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An Overview of Triple-Composed S-Metric Spaces with Few Fixed-Point Theorems

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Abstract

Metric fixed-point theory has received a lot of recent attention. The Banach fixed-point theorem served as the foundation for this theory. This theorem's generalizations have been looked at using various methodologies. One of these entails generalizing the prevalent contractive condition, while the other involves generalizing the prevalent metric space. Numerous generalized metric spaces were defined in the literature for the second generalization. As a new generalization of both a metric and an S-metric space in this context, our major goal is to present the idea of a triple-composed S-metric space. We also provide some fundamental and topological ideas about triple-composed S-metric space. We look into some of this idea's characteristics. On triple-composed S-metric spaces, we demonstrate various fixed-point theorems. Finally, we provide the system of linear equations with an application.

1. Introduction and Background

A vast array of applications in many areas of mathematics make fixed point theory a fascinating subject. In the 20th century, the fixed point theory's scientific foundation was established. The Picard-Banach-Caccioppoli contraction principle, which is the essential outcome of this theory, has led to significant research areas and applications of the theory to functional equations, differential equations, integral equations, and other types of equations. The contraction principle, commonly referred to as Banach's fixed point theory, is a crucial technique in the study of metric spaces [1]. For a variety of applications f, it ensures the existence and uniqueness of solutions to equations of the type $\rho = T\rho$, and it also offers a useful technique to find these solutions.

Diverse methodologies have been used to study fixed-point theory. These methods include the following:

- to broaden the meaning of the term "contractive condition" (as in, take a look at [2, 3, 4, 5, 6, 7, 8, 9, 10]).
- to expand the metric space that is being used (consider, as in [11, 12, 13, 14, 15] as well as any references therein).
- to study some of the fixed point set's geometric characteristics (in the sense of, look at [16, 17, 18, 19, 20, 21, 22] including any references thereto).

When using the second strategy, following is an introduction to the concepts of an S-metric space and a double-composed metric space:

Definition 1.1. *[14] Let Ω be a non-empty set. A function $S:\Omega\times\Omega\times\Omega\to[0,\infty)$ is called an S-metric if it satisfies:*

(S1) $S(\rho,\omega,\upsilon)=0$ *if and only if* $\rho=\omega=\upsilon$ *for all* $\rho,\omega,\upsilon\in\Omega$,
(S2) $S(\rho,\omega,\upsilon)\leq S(\rho,\rho,a)+S(\omega,\omega,a)+S(\upsilon,\upsilon,a)$ *for all* $\rho,\omega,\upsilon,a\in\Omega$.

The pair (Ω,S) is called an S-metric space.

Definition 1.2. *[11] Let Ω be a non-empty set and $\alpha,\beta:[0,\infty)\to[0,\infty)$ be two non-constant functions. A function $D:\Omega\times\Omega\to[0,\infty)$ is called a double-composed metric if it satisfies:*

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(D1) $D(\rho,\omega)=0$ *if and only if* $\rho=\omega$ *for all* $\rho,\omega\in\Omega$,
(D2) $D(\rho,\omega)=D(\omega,\rho)$ *for all* $\rho,\omega\in\Omega$,
(D3) $D(\rho,\omega)\leq\alpha(D(\rho,\upsilon))+\beta(D(\upsilon,\omega))$ *for all* $\rho,\omega,\upsilon\in\Omega$.

The pair (Ω,D) *is called a double-composed metric space. It is denoted by DCMS.*

Using the ideas above, the primary goal of this study is to create a new generalized metric space called a triple-composed *S*-metric space. In Section 2, we define the concept of a triple-composed S-metric space and look at some of its fundamental characteristics. We also provide some topological ideas and characteristics of this space. Additionally, we provided some crucial examples to emphasize the significance of the area we are utilizing. In Section 3, on a triple-composed *S*-metric space, we demonstrate the generalization of the Nemtyskii-Edelstein fixed-point theorem and the Banach contraction principle. In Section 4, for the system of linear equations, we provide an application.

2. Triple-Composed *S*-Metric Spaces with Basic Topological Notions

In this section, we introduce the notion of a triple-composed *S*-metric with some basic concepts and properties.

Definition 2.1. *Let* Ω *be a non-empty set and* $\gamma_i:[0,\infty)\to[0,\infty)$, $i\in\{1,2,3\}$ *be three non-constant functions. A function* $S_T:\Omega\times\Omega\times\Omega\to[0,\infty)$ *is called a triple-composed S-metric if it satisfies the following conditions:*

(S_T1) $S_T(\rho,\omega,\upsilon)=0$ *if and only if* $\rho=\omega=\upsilon$ *for all* $\rho,\omega,\upsilon\in\Omega$,
(S_T2) $S_T(\rho,\omega,\upsilon)\leq\gamma_1(S_T(\rho,\rho,a))+\gamma_2(S_T(\omega,\omega,a))+\gamma_3(S_T(\upsilon,\upsilon,a))$ *for all* $\rho,\omega,\upsilon,a\in\Omega$. *Then the pair* (Ω,S_T) *is called a triple-composed S-metric space. We denote a triple-composed S-metric space by TCSMS.*

Remark 2.2. *Every S-metric space is a TCSMS with the control functions*

$$\gamma_i(t)=t,$$

for all $t\in[0,\infty)$. *But the converse statement is not always true.*

Example 2.3. *Let* $\Omega=\{1,2,3\}$ *and the function* $S_T:\Omega\times\Omega\times\Omega\to[0,\infty)$ *be defined as*

$$S_T(1,1,1)=S_T(2,2,2)=S_T(3,3,3)=0,$$

$$S_T(1,2,3)=S_T(1,3,2)=3000,$$

$$S_T(2,1,3)=S_T(2,3,1)=2000,$$

$$S_T(3,1,2)=S_T(3,2,1)=1000,$$

$$S_T(1,1,2)=S_T(2,2,1)=200,$$

$$S_T(2,2,3)=S_T(3,3,2)=300$$

and

$$S_T(1,1,3)=S_T(3,3,1)=400.$$

Then the function S_T *is a triple-composed S-metric with non-constant control functions* $\gamma_i:[0,\infty)\to[0,\infty)$, $i\in\{1,2,3\}$ *defined as*

$$\gamma_1(t)=t+3000,\ \gamma_2(t)=t+4000 \text{ and } \gamma_3(t)=t+5000,$$

for all $t\in[0,\infty)$. *But it is not an S-metric. Indeed, for* $\rho=a=1$, $\omega=2$ *and* $\upsilon=3$, *we get*

$$S_T(1,2,3)=3000\leq S_T(1,1,1)+S_T(2,2,1)+S_T(3,3,1)=600,$$

a contradiction.

Lemma 2.4. *Let* (Ω,S_T) *be a TCSMS. Then we have*

$$S_T(\rho,\rho,\omega)=S_T(\omega,\omega,\rho), \tag{2.1}$$

for all $\rho,\omega\in\Omega$ *when* $\gamma_1(0)=\gamma_2(0)=0$ *and* $\gamma_3(t)=t$ *for all* $t\in[0,\infty)$.

Proof. Using the condition $(S_T 2)$, we get

$$\begin{aligned} S_T(\rho,\rho,\omega) &\leq \gamma_1(S_T(\rho,\rho,\rho))+\gamma_2(S_T(\rho,\rho,\rho))+\gamma_3(S_T(\omega,\omega,\rho)) \\ &= S_T(\omega,\omega,\rho) \end{aligned} \tag{2.2}$$

and

$$\begin{aligned} S_T(\omega,\omega,\rho) &\leq \gamma_1(S_T(\omega,\omega,\omega))+\gamma_2(S_T(\omega,\omega,\omega))+\gamma_3(S_T(\rho,\rho,\omega)) \\ &= S_T(\rho,\rho,\omega). \end{aligned} \tag{2.3}$$

Consequently, by the inequalities (2.2) and (2.3), we obtain

$$S_T(\rho,\rho,\omega)=S_T(\omega,\omega,\rho).$$

□

Definition 2.5. *Let (Ω,S_T) be a TCSMS. If S_T satisfies the equality (2.1), then S_T is called a symmetric triple-composed S-metric.*

Lemma 2.6. *Let (Ω,D) be a DCMS with the additive functions $\alpha,\beta:[0,\infty)\to[0,\infty)$ and the function $S_T:\Omega\times\Omega\times\Omega\to[0,\infty)$ be defined as*

$$S_T(\rho,\omega,\upsilon)=D(\rho,\upsilon)+D(\omega,\upsilon), \tag{2.4}$$

for all $\rho,\omega,\upsilon\in\Omega$. Then (Ω,S_T) is a TCSMS.

Proof. $(S_T 1)$

$$\begin{aligned} S_T(\rho,\omega,\upsilon) &= 0 \\ &\Longleftrightarrow D(\rho,\upsilon)+D(\omega,\upsilon)=0 \\ &\Longleftrightarrow D(\rho,\upsilon)=0 \text{ and } D(\omega,\upsilon)=0 \\ &\Longleftrightarrow \rho=\omega=\upsilon. \end{aligned}$$

$(S_T 2)$

$$\begin{aligned} S_T(\rho,\omega,\upsilon) &= D(\rho,\upsilon)+D(\omega,\upsilon) \\ &\leq \alpha(D(\rho,a))+\beta(D(a,\upsilon))+\alpha(D(\omega,a))+\beta(D(a,\upsilon)) \\ &\leq \alpha(D(\rho,a))+\alpha(D(\rho,a))+\alpha(D(\omega,a))+\alpha(D(\omega,a))+\beta(D(\upsilon,a))+\beta(D(\upsilon,a)) \\ &\leq \alpha(D(\rho,a)+D(\rho,a))+\alpha(D(\omega,a)+D(\omega,a))+\beta(D(\upsilon,a)+D(\upsilon,a)) \\ &= \alpha(S_T(\rho,\rho,a))+\alpha(S_T(\omega,\omega,a))+\beta(S_T(\upsilon,\upsilon,a)). \end{aligned}$$

Consequently, (Ω,S_T) is a *TCSMS* with the functions $\alpha_1=\alpha_2=\alpha$ and $\alpha_3=\beta$. □

If the triple-composed S-metric S_T satisfies the equality (2.4), then it is called a generated by a double-composed metric D. In the following example, we see that a triple-composed S-metric S_T which is not generated by any double-composed metric D.

Example 2.7. *Let $\Omega=\mathbb{R}^n$ and the function $S_T:\Omega\times\Omega\times\Omega\to[0,\infty)$ be defined as*

$$S_T(\rho,\omega,\upsilon)=\|\omega+\upsilon-2\rho\|+\|\omega-\upsilon\|,$$

for all $\rho,\omega,\upsilon\in\mathbb{R}^n$. Then the function S_T is a triple-composed S-metric with non-constant control functions $\gamma_i:[0,\infty)\to[0,\infty)$, $i\in\{1,2,3\}$ defined as

$$\gamma_1(t)=\gamma_2(t)=2t \text{ and } \gamma_3(t)=t,$$

for all $t\in[0,\infty)$. But it is not generated by any double-composed metric D. On the contrary, assume that

$$S_T(\rho,\omega,\upsilon)=D(\rho,\upsilon)+D(\omega,\upsilon),$$

for all $\rho,\omega,\upsilon\in\mathbb{R}^n$. Then we get

$$S_T(\rho,\rho,\upsilon)=2D(\rho,\upsilon)=2\|\rho-\upsilon\|\Longrightarrow D(\rho,\upsilon)=\|\rho-\upsilon\|$$

and

$$S_T(\omega,\omega,\upsilon)=2D(\omega,\upsilon)=2\|\omega-\upsilon\|\Longrightarrow D(\omega,\upsilon)=\|\omega-\upsilon\|.$$

Hence we get

$$\|\omega+\upsilon-2\rho\|+\|\omega-\upsilon\|=\|\rho-\upsilon\|+\|\omega-\upsilon\|,$$

a contradiction.

Definition 2.8. *Let (Ω, S_T) be a TCSMS, $r > 0$ and $\rho_0 \in \Omega$.*

(i) The open ball with a center ρ_0 and a radius r is defined as

$$B_{S_T}(\rho_0, r) = \{\rho \in \Omega : S_T(\rho, \rho, \rho_0) < r\}.$$

(ii) The closed ball with a center ρ_0 and a radius r is defined as

$$B_{S_T}[\rho_0, r] = \{\rho \in \Omega : S_T(\rho, \rho, \rho_0) \leq r\}.$$

(iii) The circle with a center ρ_0 and a radius r is defined as

$$C_{S_T}(\rho_0, r) = \{\rho \in \Omega : S_T(\rho, \rho, \rho_0) = r\}.$$

Remark 2.9. *The closed ball can be considered as a disc with a center ρ_0 and a radius r on a TCSMS as follows:*

$$D_{S_T}(\rho_0, r) = \{\rho \in \Omega : S_T(\rho, \rho, \rho_0) \leq r\}.$$

Definition 2.10. *Let (Ω, S_T) be a TCSMS and $A \subset \Omega$.*

(i) If there is $\rho > 0$ such that

$$B_{S_T}(\rho_0, \rho) \subset A,$$

for each $\rho \in A$ then A is called an open subset of ρ.

(ii) A is called S_T-bounded if there is $\rho > 0$ such that

$$S_T(\rho, \rho, \omega) < \rho,$$

for all $\rho, \omega \in A$.

Lemma 2.11. *Let (Ω, S_T) be a TCSMS with the symmetric S_T and non-constant control functions $\gamma_i : [0, \infty) \to [0, \infty)$, $i \in \{1, 2, 3\}$ such that $\gamma_i(t) < t$ for all $t \in (0, \infty)$. If $\rho > 0$ and $\rho \in \Omega$, then the ball $B_{S_T}(\rho, \rho)$ is an open subset of Ω.*

Proof. Let $\omega \in B_{S_T}(\rho, \rho)$. Hence we get

$$S_T(\omega, \omega, \rho) < \rho.$$

If we take $\mu = S_T(\rho, \rho, \omega)$ and $\rho' = \frac{\rho - \mu}{2}$, then we prove

$$B_{S_T}(\omega, \rho') \subseteq B_{S_T}(\rho, \rho).$$

Let $\upsilon \in B_{S_T}(\omega, \rho')$. So we have

$$S_T(\upsilon, \upsilon, \omega) < \rho'.$$

Using the definition of S_T and the hypothesis, we obtain

$$\begin{aligned} S_T(\upsilon, \upsilon, \rho) &\leq \gamma_1(S_T(\upsilon, \upsilon, \omega)) + \gamma_2(S_T(\upsilon, \upsilon, \omega)) + \gamma_3(S_T(\rho, \rho, \omega)) \\ &\leq S_T(\upsilon, \upsilon, \omega) + S_T(\upsilon, \upsilon, \omega) + S_T(\rho, \rho, \omega) \\ &< \rho - \mu + \mu = \rho \end{aligned}$$

and so

$$B_{S_T}(\omega, \rho') \subseteq B_{S_T}(\rho, \rho).$$

Consequently, $B_{S_T}(\rho, \rho)$ is an open subset of Ω. □

Definition 2.12. *Let (Ω, S_T) be a TCSMS and $A \subset \Omega$. Let τ_{S_T} be the set of all $A \subset \Omega$ with $\rho \in A$ if and only if there exists $\rho > 0$ such that*

$$B_{S_T}(\rho, \rho) \subset A.$$

Then τ_{S_T} is a topology induced by S_T on Ω.

Definition 2.13. *Let (Ω, S_T) be a TCSMS and $\{\rho_n\}$ be sequence in Ω.*

(i) $\{\rho_n\}$ *converges to* ρ *if and only if* $S_T(\rho_n,\rho_n,\rho)\to 0$ *as* $n\to\infty$*, that is, for each* $\varepsilon>0$*, there exists* $n_0\in\mathbb{N}$ *such that for all* $n\geq n_0$*,* $S_T(\rho_n,\rho_n,\rho)<\varepsilon$ *and it is denoted by*

$$\lim_{n\to\infty}\rho_n=\rho.$$

(ii) $\{\rho_n\}$ *is a Cauchy sequence if* $\lim_{n,m\to\infty} S_T(\rho_n,\rho_n,\rho_m)$ *exists and is finite, that is, for each* $\varepsilon>0$*, there exists* $n_0\in\mathbb{N}$ *such that*

$$S_T(\rho_n,\rho_n,\rho_m)<\varepsilon,$$

for each $n,m\geq n_0$*.*

(iii) (Ω,S_T) *is called complete if every Cauchy sequence is convergent to some point in* Ω*.*

Lemma 2.14. *Let* (Ω,S_T) *be a TCSMS with the symmetric* S_T *and non-constant continuous control functions* $\gamma_i:[0,\infty)\to[0,\infty)$*,* $i\in\{1,2,3\}$ *satisfying*

$$\sum_{i=1}^{3}\gamma_i(0)=0.$$

If the sequence $\{\rho_n\}$ *in* Ω *converges to* ρ*, then* ρ *is unique.*

Proof. Let $\{\rho_n\}$ converges to ρ and ω. Then we have

$$\lim_{n\to\infty}S_T(\rho_n,\rho_n,\rho)=0$$

and

$$\lim_{n\to\infty}S_T(\rho_n,\rho_n,\omega)=0.$$

Using the triangle inequality, we get

$$S_T(\rho,\rho,\omega)\leq\gamma_1(S_T(\rho,\rho,\rho_n))+\gamma_2(S_T(\rho,\rho,\rho_n))+\gamma_3(S_T(\omega,\omega,\rho_n)).$$

Since α_i are continuous, using the above inequality and the symmetry property, we obtain

$$\begin{aligned}S_T(\rho,\rho,\omega) &\leq \gamma_1\left(\lim_{n\to\infty}S_T(\rho_n,\rho_n,\rho)\right)+\gamma_2\left(\lim_{n\to\infty}S_T(\rho_n,\rho_n,\rho)\right)+\gamma_3\left(\lim_{n\to\infty}S_T(\rho_n,\rho_n,\omega)\right)\\ &= \gamma_1(0)+\gamma_2(0)+\gamma_3(0)=0.\end{aligned}$$

Therefore, we get $\rho=\omega$. □

Lemma 2.15. *Let* (Ω,S_T) *be a TCSMS with non-constant continuous control functions* $\gamma_i:[0,\infty)\to[0,\infty)$*,* $i\in\{1,2,3\}$ *satisfying*

$$\gamma_i(t)<t,$$

for all $t\in(0,\infty)$ *and* $\gamma_i(0)=0$*. If the sequence* $\{\rho_n\}$ *in* Ω *converges to* ρ*, then* $\{\rho_n\}$ *is Cauchy.*

Proof. Let

$$\lim_{n\to\infty}\rho_n=\rho.$$

Then for each $\varepsilon>0$, there exist $n_1,n_2\in\mathbb{N}$ such that

$$n\geq n_1\Longrightarrow S_T(\rho_n,\rho_n,\rho)<\frac{\varepsilon}{4}$$

and

$$n\geq n_2\Longrightarrow S_T(\rho_m,\rho_m,\rho)<\frac{\varepsilon}{2}.$$

If we take $n_0=\max\{n_1,n_2\}$, then for every $n,m\geq n_0$, using the triangle inequality of S_T, we get

$$\begin{aligned}S_T(\rho_n,\rho_n,\rho_m) &\leq \gamma_1(S_T(\rho_n,\rho_n,\rho))+\gamma_2(S_T(\rho_n,\rho_n,\rho))+\gamma_3(S_T(\rho_m,\rho_m,\rho))\\ &< \frac{\varepsilon}{4}+\frac{\varepsilon}{4}+\frac{\varepsilon}{2}=\varepsilon.\end{aligned}$$

Therefore, $\{\rho_n\}$ is Cauchy. □

Lemma 2.16. *Let (Ω, S_T) be a TCSMS with the symmetric S_T and non-constant continuous control functions $\gamma_i : [0,\infty) \to [0,\infty)$, $i \in \{1,2,3\}$ satisfying*

$$\gamma_i(t) < t,$$

for all $t \in (0,\infty)$ and $\gamma_i(0) = 0$. If there exist $\{\rho_n\}$ and $\{\omega_n\}$ such that

$$\lim_{n\to\infty} \rho_n = \rho \text{ and } \lim_{n\to\infty} \omega_n = \omega,$$

then

$$\lim_{n\to\infty} S_T(\rho_n, \rho_n, \omega_n) = S_T(\rho, \rho, \omega).$$

Proof. Let

$$\lim_{n\to\infty} \rho_n = \rho \text{ and } \lim_{n\to\infty} \omega_n = \omega.$$

Then for each $\varepsilon > 0$, there exist $n_1, n_2 \in \mathbb{N}$ such that

$$n \geq n_1 \Longrightarrow S_T(\rho_n, \rho_n, \rho) < \frac{\varepsilon}{4}$$

and

$$n \geq n_2 \Longrightarrow S_T(\omega_n, \omega_n, \omega) < \frac{\varepsilon}{4}.$$

If we take $n_0 = \max\{n_1, n_2\}$, hence for every $n \geq n_0$, we get

$$\begin{aligned} S_T(\rho_n, \rho_n, \omega_n) &\leq \gamma_1(S_T(\rho_n, \rho_n, \rho)) + \gamma_2(S_T(\rho_n, \rho_n, \rho)) + \gamma_3(S_T(\omega_n, \omega_n, \rho)) \\ &< 2S_T(\rho_n, \rho_n, \rho) + S_T(\omega_n, \omega_n, \rho) \\ &\leq 2S_T(\rho_n, \rho_n, \rho) + \gamma_1(S_T(\omega_n, \omega_n, \omega)) + \gamma_2(S_T(\omega_n, \omega_n, \omega)) + \gamma_3(S_T(\rho, \rho, \omega)) \\ &< 2S_T(\rho_n, \rho_n, \rho) + 2S_T(\omega_n, \omega_n, \omega) + S_T(\rho, \rho, \omega) \\ &< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} + S_T(\rho, \rho, \omega) = \varepsilon + S_T(\rho, \rho, \omega) \end{aligned}$$

and

$$S_T(\rho_n, \rho_n, \omega_n) - S_T(\rho, \rho, \omega) < \varepsilon. \tag{2.5}$$

On the other hand, using the symmetry property, we obtain

$$\begin{aligned} S_T(\rho, \rho, \omega) &\leq \gamma_1(S_T(\rho, \rho, \rho_n)) + \gamma_2(S_T(\rho, \rho, \rho_n)) + \gamma_3(S_T(\omega, \omega, \rho_n)) \\ &< 2S_T(\rho_n, \rho_n, \rho) + S_T(\omega, \omega, \rho_n) \\ &\leq 2S_T(\rho_n, \rho_n, \rho) + \gamma_1(S_T(\omega, \omega, \omega_n)) + \gamma_2(S_T(\omega, \omega, \omega_n)) + \gamma_3(S_T(\rho_n, \rho_n, \omega_n)) \\ &< 2S_T(\rho_n, \rho_n, \rho) + 2S_T(\omega_n, \omega_n, \omega) + S_T(\rho_n, \rho_n, \omega_n) \\ &< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} + S_T(\rho_n, \rho_n, \omega_n) = \varepsilon + S_T(\rho_n, \rho_n, \omega_n) \end{aligned}$$

and

$$S_T(\rho, \rho, \omega) - S_T(\rho_n, \rho_n, \omega_n) < \varepsilon. \tag{2.6}$$

Using the inequalities (2.5) and (2.6), we get

$$|S_T(\rho_n, \rho_n, \omega_n) - S_T(\rho, \rho, \omega)| < \varepsilon,$$

that is,

$$\lim_{n\to\infty} S_T(\rho_n, \rho_n, \omega_n) = S_T(\rho, \rho, \omega).$$

□

3. Some Fixed-Point Theorems

In this section, we prove two fixed-point theorems.

Theorem 3.1. *Let (Ω, S_T) be a TCSMS with the symmetric S_T and non-constant continuous control functions $\gamma_i : [0,\infty) \to [0,\infty)$, $i \in \{1,2,3\}$. Let $T : \Omega \to \Omega$ be a self-mapping satisfying*

$$S_T(T\rho, T\rho, T\omega) \leq hS_T(\rho, \rho, \omega), \tag{3.1}$$

for all $\rho, \omega \in \Omega$ and $h \in (0,1)$. Let us define a sequence $\{\rho_n\}$ by

$$\rho_n = T^n \rho_0,$$

for $\rho_0 \in \Omega$. Assume that the following conditions are satisfied:

(a) γ_i are continuous and non-decreasing functions with

$$\sum_{i=1}^{3} \gamma_i(0) = 0$$

and γ_3 is sub-additive.

(b) $$\lim_{m,n\to\infty} \left[\sum_{j=m}^{n-2} \gamma_3^{j-m} \gamma_1 \left(h^j S_T(\rho_0,\rho_0,\rho_1)\right) + \sum_{j=m}^{n-2} \gamma_3^{j-m} \gamma_2 \left(h^j S_T(\rho_0,\rho_0,\rho_1)\right) \gamma_3^{n-m-1} \left(h^{n-1} S_T(\rho_0,\rho_0,\rho_1)\right) \right] = 0,$$

where $\gamma_3^{j-m}\gamma_1\left(h^j S_T(\rho_0,\rho_0,\rho_1)\right)$, $\gamma_3^{j-m}\gamma_2\left(h^j S_T(\rho_0,\rho_0,\rho_1)\right)$ and $\gamma_3^{n-m-1}\left(h^{n-1} S_T(\rho_0,\rho_0,\rho_1)\right)$ are the composite functions.

Then T has a unique fixed point.

Proof. Let $\rho_0 \in \nleq$. Let us define a sequence $\{\rho_n\}$ with

$$\rho_n = T^n \rho_0 = T\rho_{n-1},$$

for all $n \in \mathbb{N}$. Using the inequality (3.1), we get

$$\begin{aligned} S_T(\rho_n,\rho_n,\rho_{n+1}) &= S_T(T\rho_{n-1}, T\rho_{n-1}, T\rho_n) \\ &\leq hS_T(\rho_{n-1},\rho_{n-1},\rho_n) \\ &= hS_T(T\rho_{n-2}, T\rho_{n-2}, T\rho_{n-1}) \\ &\leq h^2 S_T(\rho_{n-2},\rho_{n-2},\rho_{n-1}) \\ &\;\;\vdots \\ &\leq h^n S_T(\rho_0,\rho_0,\rho_1). \end{aligned}$$

For $n \geq m$, using the triangle inequality, we obtain

$$\begin{aligned} S_T(\rho_m,\rho_m,\rho_n) &\leq \gamma_1(S_T(\rho_m,\rho_m,\rho_{m+1})) + \gamma_2(S_T(\rho_m,\rho_m,\rho_{m+1})) + \gamma_3(S_T(\rho_n,\rho_n,\rho_{m+1})) \\ &= \gamma_1(S_T(\rho_m,\rho_m,\rho_{m+1})) + \gamma_2(S_T(\rho_m,\rho_m,\rho_{m+1})) + \gamma_3(S_T(\rho_{m+1},\rho_{m+1},\rho_n)) \\ &\leq \gamma_1(S_T(\rho_m,\rho_m,\rho_{m+1})) + \gamma_2(S_T(\rho_m,\rho_m,\rho_{m+1})) \\ &\quad + \gamma_3 \left[\begin{array}{c} \gamma_1(S_T(\rho_{m+1},\rho_{m+1},\rho_{m+2})) + \gamma_2(S_T(\rho_{m+1},\rho_{m+1},\rho_{m+2})) \\ + \gamma_3(S_T(\rho_{m+2},\rho_{m+2},\rho_n)) \end{array} \right] \\ &= \gamma_1(S_T(\rho_m,\rho_m,\rho_{m+1})) + \gamma_2(S_T(\rho_m,\rho_m,\rho_{m+1})) \\ &\quad + \gamma_3\gamma_1(S_T(\rho_{m+1},\rho_{m+1},\rho_{m+2})) + \gamma_3\gamma_2(S_T(\rho_{m+1},\rho_{m+1},\rho_{m+2})) \\ &\quad + \gamma_3^2(S_T(\rho_{m+2},\rho_{m+2},\rho_n)), \end{aligned}$$

$$\begin{aligned} \gamma_3^2(S_T(\rho_{m+2},\rho_{m+2},\rho_n)) &\leq \gamma_3^2 \left[\begin{array}{c} \gamma_1(S_T(\rho_{m+2},\rho_{m+2},\rho_{m+3})) + \gamma_2(S_T(\rho_{m+2},\rho_{m+2},\rho_{m+3})) \\ + \gamma_3(S_T(\rho_{m+3},\rho_{m+3},\rho_n)) \end{array} \right] \\ &= \gamma_3^2\gamma_1(S_T(\rho_{m+2},\rho_{m+2},\rho_{m+3})) + \gamma_3^2\gamma_2(S_T(\rho_{m+2},\rho_{m+2},\rho_{m+3})) + \gamma_3^3(S_T(\rho_{m+3},\rho_{m+3},\rho_n)), \end{aligned}$$

$$\begin{aligned} \gamma_3^3(S_T(\rho_{m+3},\rho_{m+3},\rho_n)) &\leq \gamma_3^3 \left[\begin{array}{c} \gamma_1(S_T(\rho_{m+3},\rho_{m+3},\rho_{m+4})) + \gamma_2(S_T(\rho_{m+3},\rho_{m+3},\rho_{m+4})) \\ + \gamma_3(S_T(\rho_{m+4},\rho_{m+4},\rho_n)) \end{array} \right] \\ &= \gamma_3^3\gamma_1(S_T(\rho_{m+3},\rho_{m+3},\rho_{m+4})) + \gamma_3^3\gamma_2(S_T(\rho_{m+3},\rho_{m+3},\rho_{m+4})) + \gamma_3^4(S_T(\rho_{m+4},\rho_{m+4},\rho_n)) \end{aligned}$$

and so we have

$$\begin{aligned}
S_T(\rho_m,\rho_m,\rho_n) &\le \gamma_1(S_T(\rho_m,\rho_m,\rho_{m+1}))+\gamma_2(S_T(\rho_m,\rho_m,\rho_{m+1})) \\
&\quad +\gamma_3\gamma_1(S_T(\rho_{m+1},\rho_{m+1},\rho_{m+2}))+\gamma_3\gamma_2(S_T(\rho_{m+1},\rho_{m+1},\rho_{m+2})) \\
&\quad +\gamma_3^2\gamma_1(S_T(\rho_{m+2},\rho_{m+2},\rho_{m+3}))+\gamma_3^2\gamma_2(S_T(\rho_{m+2},\rho_{m+2},\rho_{m+3})) \\
&\quad +\gamma_3^3\gamma_1(S_T(\rho_{m+3},\rho_{m+3},\rho_{m+4}))+\gamma_3^3\gamma_2(S_T(\rho_{m+3},\rho_{m+3},\rho_{m+4})) \\
&\quad +\cdots+\gamma_3^{n-m-2}\begin{bmatrix}\gamma_1(S_T(\rho_{n-2},\rho_{n-2},\rho_{n-1})) \\ +\gamma_2(S_T(\rho_{n-2},\rho_{n-2},\rho_{n-1})) \\ +\gamma_3(S_T(\rho_{n-1},\rho_{n-1},\rho_n))\end{bmatrix} \\
&= \gamma_1(S_T(\rho_m,\rho_m,\rho_{m+1}))+\gamma_2(S_T(\rho_m,\rho_m,\rho_{m+1})) \\
&\quad +\gamma_3\gamma_1(S_T(\rho_{m+1},\rho_{m+1},\rho_{m+2}))+\gamma_3\gamma_2(S_T(\rho_{m+1},\rho_{m+1},\rho_{m+2})) \\
&\quad +\gamma_3^2\gamma_1(S_T(\rho_{m+2},\rho_{m+2},\rho_{m+3}))+\gamma_3^2\gamma_2(S_T(\rho_{m+2},\rho_{m+2},\rho_{m+3})) \\
&\quad +\gamma_3^3\gamma_1(S_T(\rho_{m+3},\rho_{m+3},\rho_{m+4}))+\gamma_3^3\gamma_2(S_T(\rho_{m+3},\rho_{m+3},\rho_{m+4})) \\
&\quad +\cdots+\gamma_3^{n-m-2}\gamma_1(S_T(\rho_{n-2},\rho_{n-2},\rho_{n-1})) \\
&\quad +\gamma_3^{n-m-2}\gamma_2(S_T(\rho_{n-2},\rho_{n-2},\rho_{n-1}))+\gamma_3^{n-m-1}(S_T(\rho_{n-1},\rho_{n-1},\rho_n)) \\
&= \sum_{j=m}^{n-2}\gamma_3^{j-m}\gamma_1\left(S_T\left(\rho_j,\rho_j,\rho_{j+1}\right)\right)+\sum_{j=m}^{n-2}\gamma_3^{j-m}\gamma_2\left(S_T\left(\rho_j,\rho_j,\rho_{j+1}\right)\right)+\gamma_3^{n-m-1}\left(S_T(\rho_{n-1},\rho_{n-1},\rho_n)\right).
\end{aligned} \tag{3.2}$$

Using the inequalities (3.1), (3.2) and the conditions (a), (b), we have

$$S_T(\rho_m,\rho_m,\rho_n) \le \sum_{j=m}^{n-2}\gamma_3^{j-m}\gamma_1\left(h^jS_T(\rho_0,\rho_0,\rho_1)\right)+\sum_{j=m}^{n-2}\gamma_3^{j-m}\gamma_2\left(h^jS_T(\rho_0,\rho_0,\rho_1)\right)+\gamma_3^{n-m-1}\left(h^{n-1}S_T(\rho_0,\rho_0,\rho_1)\right).$$

Let $m,n\to\infty$, we get

$$\lim_{m,n\to\infty}S_T(\rho_m,\rho_m,\rho_n)=0.$$

Hence $\{\rho_n\}$ is a Cauchy sequence in Ω. Since Ω is a complete $TCSMS$, then $\{\rho_n\}$ converges to a point $\upsilon\in\Omega$, that is,

$$\lim_{n\to\infty}S_T(\rho_n,\rho_n,\upsilon)=0.$$

Now we prove that υ is a fixed point of T. Using the triangle inequality, we have

$$\begin{aligned}
S_T(\upsilon,\upsilon,T\upsilon) &\le \gamma_1(S_T(\upsilon,\upsilon,\rho_n))+\gamma_2(S_T(\upsilon,\upsilon,\rho_n))+\gamma_3(S_T(T\upsilon,T\upsilon,\rho_n)) \\
&= \gamma_1(S_T(\upsilon,\upsilon,\rho_n))+\gamma_2(S_T(\upsilon,\upsilon,\rho_n))+\gamma_3(S_T(T\upsilon,T\upsilon,T\rho_{n-1})) \\
&\le \gamma_1(S_T(\upsilon,\upsilon,\rho_n))+\gamma_2(S_T(\upsilon,\upsilon,\rho_n))+\gamma_3(hS_T(\upsilon,\upsilon,\rho_{n-1})).
\end{aligned}$$

Using the hypothesis, we obtain

$$\begin{aligned}
S_T(\upsilon,\upsilon,T\upsilon) &\le \lim_{n\to\infty}\begin{bmatrix}\gamma_1(S_T(\upsilon,\upsilon,\rho_n))+\gamma_2(S_T(\upsilon,\upsilon,\rho_n)) \\ +\gamma_3(hS_T(\upsilon,\upsilon,\rho_{n-1}))\end{bmatrix} \\
&\le \gamma_1\left[\lim_{n\to\infty}S_T(\upsilon,\upsilon,\rho_n)\right]+\gamma_2\left[\lim_{n\to\infty}S_T(\upsilon,\upsilon,\rho_n)\right]+\gamma_3\left[\lim_{n\to\infty}S_T(\upsilon,\upsilon,\rho_{n-1})\right] \\
&= \gamma_1(0)+\gamma_2(0)+\gamma_3(0)=0.
\end{aligned}$$

Finally, we show that υ is a unique fixed point of T. On the contrary, suppose that T has two fixed points $\upsilon,t\in\Omega$ such that $\upsilon\neq t$. Using (3.1), we have

$$S_T(T\upsilon,T\upsilon,Tt)=S_T(\upsilon,\upsilon,t)\le hS_T(\upsilon,\upsilon,t),$$

a contradiction with $h\in(0,1)$. It should be $S_T(\upsilon,\upsilon,t)=0$, that is, $\upsilon=t$. □

Remark 3.2. *Let us take $\gamma_i(t)=t$ for $i\in\{1,2,3\}$ and $t\in[0,\infty)$, then we obtain classical Banach fixed-point theorem on S-metric spaces (see [14] for more details).*

Definition 3.3. *Let (Ω,S_T) be a TCSMS.*
(i) A covering of Ω is a collection of sets whose union is Ω.
(ii) An open covering of Ω is a collection of open sets whose union is Ω.
(iii) The TCSMS Ω is called compact if every open covering has a finite subcovering.

Theorem 3.4. *Let* (Ω, S_T) *be a TCSMS with the symmetric* S_T *and non-constant continuous control functions* $\gamma_i : [0,\infty) \to [0,\infty)$*,* $i \in \{1,2,3\}$*. Let* $T : \Omega \to \Omega$ *be a self-mapping satisfying*

$$S_T(T\rho, T\rho, T\omega) < S_T(\rho, \rho, \omega),$$

for all $\rho, \omega \in \Omega$ *and the sequence* $\{\rho_n\}$ *be defined as in Theorem 3.1. Suppose that the following conditions are satisfied:*

(a) γ_i *are continuous and non-decreasing functions with*

$$\sum_{i=1}^{3} \gamma_i(0) = 0$$

and γ_3 *is sub-additive.*

(b) $\lim_{m,n\to\infty} \left[\sum_{j=m}^{n-2} \gamma_3^{j-m} \gamma_1(S_T(\rho_0, \rho_0, \rho_1)) + \sum_{j=m}^{n-2} \gamma_3^{j-m} \gamma_2(S_T(\rho_0, \rho_0, \rho_1)) + \gamma_3^{n-m-1}(S_T(\rho_0, \rho_0, \rho_1)) \right] = 0,$

where $\gamma_3^{j-m}\gamma_1(S_T(\rho_0, \rho_0, \rho_1))$*,* $\gamma_3^{j-m}\gamma_2(S_T(\rho_0, \rho_0, \rho_1))$ *and* $\gamma_3^{n-m-1}(S_T(\rho_0, \rho_0, \rho_1))$ *are the composite functions.*

Then T *has a unique fixed point.*

Proof. For the existence point, we note that the map

$$\rho \longmapsto S_T(\rho, \rho, T\rho)$$

attems its minimum $\rho_0 \in \Omega$. Using the symmetry property, we get

$$\rho_0 = T\rho_0,$$

since otherwise

$$S_T(T(T\rho_0), T(T\rho_0), T\rho_0) < S_T(T\rho_0, T\rho_0, \rho_0) = S_T(\rho_0, \rho_0, T\rho_0),$$

a contradiction. It is easily seen the uniqueness of $\rho_0 \in \rho$. □

Remark 3.5. *Let us take* $\gamma_i(t) = t$ *for* $i \in \{1,2,3\}$ *and* $t \in [0,\infty)$*, then we obtain classical Nemytskii-Edelstein's fixed-point theorem on S-metric spaces (see [14] for more details).*

4. An Application to the System of Linear Equations

Let $\Omega = \mathbb{R}^n$ and the function $\|.\| : \mathbb{R}^n \to [0,\infty)$ be defined as

$$\|\rho\| = |\rho|,$$

for all $\rho \in \mathbb{R}^n$. Using the function S_T given in Example 2.7, we have

$$S_T(\rho, \omega, \upsilon) = |\omega + \upsilon - 2\rho| + |\omega - \upsilon|,$$

for all $\rho, \omega, \upsilon \in \mathbb{R}^n$ (similar function was given in [23] for S-metric spaces).
The function $S_T : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \to [0,\infty)$ is defined by

$$S_T(\rho, \omega, \upsilon) = \sum_{i=1}^{n} |\omega_i + \upsilon_i - 2\rho_i| + \sum_{i=1}^{n} |\omega_i - \upsilon_i|,$$

or all $\rho, \omega, \upsilon \in \mathbb{R}^n$, where

$$\rho = (\rho_1, \rho_2, \ldots, \rho_n),$$

$$\omega = (\omega_1, \omega_2, \ldots, \omega_n)$$

and

$$\upsilon = (\upsilon_1, \upsilon_2, \ldots, \upsilon_n).$$

Then S_T is a triple-composed S-metric with non-constant control functions $\gamma_i : [0,\infty) \to [0,\infty)$, $i \in \{1,2,3\}$ defined as

$$\gamma_1(t) = \gamma_2(t) = 2t \text{ and } \gamma_3(t) = t,$$

for all $t \in [0,\infty)$.
If

$$\sum_{i=1}^{n} \left|\alpha_{ij}\right| \leq h < 1 \ (1 \leq j \leq n),$$

then the system of linear equations

$$\begin{array}{rcl} \alpha_{11}\rho_1 + \alpha_{12}\rho_2 + \cdots + \alpha_{1n}\rho_n & = & \beta_1 \\ \alpha_{21}\rho_1 + \alpha_{22}\rho_2 + \cdots + \alpha_{2n}\rho_n & = & \beta_2 \\ \vdots & \vdots & \vdots \\ \alpha_{n1}\rho_1 + \alpha_{n2}\rho_2 + \cdots + \alpha_{nn}\rho_n & = & \beta_n \end{array}$$

has a unique solution. To show this, let us define the self-mapping T by

$$T\rho = A_\alpha \rho + \beta,$$

for all $\rho \in \mathbb{R}^n$, where

$$\beta = (\beta_1, \beta_2, \ldots, \beta_n)$$

and

$$A_\alpha = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \alpha_{n1} & \alpha_{n2} & \cdots & \alpha_{nn} \end{pmatrix}.$$

Then T satisfies the conditions of Theorem 3.1. Indeed, for $\rho, \omega \in \mathbb{R}^n$, we obtain

$$\begin{aligned} S_T(T\rho, T\rho, T\omega) &= 2\sum_{i=1}^{n} \left| \sum_{j=1}^{n} \alpha_{ij}(\rho_j - \omega_j) \right| \\ &\leq 2\sum_{i=1}^{n}\sum_{j=1}^{n} \left|\alpha_{ij}\right| \left|\rho_j - \omega_j\right| \\ &= 2\sum_{j=1}^{n}\sum_{i=1}^{n} \left|\alpha_{ij}\right| \left|\rho_j - \omega_j\right| \\ &= \sum_{j=1}^{n} 2\left|\rho_j - \omega_j\right| \sum_{i=1}^{n} \left|\alpha_{ij}\right| \leq h S_T(\rho, \rho, \omega). \end{aligned}$$

Consequently,the system of linear equations has a unique solution.

5. Conclusion and Future Works

In this study, we develop triple-composed S-metric spaces, a novel generalized metric space. We examine some fundamental characteristics of this new space and demonstrate two fundamental fixed-point theorems. To demonstrate the viability of the space we propose, we provide some illustrative cases and an application to the system of linear equations. Future research could establish some new generalized fixed-point theorems and examine the geometric characteristics of the fixed-point set on a triple-composed S-metric space. On the other hand, the concept of a triple-composed S_b-metric space can be defined as a new generalized metric space.

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