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ANALYZING URBAN GROWTH DYNAMICS FOR GEBZE: GEOGRAPHIC INFORMATION SYSTEMS AND AI-BASED PARAMETER OPTIMIZATION

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Highlights

- A fractional-order urban growth model is used to study Gebze's population dynamics.
- The model parameters are optimized through genetic algorithm techniques.
- Urban population growth scenarios are simulated to project urban growth of Gebze.
- Analysing changes in land use is important in sustainable landscape planning.

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ABSTRACT: This study aims to investigate the effect of population parameters on changes in urban land cover/use in the Gebze district for the years 2010, 2015, and 2024 through spatial analyses and a fractional-order mathematical model. The fractional-order mathematical model is applied to examine the dynamics of urban growth in the district, while model parameters are optimized via an artificial intelligence–based approach. Additionally, the potential urban growth scenarios for Gebze were simulated by MATLAB to project population growth for 2038. Subsequently, changes in land use for the specified years are calculated using Remote Sensing and Geographic Information Systems. In light of the results obtained, industrialisation, among the economic indicators, emerges as a key factor influencing urban population growth and land use change in the Gebze district. The expansion of industrial areas by 95.2% between 2010 and 2024 has driven a 41.2% increase in urban growth, while projections for 2038 estimate that the population of Gebze district will range between 600.000 and over 2.000.000. While the increase in population due to industrial development in the district by 2038 is expected to bring economic benefits, it may also lead to certain ecological and socio-cultural issues.

Keywords: *Artificial Intelligence-based Optimization, Genetic Algorithm, Land Use Change Analysis, Sustainable Planning, Urban Growth*

1. INTRODUCTION

The increase in job opportunities associated with industrialisation in urban areas, access to health and education facilities, and access to social and cultural activities are important factors encouraging urbanisation [1], [2]. These factors have particularly increased migration to cities in recent years. In 1950, only 30% of the world's population lived in urban areas, while this figure is projected to reach 57% by 2024. According to global population projections, it is estimated that approximately 68% of the world's population will live in cities by 2050 [3]. Cities with populations exceeding 10 million are referred to as 'megacities,' with the world's most populous megacities being Tokyo (Japan) with 38.505.00, Jakarta (Indonesia) 35.386.000, Delhi (India) 31.190.000, and Guangzhou (China) 27.119.000 [4]. The rapid increase in the population living in urban areas in recent years is challenging the 2030 Sustainable Development Goals (SDGs) adopted by the United Nations (UN) in 2015 [5], [6], [7]. SDG 11: Sustainable Cities and Communities is defined as the 11th goal of the 2030 SDGs and directly encompasses urbanisation. With rapid population growth, the need for housing in urban areas has increased, leading to income inequality and social division, as well as traffic, infrastructure, and environmental problems [8], [9]. In Turkey, urbanisation began to emerge after the 1950s. During this period, the mechanisation of agricultural activities led to an increase in industry and trade in cities, triggering migration from rural to urban areas. After the 1980s, urban growth accelerated with the establishment of organised industrial zones and the growth of the service sector. According to records from the Turkish Statistical Institute (TSI), approximately 44% of the population lived in cities in the 1980s, while this figure rose to 94% by

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2024.

Since there is a direct relationship between population growth and land use, any change will have a two-way effect. In areas with high population growth, land use classes and land cover are affected, and changes in land use also directly affect population distribution and density. People determine where they will live and work based on land use patterns, thereby influencing the structure and growth of the population in that area. Additionally, population growth and the increasing and diversifying needs of people result in pressure on urban ecosystems, leading to improper land use and the destruction of land resources. Improper land use and damage to natural resources pave the way for various biophysical, ecological, economic, and social problems [10], [11], [12]. For this reason, it is important to ensure the sustainable use of both natural and human resources in urban areas characterised by limited land and high population density. Understanding the temporal and spatial characteristics of changes in land use/cover is one of the first and most important steps to be taken in this regard [13], [14].

Population projections are an important parameter in sustainable urban planning, particularly in terms of estimating the pressures on ecological systems. In scenarios of intensive urbanisation, biodiversity loss, reduced carbon storage capacity and increased urban heat island effects emerge as key issues. Therefore, examining and evaluating population projections alongside ecological processes will reduce cities' vulnerability to climate change, create resilient cities, and enable the development of planning strategies compatible with nature-based solutions [15]. Another important aspect of population projections in terms of sustainable urban planning is that they guide social structures. Demographic forecasts enable the future demand for housing, health, transport, education and recreational areas to be met in an equitable and balanced manner. Especially in cities experiencing rapid population growth, reliable population projections play a crucial role in preventing social inequalities, ensuring spatial justice, and improving the quality of life in cities [16], [17].

When examining scientific studies related to determining changes in land use in urban areas and creating simulations of future land use, various models are used. These models are generally categorised as mathematical models, spatial stochastic models, process-based models, static, dynamic, and integrated models.

The fractional order mathematical modeling has become remarkably important due to characteristics of nonlocal operators like memory, heredity, flexibility, and crossover. As a result of their ability to capture complex dynamic behaviors, numerous researchers have focused on modeling real-world problems using fractional derivatives, including population growth dynamics [18], [19], [20]. To this end, a fractional-order urban growth model was recently developed that explicitly considers the costs associated with maintenance, taking advantage of power-law fading memory and the inherent flexibility of fractional calculus in capturing complex urban dynamics [21]. The pioneering fractional-order mathematical model, which extends the framework of Bettencourt et al. [22] by incorporating memory effects, has demonstrated a superior fit to urban growth data from multiple countries, including the USA, China, Singapore, Canada, Switzerland, and New Zealand. This model enables the determination of optimal parameters for various urban indicators and provides decision-makers with quantitative insights to support sustainable growth [21]. In parallel, artificial-intelligence-based parameter-optimization techniques, particularly genetic algorithms, have gained prominence in recent urban growth modeling, as they enhance model flexibility, yield accurate parameter estimates, and improve data fit [23],[24],[25],[26]. Combining fractional-order modeling with such optimization approaches offers a promising avenue for capturing the complex dynamics of urban growth, improving the fit to empirical data, and guiding evidence-based policy decisions.

Spatial stochastic models are models that use probabilistic rules to model geographic(spatial) events and incorporate randomness. As emphasised by Turner [27], spatially explicit stochastic and process-based simulation models are effectively used in the analysis of various landscape dynamics and biophysical processes [28]. Within this framework, the LUCAS (Land-Use Change Analysis System) was developed, consisting of three modules integrated through a common database. The model uses socio-economic parameters to predict potential changes in land cover and generates a Transition Probability

Matrix (TPM) that represents landscape dynamics [29], [30], [31]. Dynamic models, on the other hand, simulate land use changes over time and predict how these changes will evolve over time [32], [33]. Models based on fundamental natural or anthropogenic processes (water cycle, population growth, economic pressures, etc.) that influence changes in land use are defined as process-based models [34], [35]. Process-based models create simulations that are not only time-dependent but also more detailed in terms of space and interactions, based on cause-and-effect relationships. Since urban growth is influenced by various decisions and scales, process-based models are prominent in the creation of multi-faceted and detailed models. Among the process-based modelling approaches frequently used in urban planning, Cellular Automata (CA) models, Artificial Neural Networks (ANN), Logistic Regression Analysis, and Markov Chain-based methods stand out [34], [36].

This study aims to investigate the effect of population parameters on changes in urban land cover/use through spatial analysis and the fractional-order mathematical model. The study examines population changes in Gebze, an important industrial centre in Turkey, for the years 2010, 2015, and 2024. Land use changes for the specified years were calculated using Remote Sensing (RS) and Geographic Information Systems (GIS). In addition, to predict urban growth dynamics in Gebze, the fractional-order mathematical model is used, with model parameters optimized through the genetic algorithm-an artificial intelligence-based optimization technique implemented in MATLAB. This study is significant in that it examines in detail the urban growth processes of the region, one of Turkey's major industrial parks, for the first time using both a fractional-order model calibrated with an artificial intelligence-based optimization method and the integration of mathematical and geographic models using RS and GIS. It is anticipated that this study will shed light on future scientific research on the subject.

2. MATERIAL AND METHODS

In this study, Gebze district, located within the borders of Kocaeli province and one of the industrial centres of Turkey's Marmara Region, was selected as the study area (Figure 1). The district is also close to Istanbul, one of Turkey's most important provinces in terms of economic, cultural, historical, strategic and demographic aspects. Gebze is geographically located at 40°49'56′and 40°53'38′ north latitude, and 29°30'59′ and 29°25'24′ east longitude.

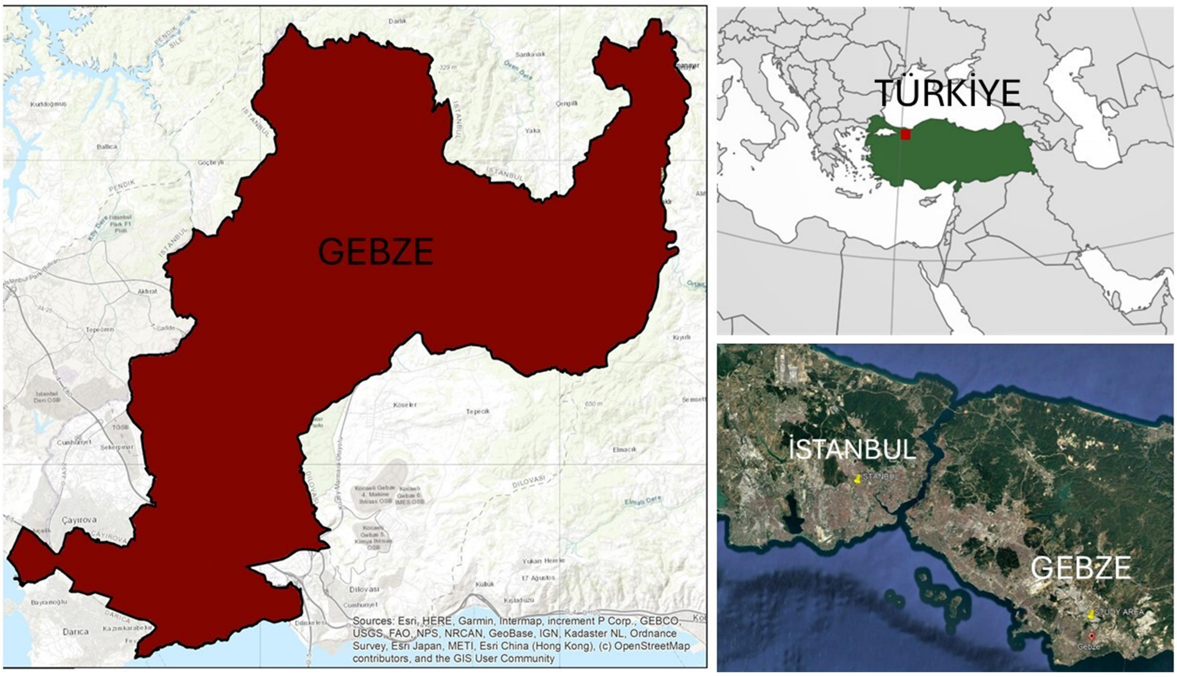

Figure 1. Location of study area

According to data from the Turkish Statistical Institute (TSI), the district's population was 305.557 in 2010, 350.115 in 2015, and 411.800 in 2024. The district, which has the transitional climate characteristics of the Marmara Region, is influenced by both the Mediterranean and Black Sea climates. Located on major roads, the district serves as a transit and buffer zone, particularly for people migrating from Anatolia to Istanbul.

Over the past 50 years, the annual increase in Istanbul's population and the intense urbanisation have led to the idea of moving new industrial and residential areas outside Istanbul. For this reason, Gebze has seen the construction of intensive industry and residential areas to meet this need. Organised Industrial Zones (OSB), established worldwide after the 1960's to support industrial development, have brought significant changes to urban land use. Gebze, which has Industrial Zones (OSB) in Turkey, is now known as an industrial city [37].

The methodology of this study, which examines the impact of population changes on land uses in Gebze city with the help of spatial analysis and the fractional-order mathematical model, consists of 3 stages. These stages are as follows:

1. Parameters optimization of the fractional-order mathematical model for urban growth dynamics using AI-based optimization approach and performing numerical simulations
2. Calculation of changes in land use through Geographic Information Systems and Remote Sensing
3. Interpretation of results

2.1. Fractional-Order Mathematical Model for Urban Growth Dynamics

The memory effect implies that when comparable changes occur repeatedly, the population growth rate may be influenced differently than in the urban growth model with classical derivative. It follows that the same growth value can correspond to a number of different previous time intervals. In addition, individuals living in the city are naturally influenced by their previous experiences in the decision-making process. To capture this behavior, Kee et al. [21] introduced the fractional order model with the

Caputo fractional derivative to incorporate a memory effect with power-law fading into the urban growth model designed by Bettencourt et al. [22].

In urban science, the power scaling that yields a natural relationship between the attribute Y of a city and its population N is given by

$$Y(t) = Y_0 N(t)^{\beta}, \tag{1}$$

where Y_0 represents a normalized constant, β is the scaling exponent, and t is the time scale of the population growth, usually in years [21], [38], [39], [40], [41]. According to an accounting principle, $Y(t) = R \times N(t) + E \times dN/dt$ is the result of a relationship between $N(t)$ and $Y(t)$. R represents the average maintenance cost per unit time per individual and E denotes the additional cost of adding a new individual. Following that, Kee et al. [21] generalized the urban growth model with classical derivatives to incorporate memory effects via the fractional derivative, as below:

$$ {}_{0}^{C}D_{t}^{\alpha} N(t) = \frac{1}{\tau}\left[\frac{Y_0}{R} N(t)^{\beta} - N(t)\right], \tag{2}$$

where the left Caputo fractional derivative is expressed as given in [42]:

$$ {}_{a}^{C}D_{t}^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \int_{a}^{t} \frac{f'(\vartheta)}{(t-\vartheta)^{\alpha}} d\vartheta, 0 < \alpha \leq 1. \tag{3}$$

A key feature of the model (2) included a new parameter, $0 < \alpha \leq 1$, to catch the degree of attenuation of memory for historical urban growth dynamics. If $\alpha < 1$, the model incorporates more memory effects, reflecting the influence of past urban dynamics on current growth processes, allowing optimal fitting to empirical population data over time by optimization. If $\alpha = 1$, the model recovers to an integer-order model without memory effects. Additionally, $\tau = E/R$ is a time scale used to measure the growth or maturity age at which an individual becomes productive for the urban system. It is determined through appropriate dimensional normalization. To investigate the dynamics of urban growth in the Gebze district based on empirical data from 2010, 2015, and 2024, the fractional-order mathematical model parameters are optimized with an AI-based optimization approach in MATLAB. Moreover, the potential urban growth scenarios in the future for Gebze are shown with various parameter-optimized sets. The flowchart in Figure 2. below summarizes the methodological sequence of phases in the process of examining population growth.

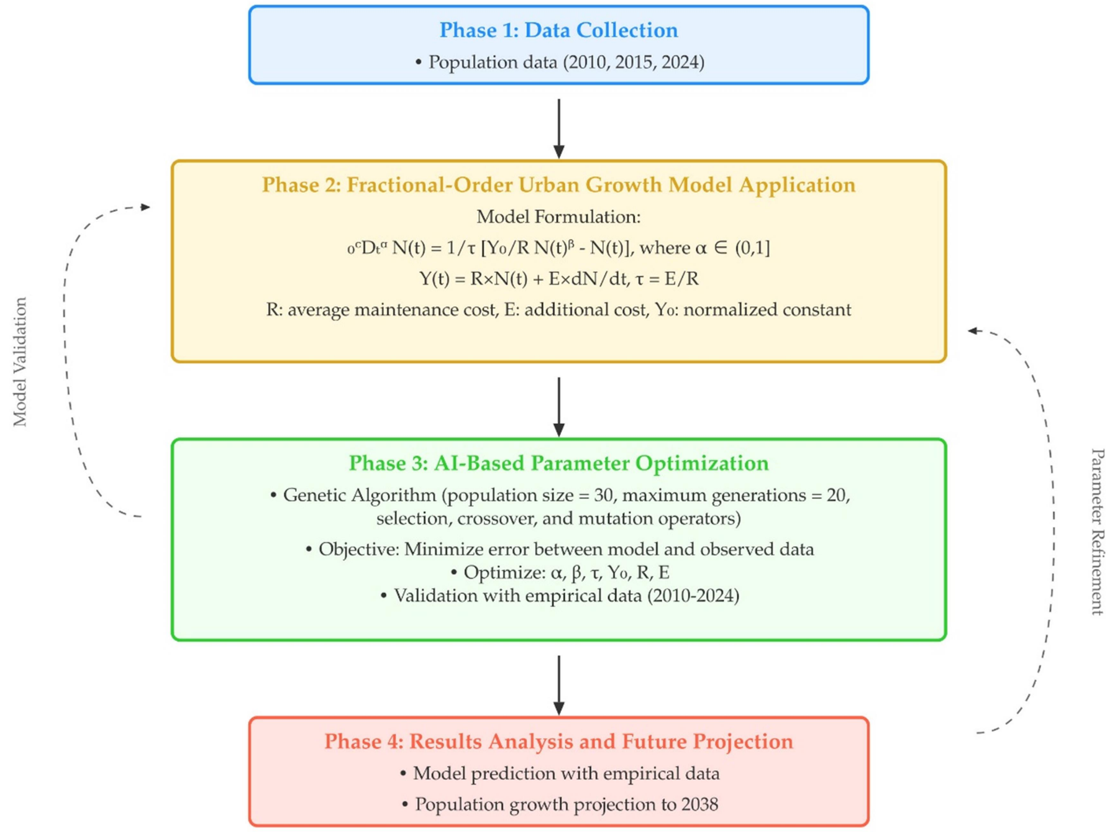

Figure 2. Methodological flowchart for examining urban growth in the Gebze district via a fractional-order mathematical model.

2.1.1. Parameter Optimization

In order to determine the model parameters and the order of fractional derivatives that best fit the model, parameter optimization is performed in MATLAB using the genetic algorithm (GA) via the ga function from the Global Optimization Toolbox [43]. GA represents a sophisticated approach within the computational intelligence subfield of artificial intelligence (AI), characterized by its evolutionary metaheuristic principles [44]. As a well-established AI-based optimization technique rooted in evolutionary computation [45], [46], [47], [48], and applicable to solving various real-world problems of complex nature [47], [48], [49], [50], [51], [52], GA mimics natural selection by iteratively evolving populations of potential solutions via selection, crossover, and mutation operations across multiple generations.

Unlike traditional gradient-based methods with local convergence properties, GA offers global optimization capabilities, making it suitable for predicting the dynamic behavior of urban growth in Gebze via a fractional-order model. For the parameter optimization problem in this model, GA is configured with a population size of 30 individuals and a maximum of 20 generations. These values are selected based on preliminary experiments, in which population sizes of 20, 30, and 40 and generation numbers of 10, 20, and 30 are tested on a representative subset of instances to achieve a balance between solution quality and computational efficiency. The objective function is specified as minimizing the mean squared error (MSE) between observed population data and model predictions. The parameter search ranges are carefully chosen with constraints applied to ensure numerically coherent results, such as maintaining the timescale parameter τ between 15 and 80, as given by Kee et al. [21].

The two-step approach described in [21] for optimizing the fractional-order urban growth model parameters is formalized in Algorithm 1, which outlines the process from initial linear regression to GA optimization.

Algorithm 1. Two-step parameter optimization for fractional order urban growth model

Input: Time series data $\{\boldsymbol{t_i}, \boldsymbol{N(t_i)}, \boldsymbol{Y(t_i)}\}$ where $\boldsymbol{i = 1, 2, \ldots, n}$
Output: Model parameters $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\tau}$, $\boldsymbol{Y_0}$, $\boldsymbol{R}$, $\boldsymbol{E}$

Step 1: Linear Regression for Estimation of $\boldsymbol{\beta}$ and $\boldsymbol{Y_0}$
1. Transform the data using logarithmic relationship:
   $\mathbf{ln}\, \boldsymbol{Y(t)} = \mathbf{ln}(\boldsymbol{Y_0}) + \boldsymbol{\beta}\, \mathbf{ln}\, \boldsymbol{N(t)}$
2. Compute $\mathbf{ln}\, \boldsymbol{N(t_i)}$ and $\mathbf{ln}\, \boldsymbol{Y(t_i)}$ for all data points
3. Perform linear regression on transformed data:
   - Fit $\mathbf{ln}\, \boldsymbol{Y(t_i)}$ against $\mathbf{ln}\, \boldsymbol{N(t_i)}$
   - Obtain slope $\boldsymbol{\beta^*}$ and intercept $\mathbf{ln}(\boldsymbol{Y_0^*})$
   - Calculate $\boldsymbol{Y_0^*} = \mathbf{exp}(\mathbf{ln}(\boldsymbol{Y_0^*}))$
4. Store estimated parameters $\boldsymbol{\beta^*}$ and $\boldsymbol{Y_0^*}$ for use in Step 2

Step 2: Genetic Algorithm Optimization for α, τ , and Y_0/R
1. Define parameter search ranges:
   - $\alpha \in [0.5, 0.99]$ %Fractional order
   - $\tau \in [15, 80]$ % Time scale (E/R)
   - $Y_0/R \in [0.1, 100]$ %Normalized rate
2. Define the objective function $F(\alpha, \tau, E/R)$:
   FUNCTION ObjectiveFunction$(\alpha, \tau, Y_0/R)$:
   a. Calculate $R = Y_0^* / (Y_0/R)$
   b. Calculate $E = \tau R$
   c. Solve the fractional-order urban growth model with initial condition $N(0) = N\_data(1)$ using predictor-corrector numerical method
   d. Compute model prediction $N(t_i)$ for each t_i
   e. Calculate MSE:
   $MSE = (1/n) \sum \big(N\_data(i) - N(t_i)\big)^2$
   f. Return MSE as the objective function value
   END FUNCTION
3. Initialize Genetic Algorithm:
   a. Set population size = 30
   b. Set maximum generations = 20
   c. Define selection, crossover, and mutation operators
4. Execute *ga* optimization:
   $[\alpha^*, \tau^*, (Y_0/R)^*] = ga(\text{ObjectiveFunction}, \text{parameter\_ranges})$
5. Calculate final parameters:
   a. $R^* = Y_0^* / ((Y_0/R)^*)$
   b. $E^* = \tau^* R^*$

Step 3: Evaluation and Validation
1. Solve the model using optimal parameters $\boldsymbol{\alpha^*}$, $\boldsymbol{\beta^*}$, $\boldsymbol{Y_0^*}$, $\boldsymbol{R^*}$, $\boldsymbol{E^*}$
2. Compare model prediction with real data:
   a. Plot model prediction $\boldsymbol{N(t)}$ vs. $\boldsymbol{N\_data(t_i)}$
   b. Calculate MSE
   c. Validate if $\boldsymbol{\tau^*} = \boldsymbol{E^*/R^*}$ is within reasonable bounds $[\mathbf{15}, \mathbf{80}]$

The optimization results yielded a set of parameters that characterize Gebze's urban growth dynamics. Table 1 below presents the optimized parameter values and the MSE, as well as the optimized maintenance cost (R) and additional cost (E) obtained through the implementation of Algorithm 1, derived using the fractional-order urban growth model.

Table 1. Optimized parameters of the fractional-order urban growth model with $\boldsymbol{\beta = 1.1667}$ and $\boldsymbol{Y_0 = 3}$ fitted to empirical population data of Gebze [53]. For each parameter set, the corresponding MSE value is quantified. *R* and *E* are measured in person-hours per person per year and person-hours per person, respectively.

α	τ	R	E	MSE
0.7613	54.0904	8.5714	463.6319	837.171289
0.7881	55.2642	8.5714	473.6931	633.217972
0.8158	59.8028	8.5714	512.5956	744.787227
0.8578	51.6232	9.7573	503.7035	680.715326
0.9320	72.5962	8.5714	622.2529	966.597708
0.9800	79.3841	8.5714	680.4353	1273.374267

2.2. Calculation of land use changes through Geographic Information Systems and Remote Sensing

Pattern changes in cities are caused by many factors such as population growth, natural disasters, and urbanisation. Measuring the impacts of these changes is complex and can be attributed to numerous interacting causes. Moreover, analysing their effects on ecosystem and socio-economic processes, in addition to predicting their future impacts through analyses conducted today, is crucial for guiding urban growth appropriately and for developing ecologically and economically viable and sustainable policies. In recent years, with the developments in GIS and RS techniques, there has been a shift from the 'monitor and repair' approach to the 'predict and prevent' approach in spatial planning studies. GIS and RS play a significant role in calculating spatial change in cities and taking measures against negative effects that may occur in the future [29], [31].

Image classification analysis is an important step in calculating the temporal and spatial changes in land cover/land use in RS. Although image classification can be divided into pixel-based and object-based methods, the Maximum Likelihood algorithm used in pixel-based image classification has been widely adopted by researchers as the most preferred method in the literature [54], [55].

In this study, the Maximum Likelihood algorithm was used to examine changes in land use in Gebze district between 2010, 2015 and 2024, and the images were divided into a total of six land use classes. These classes are forest, industry, water, agriculture, urban and other.

Satellite images obtained from the United States Geological Survey (USGS) Earth Explorer website were used to calculate the temporal and spatial changes in land use/cover in Gebze district in 2010, 2015 and 2024. The technical specifications of the images, which were formatted according to the UTM Zone 35N- WGS 1984 projection system, are presented in Table 2.

Table 2. Features of satellite images

Satellite	Sensor	Path-Row	Acquisition Date	Data Sources
Landsat 5	TM	181-032	2010/05/26	Landsat 4-5 TM C2 Level-2
Landsat 8	OLI-TIRS	181-032	2015/04/09	Landsat 8 OLI/TIRS C2 Level-2
Landsat 8	OLI-TIRS	181-032	2024/06/07	Landsat 8 OLI/TIRS C2 Level-2

3. RESULTS AND DISCUSSION

This section focuses on a fractional-order mathematical model and the results obtained from calculating land cover/use change to analyse the urban growth dynamics of Gebze.

3.1 Numerical Simulations

The urban population dynamics in Gebze from 2010 to 2024 are simulated in Figure 3 using the fractional-order urban growth model, fitted to empirical population data through an AI-based optimization approach [43], [53]. Simulations are generated by numerically solving the model with the Adams-type predictor-corrector method [56], following the optimization procedure described in Algorithm 1 in MATLAB. Optimized parameter sets, incorporating fractional derivative order to capture memory effects, are used to project population growth scenarios for 2038 (Figure 4).

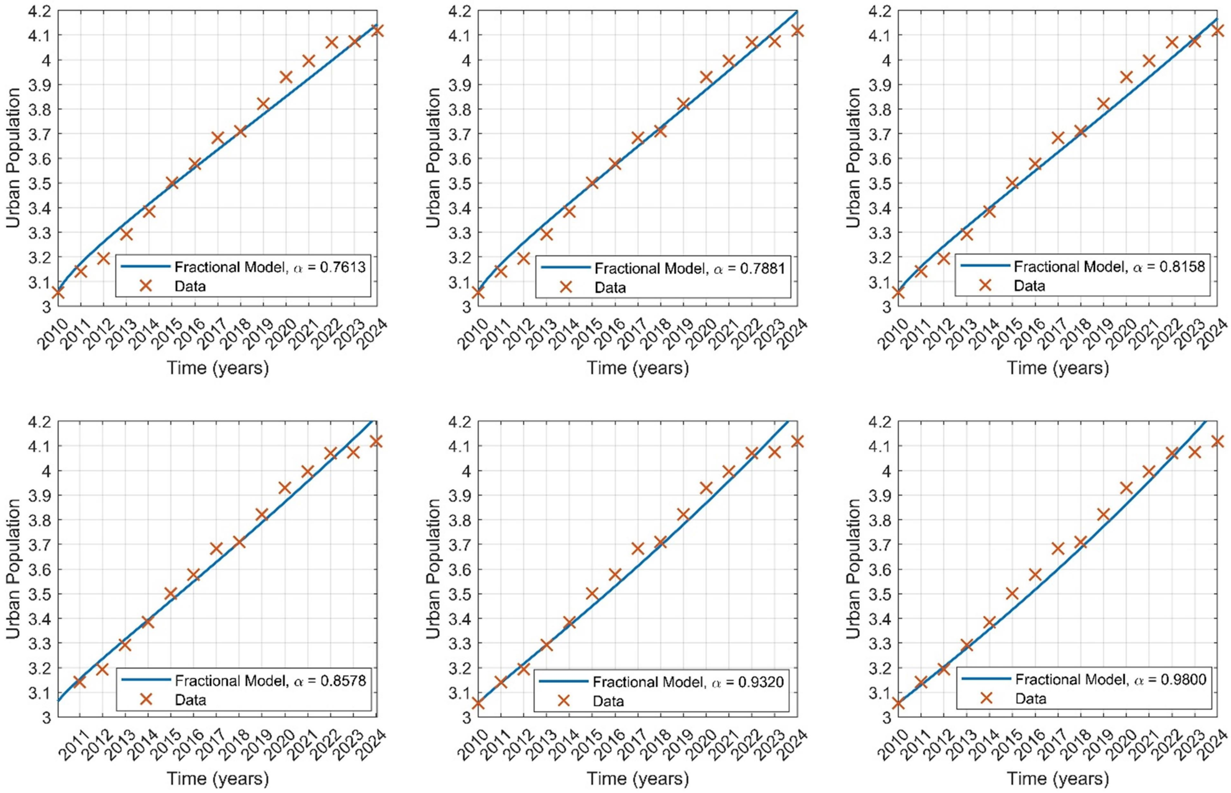

Figure 3. Fitted model to Gebze data between 2010 and 2024 by optimizing the model parameters.

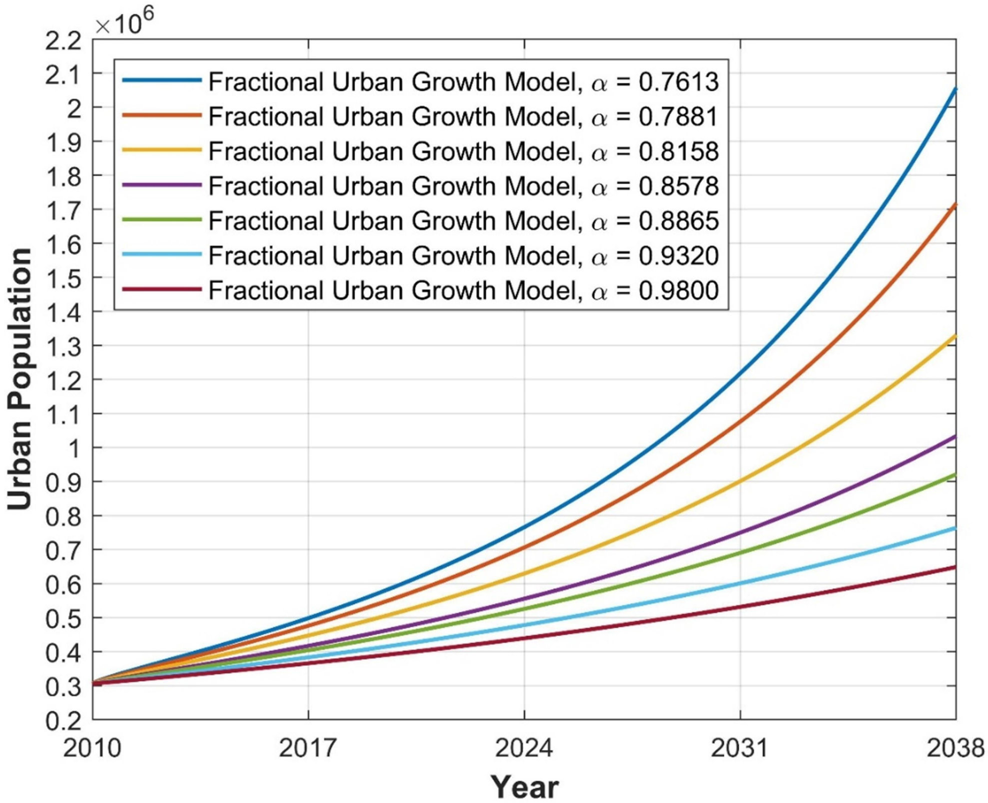

Figure 4. Simulations of future potential urban growth projections of Gebze until 2038 based on the values of the optimized parameters in Table 1.

In Figure 3, urban population dynamics in Gebze are simulated by fitting the fractional-order model to empirical population data between 2010 and 2024. Figure 4 demonstrates potential urban growth scenarios for Gebze until 2038 to investigate long-term demographic uncertainties, using the optimized parameter values in Table 1. It is observed that the MSE is lower for the optimization values where the fractional derivative is of lower order, meaning that the model is better able to capture the urban growth behavior (see Figure 4 and Table 1). Hence, it can be implied that the urban growth dynamics predicted with these lower-order values can better reveal the future course of the urban growth dynamics. This can also be attributed to the memory property of the fractional derivative, which naturally reflects the decision-making processes of an individual living in a city influenced by their past experiences. Moreover, it is observed that the MSE increases at optimization values where the order of the fractional derivative approaches one. Since the fitted model at high-order values shows greater deviation from the empirical population data, the future prediction may not be as effective as at low-order values. On the other hand, Gebze can be classified as boom/collapse growth according to Bettencourt et al.'s [22] classification for urban properties and their implications for growth, as the optimized scaling exponent ($\beta = 1.1667 > 1$ in Table 1) represents accelerating growth rates.

3.2 Result of Changes in Land Cover/Use

The land use maps of Gebze district classified using the Maximum Likelihood algorithm are shown in Figure 5.

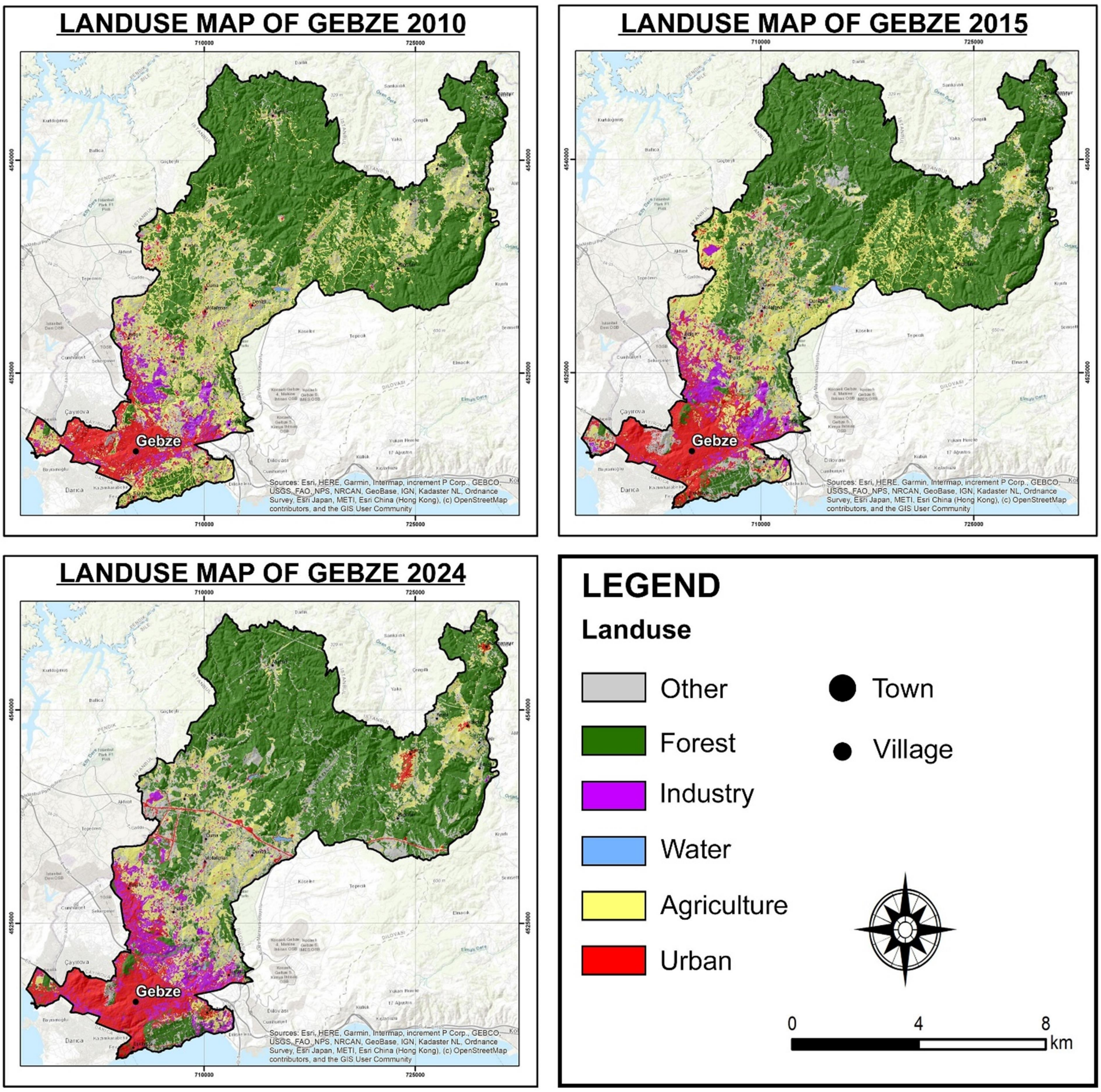

Figure 5. Classified LULC maps (2010, 2015, 2024)

When examining the temporal and spatial changes in land use in Gebze district (Table 3 and Table 4), it is observed that industrial areas increased by 23% between 2010 and 2015, by 72.1% between 2015 and 2024, and by 95.2% between 2010 and 2024. When population data is examined, the district's population increased by a total of 34.7% between 2010 and 2024. Gebze district, which has an important place in Turkey's Gross Domestic Product, is among the centres where urbanisation is intense. With the increase in industrialisation in the district, labour demand has emerged and housing demands have increased. Both industrial areas and construction increased by 41.2% between 2010 and 2024. The Organised Industrial Zones established in Gebze have contributed to the formation of new residential areas and the expansion of existing ones. The rapid urbanisation in the district has directly affected agricultural areas. It has been revealed that agricultural areas decreased by 24% between 2010 and 2015, by 22% between 2015 and 2024, and by 40.5% between 2010 and 2024.

Table 3. The area and changes of LULC classes (hectare)

Land Use Classes	2010	2015	2024
Other	4.340	4.800	6.684
Forest	22.609	23.281	22.695
Industry	1.622	1.996	3.167
Water	30	47	66
Agriculture	10.592	8.064	6.298
Urban	3.114	4.120	4.399
Total	42.308	42.308	42.308

Table 4. Area statistics of land use types in different periods in Gebze (hectare)

LULC	Amount of change (ha)			Percentage of Change (%)		
Classes	2010-2015	2015-2024	2010-2024	2010-2015	2015-2024	2010-2024
Other	+460	+1884	+2344	+10.6	+39.2	+48.8
Forest	+675	-1586	-919	+30.5	-6.8	-4.1
Industry	+374	+1171	+1545	+23	+72.1	+95.2
Water	+17	+19	+36	+56.6	+40.4	+76.5
Agriculture	-2528	-1766	-4294	-24	-22	-40.5
Urban	+1006	+279	+1285	+32.3	+6.7	+41.2

In this district, which has seen rapid growth in industry and population, urbanisation has increased not only in terms of housing but also in terms of shopping centres, entertainment and recreation areas linked to the service sector. In addition, progress has been made in the transport sector to accelerate trade, leading to an increase in roads and car parks. Urbanisation has not only affected agricultural areas but also forest areas, leading to significant changes in the district's ecological structure. It has been concluded that the forest areas of the district decreased by 30.5% between 2010 and 2015, by 6.8% between 2015 and 2024, and by 4.1% between 2010 and 2024. Scientific studies on the subject indicate that the main reasons for the decrease in forests in the district are changes in forest laws, the opening of some forest areas to construction, and forest fires. In particular, forest fires in 2020, 2023, and 2024 caused significant damage to forested areas. The increase in water surfaces in the district between 2010 and 2024 is attributed to the raising and expansion of the reservoir known as Denizli Reservoir, located within the study area. The primary reason for the raising and expansion of the Denizli Reservoir, which is used as irrigation water in agricultural areas, is the protocol signed in 2013 between the Kocaeli Metropolitan Municipality and the Organised Industrial Zones to make it available for industrial use as well.

Simulating the urban population of Gebze district between 2010 and 2024 using a fractional-order model and creating different population scenarios up to 2038 provides valuable insights into the analysis and interpretation of changes in land use during the same period. As described in [22], the scaling exponent ($\beta = 1.1667 > 1$) parameter in the fractional-order urban growth model reflects increasing returns with population size and is manifested by quantities associated with social currencies such as information, innovation, or wealth, which are related to cities' intrinsic social structure. This can also be considered a result of industrialisation, which is evident when examining Gebze's growth dynamics in recent years using its geographic information system. Furthermore, the 2038 scenarios presented with optimized parameters predict that Gebze will continue to grow.

In particular, the 2038 population scenario estimates will be effective in predicting how the district's spatial changes will be affected. Between 2010 and 2024, 1.545 ha of land in the district was converted into industrial areas, and during this period, the district's total population increased by 106.243 people. When examining the population scenarios for the district in 2038, seven different results emerge. According to the highest population scenario, represented by the blue curve in Figure 4, it is estimated that the population of Gebze district will exceed 2 million in 2038. If this scenario materialises, there will be a significant increase in housing, education, health, and infrastructure needs. Additionally, as urbanisation increases, there is a risk of increased pressure on the district's agricultural and forest ecosystems and the emergence of unplanned development. Gebze district, which had a total population of 411.800 in 2024, is projected to have an average population of 600.000 according to the 2038 red curve scenario. During the 24-year period between 2010 and 2024, 4.294 hectares of agricultural land and 919 hectares of forest land were lost. Even if the red curve scenario, which represents the lowest possible population growth by 2038, were to materialise, changes in the ecological, economic, and socio-cultural structure of the study area would be inevitable.

The estimated population scenarios for 2038, which emerged as a result of the analyses, are among the important parameters that need to be considered in Gebze's urban planning processes. The population pressure that arises with the increase in industrial areas indicates that urban development must be managed not only economically but also ecologically and socially. If the high population scenario materialises in the future, the increased demand for housing, education, healthcare and infrastructure will necessitate an effective and holistic approach to urban planning processes. Therefore, in order to counter the risk of uncontrolled and unplanned growth, it is crucial to adopt sustainable planning principles, protect the ecology, ensure the continuity of agricultural land, and strengthen green infrastructure systems in order to create a resilient city. Based on all this, urban growth is not only a spatial expansion but also a process that transforms social and ecological balances. Therefore, in meeting the new needs that will arise with population growth, planning approaches that are socially just and ecologically based should be adopted. Furthermore, involving the people of Gebze in decision-making mechanisms through participatory planning processes will not only strengthen social acceptance but also ensure that urban development is more inclusive and resilient. Therefore, it is imperative and critical that policies for Gebze's future be based on forward-looking, long-term strategies that strike a balance between economic growth and ecological sustainability. The balanced implementation of Gebze's socio-economic growth, while also preserving ecological integrity and implementing climate-friendly policies, must be carefully addressed in sustainable urban planning efforts.

4. CONCLUSIONS

The Industrial Revolution, which has emerged in the late 18th century, has spread from England to Europe and America, has caused changes in the world's economic and political systems, and has given rise to the concept of globalisation. With the dominance of mass production in the early 20th century, organised industrial zones have been established, and migration from rural areas to cities has begun in order to meet the labour demand in these zones. Towards the middle and end of the 20th century, the increase in the number of people living in cities has led to the emergence of housing, education, health, social and cultural needs, and new spaces have been created to meet these needs. The new spaces that have been built have caused changes in the urban land cover. With the beginning of the 21st century, urbanisation has become one of the most important issues discussed on a global scale.

In this study, the fractional-order urban growth model has been performed to examine the dynamics of population in Gebze, with model parameters and order of fractional derivative having been optimized by genetic algorithm, a well-established AI-based optimization technique. Since the optimized scaling exponent as $\beta = 1.1667 > 1$ has represented accelerating growth rates, Gebze could be classified as exhibiting boom/collapse growth. It has been concluded that the indicated growth trends may be a consequence of the increasing industrialisation of Gebze as a result of increasing returns with population size. Furthermore, the potential urban growth scenarios for Gebze have been demonstrated to project population growth for 2038. Analyses conducted through RS and GIS indicate that industrial activities, which have gained momentum particularly in recent years, are at the root of this population increase. The expansion of industrial areas has not only increased economic activity and employment but has also led to urban growth and changes in natural land cover. The shrinking of agricultural areas and forest ecosystems during this process has resulted in significant losses in terms of the biological diversity of the working area and its immediate surroundings, food security, and ecosystem services.

These changes over time pose multifaceted risks in the context of spatial planning and sustainability. In particular, the reduction in agricultural production areas puts pressure on the food supply of urban communities, while the shrinking of forested areas contributes to the weakening of carbon sinks and increased greenhouse gas emissions. This situation increases vulnerabilities in the fight against climate change and leads to a more intense urban heat island effect in the future. Furthermore, the reduction in natural and near-natural areas creates socio-ecological impacts that could have negative consequences on the health, well-being, and quality of life of people living in the region.

The study's population projections for 2038 indicate that the district could grow within a fairly wide demographic range in the future. These projected outcomes raise concerns about how urban development should be managed. A scenario of unplanned and uncontrolled growth could lead to ecosystem degradation, accelerated depletion of natural resources, and deepening social inequalities. In contrast, comprehensive planning approaches and efforts developed within the framework of sustainability principles, involving public institutions, civil society organisations and universities, can both support economic development and contribute to the preservation of ecological integrity. In this context, the adoption of socially and ecologically based strategies in decision-making processes regarding Gebze's future will not only reduce environmental threats but also play a significant role in increasing social resilience and quality of life

Investigating the relationship between population growth and land use is one of the important stages in urban planning. Creating future population simulations plays a major role in shaping the physical, economic and social structure of cities in a healthy manner, strengthening infrastructure, managing disasters and risks, and planning education, health and transportation systems. Additionally, identifying changes in land cover/use and interpreting future population simulations together are effective in creating sustainable and resilient cities.

Declaration of Ethical Standards

The authors conducted this study in accordance with all ethical standards.

Credit Authorship Contribution Statement

Author-1: Conceptualization, Resources, Methodology, Investigation, Writing-Review&Editing,

Software, Visualization, Supervision.

Author-2: Methodology, Investigation, Formal Analysis, Writing-Original draft preparation, Software, Visualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data Availability

The authors confirm that the data supporting the findings of this study are contained in the paper. If additional data is required, the corresponding author can provide it upon reasonable request.

Use of Artificial Intelligence

The authors used generative artificial intelligence (AI) tools solely to improve the language quality and readability of this manuscript. No AI tools were used to generate scientific content, data, analyses, or interpretations. The authors remain fully responsible for the accuracy, originality, and integrity of the manuscript.

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