5}$$

The position (2) and velocity (4) of the car, and the position (3) and velocity (5) of the pendulum are shown in the equations.

$$V = mgY_m \ = mg\ell\cos\theta \tag{6}$$

The potential energy of the system is shown in equation (6).

$$T = \frac{1}{2}M\dot{X}^2 \ + \frac{1}{2}m(\dot{X}_m^2 \ + \ \dot{Y}_m^2) \tag{7}$$

$$T = \frac{1}{2}M\dot{X}^2 \ + \frac{1}{2}m[\dot{X}^2 \ - 2\dot{X}\ell\theta\cos\dot{\theta} \ + \ \ell^2\dot{\theta}^2(\cos^2\theta + \sin^2\theta)] \tag{8}$$

$$T = \frac{1}{2}(M \ + m)\dot{X}^2 - m\dot{X}\ell\dot{\theta}\cos\theta \ + \frac{1}{2}m\ell^2\dot{\theta}^2 \tag{9}$$

The total kinetic energy of the system is found in equation (9).

$$L \ = T \ - V \tag{10}$$

$$L = \frac{1}{2}(M \ + m)\dot{X}^2 - m\dot{X}\ell\dot{\theta}\cos\theta \ + \frac{1}{2}m\ell^2\dot{\theta}^2 - mg\ell\cos\theta \tag{11}$$

The Lagrange variable is formed as in equation (11). Applying the Euler-Lagrange method in equation (1) yields the characteristic equations of the system shown in equations (12) and (13).

$$(M + m)\ddot{X} - \left(m\ell\ddot{\theta}\cos\theta - m\ell\dot{\theta}^2\sin\theta\right) = F(t) \tag{12}$$

$$\ell^2\ddot{\theta} - \ddot{X}\cos\theta - g\sin\theta = 0 \tag{13}$$

The term F(t) in equation (12) is the force applied by the DC motor moving the cart and can be calculated as in our previous work[23] and in equation (14).

$$F_{motor} = A \cdot V - B \cdot \dot{X} - C \cdot \ddot{X} \tag{14}$$

$$A = \frac{N \cdot \eta \cdot K_t}{R_a \cdot r^2}; B = \frac{N^2 \cdot \eta \cdot K_t \cdot K_b}{R_a \cdot r^2}; C = \frac{N^2 \cdot \eta \cdot J_m}{r^2} \tag{15}$$

In equation (15); r is the radius of the pulley mounted on the motor shaft, N is the gear ratio, η is the efficiency of the DC motor, K_t is the torque constant, K_b s the reverse electromotive force constant, $R_a$is the armature resistance of the motor, $J_m$is the moment

of inertia of the motor, X is the position of the cart and V is the voltage applied to the DC motor.

2.2. Rotary inverted pendulum

In linear inverted pendulums, which have been in use since the 1960s, the limited range in which the device can move restricts the control system. To overcome this limitation, Furuta et al. created an inverted pendulum with a new design [7]. It has a structure consisting of a direct-drive motor and a pendulum attached to the shaft of this motor, and requires less space than a traditional inverted pendulum.

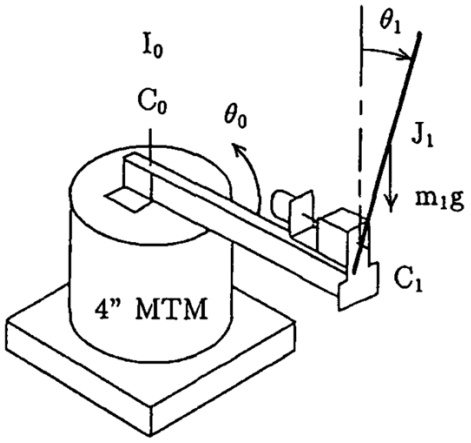


Figure 2: Model of rotary pendulum [7]

We can use the Euler-Lagrangian equation to find the mathematical model of the pendulum [39]. For the arm to which the motor is attached,

$$T_0 = \frac{1}{2} I_0 \dot{\theta}_0^2 \; ; \; V_0 = 0 \tag{16}$$

For the pendulum,

$$T_1 = \frac{1}{2} I_1 \dot{\theta}_1^2 + \frac{1}{2} m_1 \left[\left\{ \frac{d}{dt} (L_0 \sin\theta_0 + \ell_1 \sin\theta_1 \cos\theta_0) \right\}^2 + \left\{ \frac{d}{dt} (L_0 \cos\theta_0 - \ell_1 \sin\theta_1 \cos\theta_0) \right\}^2 + \left\{ \frac{d}{dt} (\ell_1 \cos\theta_1) \right\}^2 \right] \tag{17}$$

$$V_1 = m_1 g \ell_1 \cos\theta_1 \tag{18}$$

Using the calculated energies, the following equations are obtained.

$$\begin{bmatrix} I_0 + m_1(L_0^2 + \ell_1^2 \sin^2\theta_1) & m_1\ell_1 L_0 \cos\theta_1 \\ m_1\ell_1 L_0 \cos\theta_1 & J_1 + m_1\ell_1^2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_0 \\ \ddot{\theta}_1 \end{bmatrix} + \begin{bmatrix} c_0 + 2m_1\ell_1^2 \sin 2\theta_1\dot{\theta}_1 & -m_1\ell_1 L_0 \cos\theta_1 \\ -m_1\ell_1^2 \sin\theta_1\dot{\theta}_0 & J_1 + m_1\ell_1^2 \end{bmatrix} \begin{bmatrix} \dot{\theta}_0 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ -m_1 g\ell_1 \sin\theta_1 \end{bmatrix} = \begin{bmatrix} V \\ 0 \end{bmatrix} \quad (19)$$

The linearized version of the model near the vertical equilibrium position can be seen in equation (20).

$$\begin{bmatrix} I_0 + m_1 L_0^2 & m_1\ell_1 L_0 \\ m_1\ell_1 L_0 & J_1 + m_1\ell_1^2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_0 \\ \ddot{\theta}_1 \end{bmatrix} + \begin{bmatrix} C_0 & 0 \\ 0 & C_1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_0 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -m_1 g\ell_1 \end{bmatrix} \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} V \\ 0 \end{bmatrix} \quad (20)$$

The structural and operational dynamics of the rotary inverted pendulum differ fundamentally from the classical linear pendulum model. While the linear inverted pendulum requires a horizontal slide or rail framework that imposes physical constraints on the cart's maximum displacement, the rotary inverted pendulum operates via a direct-drive motor attached to a rotating arm, eliminating track length limitations completely. This angular workspace allows for continuous rotation and a significantly more compact hardware footprint. However, from a control theory perspective, the coupling between the rotational motion of the driven arm and the unactuated pendulum link introduces more complex Coriolis and centrifugal forces into the system equations, making the mathematical modelling and nonlinear swing-up control of the rotary system structurally more intricate than its linear counterpart.

3. Balance control

Before studies on swing-up control in inverted pendulum systems began, research was being conducted on control methods for balancing the pendulum. Since examining these methods is not included in this study, only studies utilizing specific methods will be briefly listed in this section. In the literature, the Linear Quadratic Regulator stands out as the most frequently used method for balance control. It has been reported that the balance controller was successfully applied using the LQR method in the studies [18], [29], [30], [34], [39], [40], [41], [42], [43], [44], [45], [46]. Aside from LQR, the most commonly encountered method is Fuzzy Logic.

In studies [21], [33], [47], [48], [49], [50] it was reported that the inverted pendulum was stabilized at the vertical equilibrium point using Fuzzy Logic. Sliding Mode Method was also used as a balance controller in studies [8], [16], [35], [51]. For pendulum balance control, PD or PID control methods were used in studies [3], [13], [52], [53] and it was reported that it was successful. Saifizul et al.[25], performed the ANFIS control of the inverted pendulum, Huang and Huang [23] performed the Grey Estimation control, Chang and Lee [54] performed the H_∞ controller, and Bıçakçı [24] performed the Virtual Element Control method.

4. Swing-up methods

4.1.Energy based method

One of the methods frequently used for swinging-up an inverted pendulum is the Energy-Based Method. First introduced by Wiklund et al. in 1993 [27] , this method involves finding the total energy of the system in the vertical equilibrium position and designing a control method that brings the total energy of the system to the energy found. If the ratio of the maximum acceleration at the pivot point (in the case of a linear inverted pendulum, the car) to the acceleration due to gravity is n = 4/3 or greater, it is possible to raise the pendulum with a single swing. Compared to Bang-Bang control, the minimum time method frequently used for swing-up before the introduction of the energy-based method, the energy injected into the system is controlled, allowing the system to be brought to the desired position without exceeding the limit [28]. Equation (21) gives the control equation required to bring the inverted pendulum close to the vertical equilibrium position.

$$u = sat_{ng}\left(k(E - E_0)sign\left(\dot{\theta}\cos\theta\right)\right) \tag{21}$$

Here, *u* is given as the acceleration of the cart, and the product of the pendulum's angular velocity and the cosine of its angle gives the direction of the applied acceleration. This control rule aims to make the derivative of the chosen Lyapunov function ($\frac{dV}{dt} = -mlk\left((E - E_0)\dot{\theta}\cos\theta\right)^2$) negative, while bringing the system's energy (E) to the desired energy value (E_0), i.e., the potential energy of the pendulum at its vertical equilibrium position. The requirement for the time derivative of the Lyapunov function ($\dot{V}$) to be negative or semi-negative is a fundamental condition for proving system stability under Lyapunov's direct method. In physical terms, a negative derivative guarantees that the system's total energy continuously dissipates or accumulates strictly toward the target equilibrium energy state (E_0). As long as $\dot{V} \leq 0$, the control law ensures that the energy error ($E - E_0$) approaches zero asymptotically, preventing unchecked energy growth or chaotic oscillations. This mathematical guarantee confirms that the system is driven systematically from its suspended position to the desired homoclinic orbit near the upright vertical position, where the balance controller can safely take over. Furthermore, in this equation, *n* is the ratio of the cart's maximum acceleration to its gravitational acceleration, and the saturation function (*sat*) aims to prevent chattering by ensuring the control rule behaves like a linear controller for small errors. Åström and Furuta, using this control rule they developed, obtained the simulation result shown in Figure 3 for the case where n ≥ 4/3, i.e., it is possible to raise the pendulum with a single oscillation.

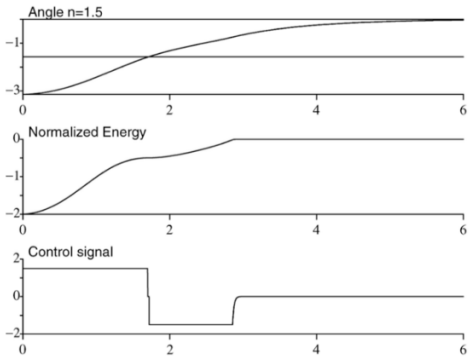


Figure 3: Single oscillation triple change simulation result[28]

In their study, Barbosa et al. [18] designed an energy-based control method using Simulink to swing-up rotary pendulum systems and swung the pendulum up in a single oscillation. They designed a nonlinear model, a dataset, and an algorithm based on the nonlinear least squares method to estimate unmeasurable parameters. Sukontanakarn and Parnichkun [34] designed an energy-based controller based on a PD controller for a rotating inverted pendulum system, and demonstrated in simulation and experimental outputs that they were able to amplify the pendulum.

Since the unlimited path length provided by a rotary pendulum cannot be achieved in a linear inverted pendulum, this limitation prevents the direct application of the energy-based method in linear inverted pendulum systems. Since the control system obtained using the method did not include track (or rail) length as a constraint, in 2002 Chatterjee et al. proposed a method to add this constraint to the energy-based method [55]. In this method, an "energy well" is embedded in the track, preventing the train from going outside the track boundaries. This well is designed to cause the car to experience a pushing force when it reaches its limit. To apply this thrust force, they defined the function $\Phi(x)$ which includes the position of the cart to impose restrictions on the dynamics of the system without changing system energies. The defined function restricts the system's dynamics but does not interfere with mechanical energies. Since the method aims to achieve a situation where the car oscillates with high amplitude within the rail boundaries, its acceleration must be designed to be minimal at the center of the rail and maximal near the endpoints. This acceleration function is shown in the logarithmic part of equation 22, where the center of the rail is defined as 0 and the distance it can travel in either direction is defined as L. Since $sgn(\dot{\theta}\cos\theta)$, located on the right side of the energy equation, must always have a negative factor, they added this equation as an additive to the energy equation, creating the new control equation which can be seen in equation 23. In swing-up mode, when the system reaches sufficient total energy to assume a vertical position, or when the system is close to a vertical position, it is switched to cruise mode, thus ensuring a smooth transition to the control method that will keep the system in the vertical position. A comparison of the swing-up time between the classical energy control method (a) and the method they developed in their publications (b) can be seen in Figure 4. For a system with limited track spacing, it can be seen that the control method using an energy well achieves the swing-up much faster. When the application results given in the study are examined, it is stated that although the simulation and experimental results obtained are similar, the balance point was reached relatively later in the experimental results and results including more noise were obtained. Examination of the resulting graphs shows that it takes approximately 10 seconds for the pendulum to balance.

$$u_{cart\ well} = k_{cw} sgn(x) \log\left(1 - \frac{|x|}{L}\right) \tag{22}$$

$$\ddot{x} = -k_{su} sgn(\dot{\theta}\cos\theta) + k_{cw} sgn(x) \log\left(1 - \frac{|x|}{L}\right) \tag{23}$$

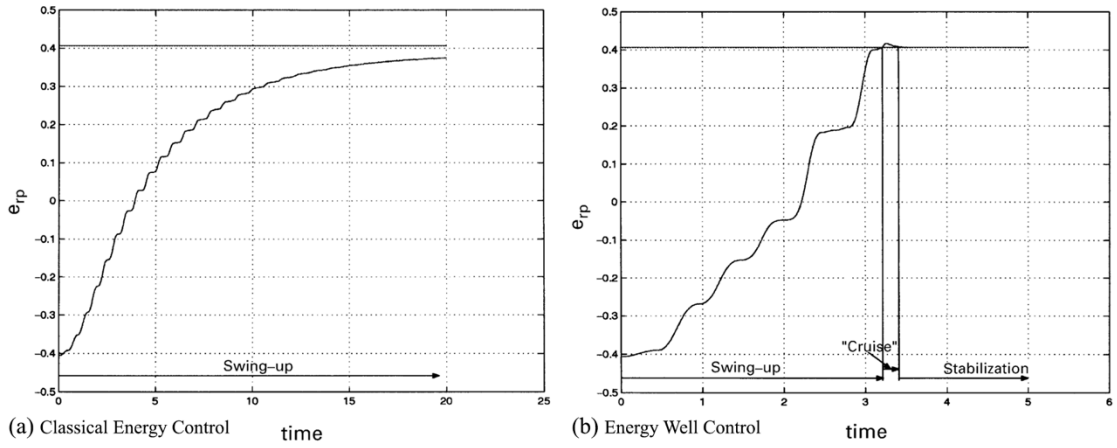


Figure 4: Comparison of classical energy control and energy well design[55]

In their publication [1], Muskinja and Tovornik worked with a linear inverted pendulum and tried to operate the energy-based method by adding an energy well in a system with a limited rail length (0.5 m). Although successful in simulation, the swing-up failed in the real system because they could not measure all of the state variables (cart and angle velocities were calculated from positions using real differentiators). They then injected energy into the system using a simple fuzzy logic controller and successfully swung-up the actual system.

Sakurama et al. used an energy-based method in a linear inverted pendulum and designed a Controlled Lagrangian control method instead of a linear controller to compensate for the error margins that might arise from friction forces ignored by the method when changing the direction of the applied force and selecting the angles for switching from the amplification method to the control method [56]. They successfully applied their control system to a real experimental setup, swinging-up and balancing the pendulum.

In their prominent publication presented in 2005, Xin and Kaneda [57] extended the boundary of energy-based control by theoretically proving and validating the simultaneous swing-up of two independent, non-identical pendulums mounted on a single cart. This system represents a highly complex underactuated configuration with a severe degree of underactuation. The authors constructed a composite Lyapunov function candidate based on the square of individual pendulum energies, defined as $V = \frac{1}{2}E_1^2 + \frac{1}{2}E_2^2$. By analysing the time derivative of this function ($\dot{V} = -Gu$), they demonstrated that under the condition of distinct natural frequencies ($\omega_1 \neq \omega_2$), the energy of each pendulum systematically converges either to zero (the upright vertical position) or to its absolute potential energy minimum. Through a rigorous stability analysis utilizing LaSalle's Invariance Principle and the concept of invariant set stability, they mathematically demonstrated that only the invariant set corresponding to the fully upright equilibrium point ($E_1 = 0, E_2 = 0$) is asymptotically stable. The remaining unstable invariant sets—where one or both pendulums remain hanging downward—cannot be sustained in practice, ensuring that the energy-based control law successfully drives almost all initial states into the desired upright balance zone.

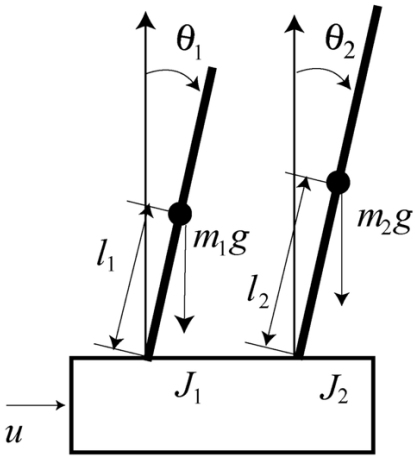


Figure 5: Schematic model of the single-cart dual-pendulum system [57]

The inverted pendulum system, which Spong and Block first introduced in their publication in 1995 [46] and named Pendubot, is also included as an experimental system used in swing-up studies. Similar to Furuta's rotary inverted pendulum, the car is again in the form of an arm and performs rotational motion, but in Pendubot, the axes of the arms are positioned parallel to each other. Fantoni et al. [11] designed a swing-up controller for Pendubot that would bring the pendulum very close to the up equilibrium position or into a homoclinic trajectory that would enter the gravitational field of a balance controller using an energy-based method, and demonstrated their success with simulation results. Albahkali et al., after bringing Pendubot's first link to an upright position, increased the system's energy to its maximum potential energy by making small oscillations from stop to stop, and brought it to a state where balance control could be performed with a linear controller [58]. Using parameters from publications in the literature, they tested control systems in a simulation environment and reported the results. Xin and colleagues re-evaluated Pendubot's energy-based swing-up control studies, expanding the range of control parameters that enable achieving the control objective, and reported the simulation results [30].

Recent advancements in soft robotics have moved beyond classic rigid inverted pendulum models, bringing to the forefront the control of flexible systems. In this context, Wang and Xin [12], proposed an energy-based boost controller for 'Soft Inverted Pendulum' systems, whose curvature is defined by an affine function and which have an underactuated structure. The authors theoretically determined the necessary and sufficient conditions for the linear controllability of the system around the vertical equilibrium point; they derived an energy control strategy that brings the pendulum to a vertical position without any stiffness constraints and is free from singular points. This study fills a significant gap in the literature by demonstrating that energy-based approaches can be effectively applied not only to traditional rigid bodies but also to flexible robotic platforms with complex and flexible dynamics.

4.2. Intelligent and learning-based control methods

Classical control methods (LQR, PID, etc.) rely on linearized models at specific operating points but can experience performance loss in highly nonlinear and uncertain systems such as inverted pendulums. To overcome these limitations, intelligent control methods that can operate independently of the model and are more resilient to uncertainties are gaining prominence in the literature.

4.2.1. Fuzzy logic approaches

A Fuzzy Logic System (FLS) involves the knowledge and experience of an expert in designing a system that controls a process where input-output relationships are defined by a set of fuzzy control rules, such as IF-THEN rules [59]. While traditional Sugeno or Mamdani type fuzzy controllers yield successful results, the complexity of the rule base can increase the computational burden. An FLS consists of three blocks. First, fuzzification, which converts the precise input value into a linguistic variable using membership functions held in the knowledge base. The second block, the inference engine, is tasked with evaluating the degree of membership of the input to fuzzy output sets using fuzzy rules. Finally, the clarifying block converts the fuzzy output into a precise value. The inference engine is central to the FLS and can reproduce the human decision-making process by using approximate reasoning to arrive at a control strategy [60]. The inference phase uses the fuzzy input values to activate the inference rules and generate the fuzzy output value.

In a study conducted in 2001, Yi et al. [49] determined fuzzy logic rules using Single Input Rule Module(SIRM) on a linear pendulum, equated each input (the speed and position of the pendulum and the car) to independent outputs, and then assigned dynamic variables to these outputs to obtain a single output. Here, separate SIRMs are defined for each input as in equation 24.

$$SIRM - i: \{R_i^j : if\ x_i = A_i^j\ then\ f_i = C_i^j\}_{j=1}^{m_i} \quad \textbf{(24)}$$

Here, x_i is the single variable in the antecedent, and the outcome variable f_i is an intermediate value for the actual output f. A_i^j and C_i^j are membership functions of the antecedent variable x_i and the outcome variable f_i. Finally, $i = 1, 2, \ldots, n$ is the index value of the SIRMs, and $j = 1, 2, \ldots, n$ is the index value of the rules in SIRM-i. To explicitly specify the different role of each input value on system performance, a dynamic significance level, called w_i^D is defined independently for each x_i input element as in equation 25.

$$w_i^D = w_i + B_i \Delta w_i^0 \quad \textbf{(25)}$$

Here, the base value w_i ensures that the relevant input value satisfies its required function by passing it through a control process. The dynamic value, defined as the product of the width B_i and Δw_i^0, the inference result of the dynamic variable Δw_i, plays a role in adjusting the effect of the input element on the system performance according to the changes in the control system. Considering these equations, the output element f, obtained by summing the fuzzy inference result of each SIRM and the dynamic significance level for each input element, can be seen in equation 26. The publication states that the magnitudes of the dynamic variables were determined using a random genetic algorithm search.

$$f = \sum_{i=1}^{n} w_i^D f_i^0 \quad \textbf{(26)}$$

The block diagram used to control the system is shown in Figure 5. Here, the Pendulum Angular Transformation Device (PATD) controls the angle of the pendulum. The subsequent Normalizer block normalizes the state variables of the pendulum system with scaling factors to obtain the x_i input elements. The subsequent Normalizer block normalizes the state variables of the pendulum system with scaling factors to obtain the xi input elements. Each i-th dynamic importance degree (DID-*i*) block calculates the dynamic importance value of the x_i input element. The output *f* of the fuzzy controller is obtained by multiplying all SIRM-*i* blocks by the outputs of the DID-*i* blocks and summing them together. The Output Scaling Factor (OSF) block multiplies the output element *f* by the scaling factor to find the actual drive power, *F*.

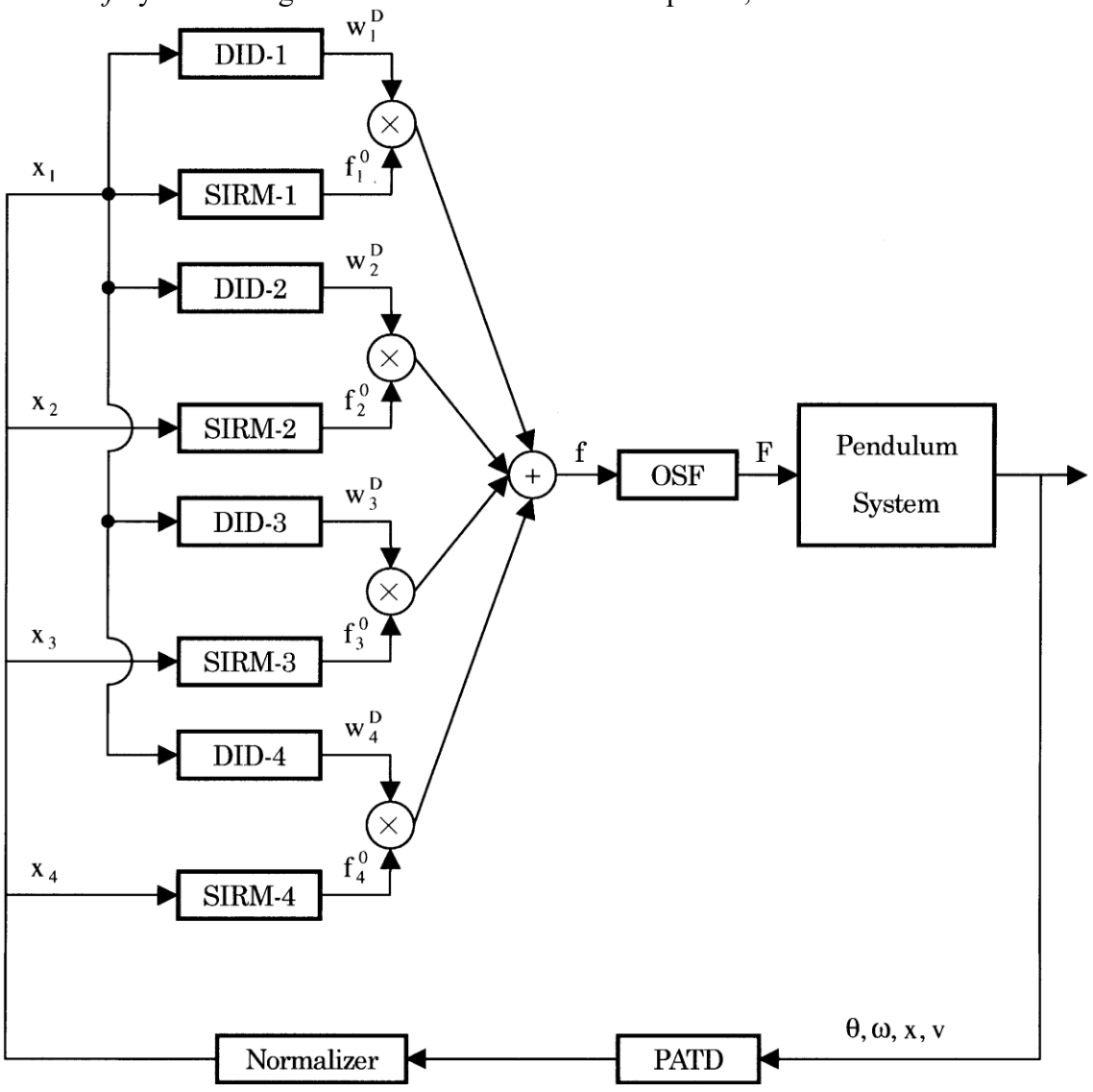


Figure 6: SIRM fuzzy logic system swing-up and balance control block diagram[49]

Using the dimensions of a system described in the literature, the control parameters they obtained enabled them to bring the pendulum from the lower equilibrium position to the upper equilibrium position and stabilize it in 5.66 seconds (Figure 6). It was also stated that they were able to control the pendulum without changing these control parameters at

all, unaffected by changes in the pendulum's length between 0.20 and 0.26 m, or the pendulum's mass between 0.03 and 0.10 kg, or the cart's mass between 0.05 and 0.15 kg.

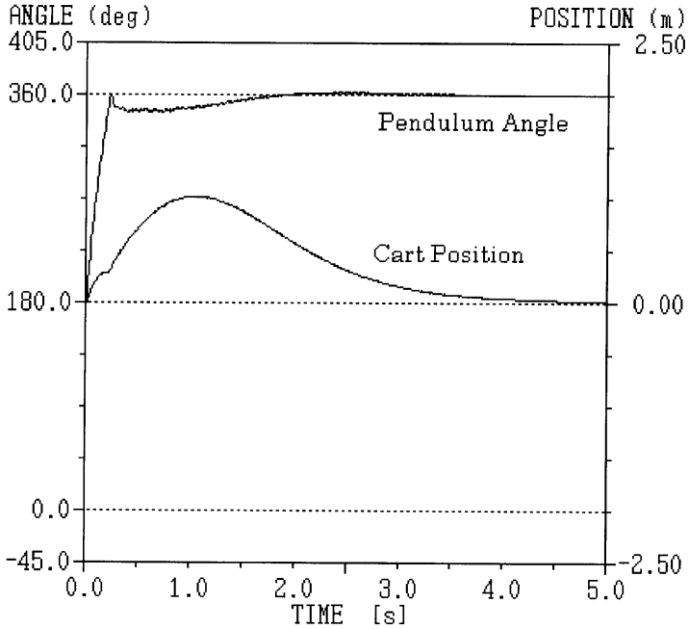


Figure 7: Swing-up and balance test results for a 0.20 m pendulum[49]

In the study conducted in 2006 [1] , the Mamdani fuzzy logic method was used to control the energy of the system. Designing a controller using the constrained rail formula in [55], Muskinja and Tovornik found that this controller, which was successful in simulation and raised the pendulum in 6 oscillations (Figure 7a), failed in the raising process when adapted to the real system (Figure 7b) because it was not possible to directly measure the carriage and pendulum speeds that should be used in the controller. To overcome this problem, instead of precise measurement of the parameters, they designed a Fuzzy Logic Controller that uses cart position, pendulum position and speed as inputs and the voltage applied to the motor as output. In this way, they managed to raise the pendulum in 5 oscillations (Figure 7c). Radhamohan et al. [50], used a fuzzy logic controller consisting of a single set of rules as a balance controller by first determining its gains for swing-up control and then changing them when the pendulum comes close to ±30° of the vertical balance position. Experimental results have shown that they can achieve swinging-up and balancing control in a linear inverted pendulum with a limited track spacing. In their study presented in 2014, Urniezius and Geguzis [43] divided a very small pendulum (11 cm) into four areas according to its angular position, unlike the linear pendulums in the literature, and used hybrid fuzzy logic and adaptive LQR control methods. In their experimental results, they stated that they were successful in raising and balancing the pendulum despite the limited rail size. In their study, Susanto et al. controlled the system's energy using the Mamdani fuzzy logic method [32]. They used the function in [61] for the system energy equation with limited rail length. They stated that the control system was successful with experimental results.

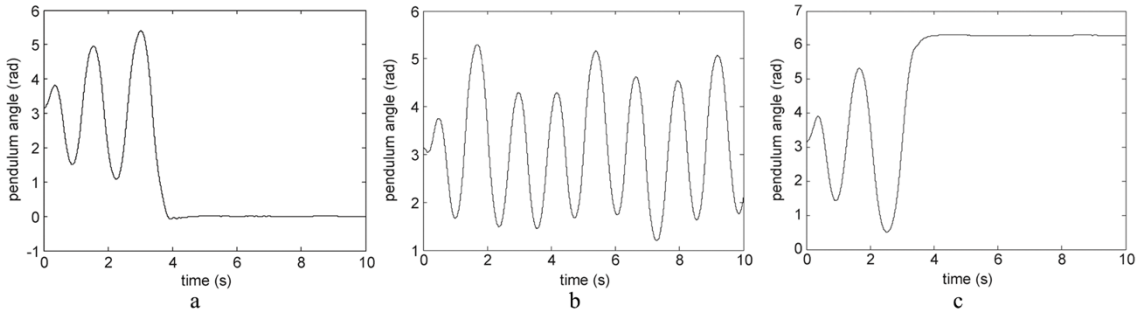


Figure 8: Comparison of energy well and FLC

In their study presented in 2021, Jain et al. first designed a fuzzy logic controller using Gaussian membership functions for swinging-up and controlling a coupled twin-arm linear pendulum and then optimized it by maximizing the entropy of the membership functions using system standard deviation data obtained from a PID controller they also designed. By comparing PID, standard FL, and optimized FL controllers, researchers have shown that the optimization is successful [33].

Tao et al. [8] designed a Fuzzy Logic-based control method for swing-up, as well as a switching method using Fuzzy Logic to transition to the balance controller for linear double inverted pendulums. They demonstrated, through the simulation results they presented, that they had successfully upgraded the system.

Khashayar and colleagues, in their study using a rotary inverted pendulum, developed a swing-up controller using the Mamdani Fuzzy Logic method and reported that they were successful in swinging the rotating inverted pendulum up with simulation results [42].

One of the current studies in this field, Bui et al. [36], proposed a new approach based on Hedge Algebra theory to lighten the computational load of classical fuzzy logic. In this method, the controller is divided into intermediate controllers that process four input state variables, and these state variables are combined according to their level of influence on stability. Simulation results have shown that the HA-based controller significantly reduces computation time and improves the system's stability and robustness performance compared to classic Sugeno-type controllers.

4.2.2. Reinforcement learning

Reinforcement Learning (RL) offers a powerful alternative to classical model-based methods in inverted pendulum control because it does not require prior knowledge of system dynamics and can discover optimal control strategies through trial and error. Studies in the literature vary widely, from data efficiency to hybrid algorithms [62].

Regarding data efficiency, which is the biggest challenge in applying RL algorithms to real systems, Riedmiller [63] presented a pioneering study with the Neural Reinforcement Learning (Neural RL) approach. In this study, which utilized the "Neural Fitted Q iteration" (NFQ) algorithm, the pendulum was successfully swung-up and balanced from zero through only 14 minutes of real-time interaction, without any support from mathematical models or simulations. This study has demonstrated the feasibility of using neural networks in the data economy and on real hardware. Another work on data efficiency was made by Baek et al. in [64]. In their work they proposed Virtual Experience

Replay (VER) which is a technique used in reinforcement learning that exploits the inherent geometrically symmetric properties of systems like inverted pendulums to generate new, virtual training data from existing measured data. By generating virtual trajectories on-the-fly, VER effectively doubles the training data without requiring any additional real-world trials or simulations which is particularly critical in real-world environments because it minimizes the continuous operation time of physical hardware, thereby preventing physical strain and potential damage to the system an also improves data efficiency. They used this technique to train an agent that can swing-up a triple inverted pendulum. Oh et al. [65] also adopted the same data efficiency method to train an RL controller to swing-up a quadruple inverted pendulum.

Taking the place of RL in control theory from an educational perspective, Israilov et al. [66], proposed an open-source framework including Q-Learning and Deep Q-Networks (DQN) algorithms. Similarly, in the study by Ben Hazem [67] , table-based Q-learning and DQN algorithms were quantitatively compared on the rotating inverted pendulum system. The findings revealed that classical Q-learning, where the state space is discretized, is sufficient for simple scenarios, but DQN exhibits much higher stability in managing the continuous and complex dynamics of the inverted pendulum.

In order to provide more precise control in continuous action spaces, Dai et al. [68], optimized the Double Deep Q-Network (DDQN) algorithm with the Prioritized Experience Replay technique. The Hardware-in-the-loop (HIL) platform used in the study minimized the performance losses encountered when transferring the agent trained in the simulation to the real hardware. On the other hand, Bhourji et al. [69] proposed a more complex structure, developing a hybrid controller consisting of DDPG algorithms for the swing-up phase and PPO algorithms for the balancing/mode selection phase. This approach has improved overall success by enabling the specialization of different RL agents in different dynamic regions of the system. Hernandez et al. [70] have used DDPG to train a rotary inverted pendulum controller.

Recent research has focused on "reward engineering" beyond algorithmic improvements. Liu et al. [37], developed a Detailed Reward Function (DRF) that simultaneously optimizes the pendulum angle and car position to accelerate the agent's learning process and a Two-Stage Learning Protocol that closes the simulation-reality gap. Bajrami et al. [38] managed to reduce the training time by 45% using a modified DDPG algorithm on the Quanser IP02 system, thus contributing to the industrial-scale applicability of RL. Böhm et al. in [71] has explored the sim-to-real gap by creating a low-cost physical pendulum apparatus designed to expose the imperfections of real-world environments, such as computation and communication delays, inaccurate sensor measurements, and continuous time evolution. To bridge the sim-to-real gap and make the real-world system solvable, they had to implement asynchronous training architectures and use dimension reduction techniques (a GRU network) to pre-process noisy observation sequences. In his dissertation Xu [72] has tackled the sim-to-real gap caused by model mismatches, sensor noise, and actuator bandwidth limitations. To mitigate the gap and achieve "zero-shot transfer", the author has outlined a practical pipeline emphasizing using high-fidelity models during training, applying a simple low-pass filter to the RL controller's actions to smooth out destructive, high-frequency chattering, and employing sensitivity-guided domain randomization.

4.3. Bang-bang control

Bang-bang control is essentially controlling a system using only two endpoints. In their work, which represents one of the earliest examples of amplifier control, Mori et al. designed a swing-up controller using the bang-bang control method in a linear inverted pendulum system and conducted experiments on a real system. Although it has been reported that the controller sometimes fails to raise the pendulum because nonlinear friction and dead-zone effects are not modelled in the system equation, the study is quite important as it is a pioneering work in swing-up control [6]. In their work in which they first proposed the rotary inverted pendulum system [39], Furuta et al. designed a feedforward controller based on a bang-bang control sequence and a state feedback controller for swinging-up the pendulum system. They experimentally tested these two control systems under conditions where the parameters were both certain and uncertain (with added disturbances). It was noted that while both controllers successfully swung-up the system when parameters were defined, only the state feedback controller was successful when a disturbance was added. In their study presented in 2011, Park and colleagues chose the bang-bang based method for swing-up control [35]. Due to the nature of the bang-bang control method, they have included a weighting factor in the controller that dampens the applied torque depending on the pendulum's speed to prevent the pendulum from reaching high speeds and starting to rotate. They reported that the system was rapidly swung-up, along with experimental results. Awrejcewicz et al. also used a bang-bang controller to amplify the double inverted pendulum system in their work [73]. In their experimental results, they stated that the pendulum was successfully brought to the point where it would switch to the balance controller.

4.4. Other control methods

Besides the control methods predominantly found in the literature, many other studies have been conducted on swinging-up an inverted pendulum using various other methods. In their publication [74] , Adhikary and Mahanta worked on a linear inverted pendulum and combined sliding mode control with a backstepping algorithm and finally created a chatter-free and continuous control signal by integrating the output. In this way, they have also managed to overcome the problems where sliding mode control fails to handle unmatched uncertainties. In their simulation results, they reported that their improved controller, compared to the standard sliding mode method, achieved faster rise and settling times for inverted pendulum swing-up control, as well as being more successful in handling unmatched uncertainties. In their work using a rotary inverted pendulum, Park and Chwa designed a sliding mode control method that forces the combined use of sliding surfaces of actuated and unactuated subsystems [35]. Through this control method, they combined inverted pendulum swing-up control with balance control, stabilizing the system without the need for a transition method. By presenting simulations for a linear inverted pendulum and experimental results for a rotary inverted pendulum, they demonstrated that the designed controller quickly stabilizes the pendulum at its vertical equilibrium point. In their study presented in 2008, Qian and colleagues reported that they had successfully designed a hierarchical sliding mode controller and presented simulation results on Pendubot [16].

In their study, Melba and Marimuthu designed a positive feedback Proportional-Derivative (PD) controller to swing-up a rotary inverted pendulum and reported that they were successful with the simulation results they presented [47]. Huang and Huang experimentally demonstrated that by combining the PD controller they designed for linear inverted pendulum experimental systems with the Grey Estimation method, they achieved

better performance than a pure PD controller [23]. Polo and colleagues subjected an inverted pendulum mounted on a wheeled vehicle platform to chaotic oscillation with the vehicle's predetermined harmonic oscillations, and reported, based on simulation results, that they were able to raise the pendulum [3]. Glück and colleagues succeeded in upgrading a three-way linear inverted pendulum system using a nonlinear feedforward controller, and they published the experimental results of the controller in their publications [10].

5. Results and discussion

This study examines existing and innovative methods in the literature for the problem of swinging-up and balancing inverted pendulum systems, which, due to their nonlinear, underactuated, and unstable nature, are one of the most important testing platforms in control theory. The literature review shows that inverted pendulum control has evolved from a purely mathematical problem to a multidisciplinary field where data-driven and learning systems are tested.

Classical control strategies (energy-based methods, sliding mode control, LQR, and PID) still serve as a fundamental point of reference due to their ability to guarantee mathematical stability around the system's equilibrium points. However, the biggest drawbacks of these methods are that they require a precise mathematical model and are sensitive to external disturbances and unmodelable dynamics such as friction. Current literature has demonstrated that energy-based methods can be successfully applied not only to rigid models but also to flexible "Soft Inverted Pendulum" systems, proving that these classical methods can be transferred to the field of autonomous soft robotics.

On the other hand, Intelligent Control Methods, developed to cope with model uncertainties, have created a paradigm shift in the literature. The computational burden imposed by classical Fuzzy Logic methods has been overcome in the literature with innovative approaches such as Hedge Algebra, enabling the design of more robust and faster-responding controllers.

The most notable development in recent years has been the integration of Reinforcement Learning and Deep Reinforcement Learning algorithms into this field. Algorithms such as Q-Learning, DQN, DDPG, and PPO, which operate completely independently of the model, have successfully managed complex nonlinear dynamics where classical methods falter by learning the optimum control policy through trial and error. Recent studies reveal that the biggest drawbacks of RL agents, namely data/time inefficiency and the "Simulation-Reality Gap" problem, are largely overcome thanks to detailed reward functions, two-stage learning protocols, and Hardware-in-the-Loop (HIL) systems.

6. Future studies and directions

In light of current trends in the literature, future research in inverted pendulum control is expected to progress in three main directions: hybrid systems, reward engineering and energy efficiency, and complex and flexible dynamics.

By developing hybrid controllers that combine classical robust control methods (e.g., SMC) with Deep Reinforcement Learning algorithms, both the model-independent flexibility of RL and the mathematical stability guarantee of classical methods can be found in the same system.

In RL algorithms, it is possible to reduce both training time and control costs by designing complex reward functions that are not only focused on time or angular error but also minimize the energy consumed by the motor (energy-aware).

Increasing the use of AI-powered autonomous control applications on dual/multiple inverted pendulum systems and soft pendulums with affine curvature will enhance the diversity and adaptability of real-life applications.

Consequently, it is not possible to speak of a "single universal solution" for the inverted pendulum swing-up problem. While classical and energy-based methods remain valid in situations where a precise mathematical model can be obtained and hardware limitations exist, Deep Reinforcement Learning algorithms emerge as an inevitable and powerful choice in modern robotics applications where system parameters are variable, external disturbances are high, and model derivation is difficult.

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