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A Novel Q-Rung Orthopair Fuzzy Distance Metric for Energy Source Selection

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Abstract

The process of selecting a dependable energy source is essential because of the adverse effects of adopting an inefficient energy source. This research presents the notion of Q-Rung Orthopair Fuzzy Sets (Q-ROFSs) embedded with a distance metric to support the selection of an eco-friendly energy source, since the theory of Q-ROFSs is a reliable means for decision-making. An efficient Q-Rung Orthopair Fuzzy Distance Measure (Q-ROFDM) is constructed in this research. The novel Q-ROFDM is described to show superiority over the extant Q-ROFDMs. In addition, the novel distance technique is utilized in the selection of an efficient energy source alternative from among several energy source alternatives using assorted energy source selection criteria based on decision-making methods. Finally, to buttress the effectiveness of the novel Q-ROFDM, a comparative analysis is shown, presenting the strength and reliability of the new technique over the extant Q-ROFDMs. This efficient technique of energy source selection can be adopted in other uncertainty fields due to its effectiveness.

Keywords: Energy source alternatives, Q-rung Orthopair fuzzy sets, Energy source selection, Q-rung Orthopair fuzzy distance metric, Decision-making.

1 | Introduction

An energy source is any means through which electricity can be produced or heat can be generated [1]. In this day and age, many sustainable energy technologies are necessary to substitute the orthodox electricity generation means like fossil fuel because of the global demands, particularly in developed and developing nations [2]. Fossil fuel-based energy sources are the major causes of global warming. The emission of

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greenhouse gas into the open air has geometrically increased in recent decades [3]. For this reason, renewable energy sources such as biomass energy, solar energy, wind energy, hydropower, and geothermal energy have been used to generate electricity, ameliorating global warming [4]. Roughly speaking, energy sources are classified into renewable sources and non-renewable sources. The renewable sources are biomass energy, solar energy, wind energy, hydropower, and geothermal energy. The nonrenewable sources consist of hydrocarbon fuels (Such as methane gas, fossil fuels, and crude oil) and nuclear energy [5]. These energy sources are crucial for fulfilling basic human needs and supporting modern life.

However, identifying a preferred eco-friendly energy source presents enormous challenges, including climatic and geographical constraints, sometimes with high initial investment costs, intermittency issues and energy storage, regulatory barriers, maintenance, technological complexities, and spatial requirements. Selecting more eco-friendly energy sources is inherently complex due to the fuzziness involved in the Decision-Making Process (DMP), which emanates from the uncertainties and the hybrid energy approach to proffering long-term energy and sustainability security.

It is imperative to note that these factors introduce vague and imprecise parameters, thereby making traditional crisp decision-making models inadequate. To navigate these uncertainties, Fuzzy Set Theory (FST) offers a more effective approach by permitting a Degree of Membership (DM) rather than strict classification [6]. Unlike the classical binary logic, which groups energy sources as either "fitting" or "unfitting," fuzzy sets accommodate partial truths, enabling more flexible and adaptive decision-making.

However, FST may not be able to handle many complex problems because it makes use of a single DM. It is on this premise that Q-Rung Orthopair Fuzzy Sets (Q-ROFSs), which were presented in [7], stand a better chance because the concept provides greater flexibility in handling uncertainty by letting the addition of the qth power of DM and Degree of Non-Membership (DNM) not surpass one, enabling more nuanced decision-making in complex environments. Numerous applications of Q-ROFSs have been discussed via Q-Rung Orthopair Fuzzy Distance Measure (Q-ROFDMs), several of which are deficient in terms of accuracy, fulfillment of distance axioms, omission of hesitation margins of the Q-ROFSs, and reliability. These lapses justify the essence of this work, which presents a new Q-ROFDM based on a 3-dimensional approach, whose performances outweigh the existing techniques.

2 | Literature Review

The ordinary data is insufficient to model real-life problems, and vague ideas are often exemplified in multi-criteria decision aid analysis [8]. In fact, it is challenging to handle the assessment criteria accurately and to fix precise preference grades. This is where the idea of Q-ROFSs comes in to treat this type of ambiguity in decision-making. Many selection problems have been resolved based on soft computing devices like Fermatean Fuzzy Sets (FFSs), Pythagorean Fuzzy Sets (PFSs), Intuitionistic Fuzzy Sets (IFSs), Q-ROFSs, etc., which are the advanced forms of the FST.

Atanassov [9] initiated the concept of IFSs by considering both DNM and DM, such that their addition does not exceed one. Due to the usefulness of IFS, it has been used to discuss practical human problems, namely, recognition of patterns [10–14], diagnostic analysis [12–15], emergency control [16], [17] and decision making [18–21].

Nonetheless, the limitation of IFSs to model cases where the addition of DNM and DM exceeds one prompted the introduction of PFSs by [22], which permits the sum of DM and DNM to exceed one, with a caveat that the summation of the square of DNM and DM must not exceed one. PFSs have been applied to so many practical problems like selection problems [23], [24] decision making [25–30], transportation problems [31] pattern recognition [32], [33] etc.

In the same way, the deficiency of PFS in the sense that the summation of the square of DNM and DM must not exceed one prompted ref. Senapati and Yager [34] to develop the FFSs, which permits the addition of the square of DNM and DM to exceed one, with a caveat that the addition of the cube of DNM and DM must not exceed one. The applications of FFSs have been discussed in diagnostic medicine [35–37] Multi-Criteria Decision-Making (MCDM) [38–40] the admission process [41] insecurity evaluation [42–44] clustering analysis [45] and transportation problems [46], [47].

To efficiently address the setbacks of IFSs, PFSs, and FFSs, Yager [7] introduced Q-ROFSs, which generalized IFSs, PFSs, and FFSs, respectively. Because Q-ROFS is appropriate for the resolution of complex soft computing problems, it has been utilized in healthcare [48–52], decision making [53–58], MCDM [59–61], etc. The idea of Q-ROFSs has been applied in DMP using the TOPSIS approach [59].

The Multi-Attribute Decision Making (MADM) is a crucial component of operational research and modern decision science, which makes preference decisions via evaluation and prioritization of finite alternatives through multiple conflict attributes, and techniques have been utilized in many decision-making problems under the Q-ROF setting [62–65]. In the same vein, varied variants of fuzzy systems have been used to discuss medical uncertainty [66–68] and optimization of climate change [69].

Many applications of Q-ROFSs have been buttressed based on Q-ROFDMs. The Q-ROFDM evaluates the distance between any two arbitrary Q-ROFSs. Du [70] developed a Q-ROFDM using the Minkowski distance metric and used it to discuss practical problems. Although the method of distance measure included all the parameters of Q-ROFSs, it does not present the property of Q-ROFSs.

Peng and Liu [71] presented some Q-ROFDMs with applications, but some of the methods omitted the Q-Rung Orthopair Fuzzy Hesitation Margins (Q-ROFHMs), which are significant in decision-making. Pinar and Boran [72] developed a Q-ROFDM using the Lp norm and utilized it in supplier selection via the MCDM method.

However, its results can be misleading because the Q-ROFHMs are discarded. Some distance methods between Q-ROFSs were presented in [73] based on the weighted induced logarithmic function with utilization in MCDM. Yin et al. [74] discussed some aggregation operators and distance methods between the hybrid sets of Q-ROFSs and probabilistic hesitant fuzzy sets, Liu et al. [75] presented some cosine similarity and dissimilarity methods for complex Q-ROFSs, and Kamacı and Petchimuthu [76] presented some Soergel-based Q-ROFDMs and buttressed their applications in real-life problems.

Hussain et al. [77] presented some Q-ROFDMs via the Hausdorff metric and applied them in decision-making problems. However, the methods are lacking because they are based on extreme values and disregard Q-ROFHMs. Ejegwa [78] constructed an efficient Q-ROFDM and applied it to discuss clustering analysis, investment analysis, and pattern recognition.

In addition, Ali [79] presented a distance method for Q-ROFSs and used it to discuss DMP of energy sources. Hussain et al. [80] constructed some dissimilarity and similarity methods under Q-ROFSs. The approaches in Aggregation operators and distance measures for probabilistic Q-Rung Orthopair hesitant fuzzy sets and their applications [74], [75], [77] were used to discuss varied applications.

Rani and Kumar [81] presented a sine-based distance measure method for Q-ROFSs and used it to discuss MCDM. The accuracy of the distance value from this method could be affected because the method is based on 2-dimensional and maximum values. Wang et al. [82] presented 2-dimensional and 3-dimensional logarithmic-based distance measure methods and applied them in pattern recognition and DMP via the MCDM technique. But the outcome of the 2-dimensional approach cannot be trusted due to information loss.

Dutta et al. [83] proposed a method of distance measure between Q-ROFSs by considering incomplete Q-ROF parameters and applied the method in the discussion of transportation problems. The fact that this method does not consider the complete Q-Rung Orthopair fuzzy parameters means its outcome cannot be trusted for reliable interpretation. Suri et al. [84] developed a logarithmic distance metric for Q-ROFSs and used it for the selection of financial investment funds via a decision-making algorithm. The method is deficient in the sense that it discards the impact of the Q-ROF index, which is essential for reliable decision-making. Finally, Basu et al. [85] presented a Q-ROFDM via score function and similarity measure and used it to discuss practical decision-making.

2.1 | Motivation and Contributions

Because of the significance of a Q-ROFDM, several approaches of Q-ROFDMs have been constructed and utilized in different areas, viz., transportation problem, selection analysis, investment analysis, clustering analysis, pattern recognition, etc. [70-85]. The majority of the existing Q-ROFDMs are deficient in terms of accuracy, fulfillment of distance axioms, omission of hesitation margins of the Q-ROFSs, and reliability. The lapses of the existing Q-ROFDMs are the motivation for this article, which presents a new Q-ROFDM based on a 3-dimensional approach and the elements' weights.

In addition, the concept of Q-ROFSs is considered for energy source selection over IFSs, PFSs, and FFSs because it is more flexible and adaptable to many cases of decision-making. In a few words, the aim of this work is the development of an innovative Q-ROFDM and its utilization in the selection process of an eco-friendly energy source by considering many energy source alternatives alongside several criteria. The article seeks to contribute in the following ways:

I. Construction of an innovative 3-dimensional Q-ROFDM, which is well structured to enhance reliability.

II. Description of the innovative 3-dimensional Q-ROFDM based on the distance metric conditions for the aim of validation.

III. Utilization of the innovative 3-dimensional Q-ROFDM to enhance proficient selection of eco-friendly energy source alternatives based on Multi Criteria Group Decision Making (MCGDM) and MADM, respectively.

IV. Comparative analysis of the innovative 3-dimensional Q-ROFDM to buttress its advantages over the existing Q-ROFDMs.

In summary, Section 1 presents the introduction, literature review, and motivation for the work. Section 2 discusses Q-ROFSs and the existing Q-ROFDMs. Section 3 discusses the innovative 3-dimensional Q-ROFDM and elaborates on its metric properties. Section 4 discusses the use of the innovative 3-dimensional Q-ROFDM in the selection process of an eco-friendly energy source alternative based on MCGDM and MADM techniques and presents a comparative analysis for the purpose of validation. Finally, Section 5 concludes the article with suggestions for further investigations.

3 | Preliminaries

A number of basic concepts relating to Q-ROFSs, distance measures under Q-ROFSs, and various existing Q-ROFDMs are presented in this section, which will enhance our discussion in the upcoming sections. All through this work, $\mathfrak{D} = \{b_1, b_2, \ldots, b_k\}$ is taken as a nonempty set with as its cardinality, and ROFS($\mathfrak{D}$) is the collection of all the possible Q-ROFSs in $\mathfrak{D}$.

Definition 1 ([7]). A Q-ROFS $\mathfrak{Y}$ on $\mathfrak{D}$ is described as follows:

$$\mathfrak{Y} = \{(b_i, \mathfrak{Y}_m(b_i), \mathfrak{Y}_n(b_i)) | b_i \in \mathfrak{D}\}, \tag{1}$$

where $\mathfrak{Y}_m, \mathfrak{D} \to [0,1]$, is the MD and $\mathfrak{Y}_n, \mathfrak{D} \to [0,1]$, is the NMD such that $0 \le \mathfrak{Y}_m^q(b_i) + \mathfrak{Y}_n^q(b_i) \le 1$ for all $b_i \in \mathfrak{D}$ and $q \ge 1$. The HD is denoted as $\mathfrak{Y}_h$ defined by $\mathfrak{Y}_h(b_i) = \left(1 - \mathfrak{Y}_m^q(b_i) - \mathfrak{Y}_n^q(b_i)\right)^{\frac{1}{q}}$ is the degree of non-determinacy of $b_i \in \mathfrak{D}$, to the set $\mathfrak{Y}$ and $\mathfrak{Y}_h(b_i) \in [0,1]$, where $i = 1, 2, \ldots, k$. For easiness, $\mathfrak{Y} = (\mathfrak{Y}_m(b_i), \mathfrak{Y}_n(b_i))$ or $\mathfrak{Y} = \langle \mathfrak{Y}_m(b_i), \mathfrak{Y}_n(b_i) \rangle$ is taken as the Q-ROF number (Q-ROFN).

Definition 2 ([7]). Given two Q-ROFNs $\mathfrak{Y} = (\mathfrak{Y}_m(b_i), \mathfrak{Y}_n(b_i))$ and $\hat{\mathfrak{Y}} = \left(\hat{\mathfrak{Y}}_m(b_i), \hat{\mathfrak{Y}}_n(b_i)\right)$, we present some basic operations on Q-ROFSs as follows:

I. $\mathfrak{Y}^c = (\mathfrak{Y}_n(\ell_i), \mathfrak{Y}_m(\ell_i))$ and $\hat{\mathfrak{Y}}^c = (\hat{\mathfrak{Y}}_n(\ell_i), \hat{\mathfrak{Y}}_m(\ell_i))$.

II. $\mathfrak{Y} = \hat{\mathfrak{Y}}$ if and only if, for all $\ell_i \in \mathfrak{D}$, $\mathfrak{Y}_m(\ell_i) = \hat{\mathfrak{Y}}_m(\ell_i)$ and $\mathfrak{Y}_n(\ell_i) = \hat{\mathfrak{Y}}_n(\ell_i)$.

III. $\mathfrak{Y} \cup \hat{\mathfrak{Y}} = (\max\{\mathfrak{Y}_m(\ell_i), \hat{\mathfrak{Y}}_m(\ell_i)\}, \min\{\mathfrak{Y}_n(\ell_i), \hat{\mathfrak{Y}}_n(\ell_i)\})$.

IV. $\mathfrak{Y} \cap \hat{\mathfrak{Y}} = (\min\{\mathfrak{Y}_m(\ell_i), \hat{\mathfrak{Y}}_m(\ell_i)\}, \max\{\mathfrak{Y}_n(\ell_i), \hat{\mathfrak{Y}}_n(\ell_i)\})$.

V. $\mathfrak{Y} \subseteq \hat{\mathfrak{Y}}$ if and only if, for all $\ell_i \in \mathfrak{D}$, $\mathfrak{Y}_m(\ell_i) \leq \hat{\mathfrak{Y}}_m(\ell_i)$ and $\mathfrak{Y}_n(\ell_i) \geq \hat{\mathfrak{Y}}_n(\ell_i)$.

VI. $\mathfrak{Y} \otimes \hat{\mathfrak{Y}} = \left(\mathfrak{Y}_m^q(\ell_i)\hat{\mathfrak{Y}}_m^q(\ell_i),\ \left(\mathfrak{Y}_n^q(\ell_i) + \hat{\mathfrak{Y}}_n^q(\ell_i) - \mathfrak{Y}_n^q(\ell_i)\hat{\mathfrak{Y}}_n^q(\ell_i)\right)^{1/q}\right)$.

VII. $\mathfrak{Y} \oplus \hat{\mathfrak{Y}} = \left(\left(\mathfrak{Y}_m^q(\ell_i) + \hat{\mathfrak{Y}}_m^q(\ell_i) - \mathfrak{Y}_m^q(\ell_i)\hat{\mathfrak{Y}}_m^q(\ell_i)\right)^{1/q},\ \mathfrak{Y}_n^q(\ell_i)\hat{\mathfrak{Y}}_n^q(\ell_i)\right)$.

Definition 3 ([70]). Let $\mathfrak{Y}$, $\hat{\mathfrak{Y}}$, $\tilde{\mathfrak{Y}}$ ∈Q-ROFS ($\mathfrak{D}$) and $d\colon \mathrm{Q-ROFSs}(\mathfrak{D}) \times \mathrm{Q-ROFSs}(\mathfrak{D}) \to [0,1]$, then $d(\mathfrak{Y}, \hat{\mathfrak{Y}})$ is a distance metric between $\mathfrak{Y}$ and $\hat{\mathfrak{Y}}$ if it fulfills the following conditions:

I. $0 \leq d(\mathfrak{Y}, \hat{\mathfrak{Y}}) \leq 1$.

II. $d(\mathfrak{Y}, \hat{\mathfrak{Y}}) = 0$ iff $\mathfrak{Y} = \hat{\mathfrak{Y}}$.

III. $d(\mathfrak{Y}, \hat{\mathfrak{Y}}) = d(\hat{\mathfrak{Y}}, \mathfrak{Y})$.

IV. $d(\mathfrak{Y}, \tilde{\mathfrak{Y}}) \leq d(\mathfrak{Y}, \hat{\mathfrak{Y}}) + d(\hat{\mathfrak{Y}}, \tilde{\mathfrak{Y}})$.

Definition 4. Let $\mathfrak{Y}$, $\hat{\mathfrak{Y}}$ ∈Q-ROFS($\mathfrak{D}$). Then, the weights of $\mathfrak{D} = \{\ell_1, \ell_2, \dots, \ell_k\}$ in $\mathfrak{Y}$ and $\hat{\mathfrak{Y}}$ are defined in *Eq. (1)*.

$$\varpi_i = \frac{\frac{3\mathfrak{Y}_m^q(\ell_i) + \mathfrak{Y}_n^q(\ell_i) + \mathfrak{Y}_h^q(\ell_i)}{3} + \frac{3\hat{\mathfrak{Y}}_m^q(\ell_i) + \hat{\mathfrak{Y}}_n^q(\ell_i) + \hat{\mathfrak{Y}}_h^q(\ell_i)}{3}}{\sum_{j=1}^{k}\left(\frac{3\mathfrak{Y}_m^q(\ell_i) + \mathfrak{Y}_n^q(\ell_i) + \mathfrak{Y}_h^q(\ell_i)}{3} + \frac{3\hat{\mathfrak{Y}}_m^q(\ell_i) + \hat{\mathfrak{Y}}_n^q(\ell_i) + \hat{\mathfrak{Y}}_h^q(\ell_i)}{3}\right)}.$$

For $\varpi_i \in [0,1]$ and $\sum_{i=1}^{k} \varpi_i = 1$.

3.1 | Existing Distance Measures on Q-Rung Orthopair Fuzzy Sets

Next, we outline some existing Q-ROFDMs. Given two Q-ROFSs, $\mathfrak{Y}, \hat{\mathfrak{Y}}$ ∈Q-ROFS ($\mathfrak{D}$), the following Q-ROFDMs are recalled in *Table 1*.

Table 1. Existing Q-ROFDMs.

Q-ROFDMs	Distance Schemes
Du [70]	$d_D(\mathfrak{Y}, \hat{\mathfrak{Y}}) = \left(\frac{1}{3k}\sum_{i=1}^{k}\left(\begin{matrix}\lvert\mathfrak{Y}_m(\ell_i) - \hat{\mathfrak{Y}}_m(\ell_i)\rvert^p + \lvert\mathfrak{Y}_n(\ell_i) - \hat{\mathfrak{Y}}_n(\ell_i)\rvert^p \\ +\lvert\mathfrak{Y}_h(\ell_i) - \hat{\mathfrak{Y}}_h(\ell_i)\rvert^p\end{matrix}\right)\right)^{1/p}$, where $p \geq 1$.
Peng and Liu [71]	$d_{PL1}(\mathfrak{Y}, \hat{\mathfrak{Y}}) = \frac{1}{k}\sum_{i=1}^{k}\left(\max\{\lvert\mathfrak{Y}_m^q(\ell_i) - \hat{\mathfrak{Y}}_m^q(\ell_i)\rvert, \lvert\mathfrak{Y}_n^q(\ell_i) - \hat{\mathfrak{Y}}_n^q(\ell_i)\rvert\}\right)$, $d_{PL2}(\mathfrak{Y}, \hat{\mathfrak{Y}}) = \frac{2}{k}\sum_{i=1}^{k}\frac{\max\{\lvert\mathfrak{Y}_m^q(\ell_i) - \hat{\mathfrak{Y}}_m^q(\ell_i)\rvert, \lvert\mathfrak{Y}_n^q(\ell_i) - \hat{\mathfrak{Y}}_n^q(\ell_i)\rvert\}}{1 + \max\{\lvert\mathfrak{Y}_m^q(\ell_i) - \hat{\mathfrak{Y}}_m^q(\ell_i)\rvert, \lvert\mathfrak{Y}_n^q(\ell_i) - \hat{\mathfrak{Y}}_n^q(\ell_i)\rvert\}}$,

Table 1. Continued.

Q-ROFDMs	Distance Schemes
Peng and Liu [71]	$d_{PL3}(\mathfrak{Y},\hat{\mathfrak{Y}}) = \frac{2\sum_{i=1}^{k}\left(\max\left\{\left\|\mathfrak{Y}_m^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\right\|,\left\|\mathfrak{Y}_n^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\right\|\right\}\right)}{\sum_{i=1}^{k}\left(1+\max\left\{\left\|\mathfrak{Y}_m^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\right\|,\left\|\mathfrak{Y}_n^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\right\|\right\}\right)}$, $d_{PL4}(\mathfrak{Y},\hat{\mathfrak{Y}}) = 1-\beta\left(\frac{\sum_{i=1}^{k}\min\{\|\mathfrak{Y}_m^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\|\}}{\sum_{i=1}^{k}\max\{\|\mathfrak{Y}_m^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\|\}}\right)-\gamma\left(\frac{\sum_{i=1}^{k}\min\{\|\mathfrak{Y}_n^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\|\}}{\sum_{i=1}^{k}\max\{\|\mathfrak{Y}_n^q(\eth_i),\hat{\mathfrak{Y}}_n^q(\eth_i)\|\}}\right)$, $d_{PL5}(\mathfrak{Y},\hat{\mathfrak{Y}}) = 1-\frac{\beta}{k}\left(\frac{\sum_{i=1}^{k}\min\{\|\mathfrak{Y}_m^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\|\}}{\sum_{i=1}^{k}\max\{\|\mathfrak{Y}_m^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\|\}}\right)-\frac{\gamma}{k}\left(\frac{\sum_{i=1}^{k}\min\{\|\mathfrak{Y}_n^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\|\}}{\sum_{i=1}^{k}\max\{\|\mathfrak{Y}_n^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\|\}}\right)$, for $\beta,\gamma\in[0,1]$ and $\beta+\gamma=1$, $d_{PL6}(\mathfrak{Y},\hat{\mathfrak{Y}}) = 1-\frac{1}{k}\sum_{i=1}^{k}\left(\frac{\min\{\mathfrak{Y}_m^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\}+\min\{\mathfrak{Y}_n^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\}}{\max\{\mathfrak{Y}_m^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\}+\max\{\mathfrak{Y}_n^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\}}\right)$, $d_{PL7}(\mathfrak{Y},\hat{\mathfrak{Y}}) = 1-\frac{1}{k}\sum_{i=1}^{k}\left(\frac{\min\{\mathfrak{Y}_m^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\}+\min\{1-\mathfrak{Y}_n^q(\mathscr{b}_i),1-\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\}}{\max\{\mathfrak{Y}_m^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\}+\max\{1-\mathfrak{Y}_n^q(\mathscr{b}_i),1-\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\}}\right)$, $d_{PL8}(\mathfrak{Y},\hat{\mathfrak{Y}}) = 1-\frac{\sum_{i=1}^{k}\left(\min\{\mathfrak{Y}_m^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\}+\min\{\mathfrak{Y}_n^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\}\right)}{\sum_{i=1}^{k}\left(\max\{\mathfrak{Y}_m^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\}+\max\{\mathfrak{Y}_n^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\}\right)}$, $d_{PL9}(\mathfrak{Y},\hat{\mathfrak{Y}}) = 1-\frac{\sum_{i=1}^{k}\left(\min\{\mathfrak{Y}_m^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\}+\min\{1-\mathfrak{Y}_n^q(\mathscr{b}_i),1-\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\}\right)}{\sum_{i=1}^{k}\left(\max\{\mathfrak{Y}_m^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\}+\max\{1-\mathfrak{Y}_n^q(\mathscr{b}_i),1-\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\}\right)}$, $d_{PL10}(\mathfrak{Y},\hat{\mathfrak{Y}}) = \frac{1}{2k}\sum_{i=1}^{k}\left(\begin{array}{c}\left\|\mathfrak{Y}_m^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\right\|+\left\|\mathfrak{Y}_n^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\right\|+\\ \left\|\mathfrak{Y}_h^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_h^q(\mathscr{b}_i)\right\|\end{array}\right)$, $d_{PL11}(\mathfrak{Y},\hat{\mathfrak{Y}}) = \frac{1}{2k}\sum_{i=1}^{k}\left(\begin{array}{c}\left\|\mathfrak{Y}_m^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\right\|+\left\|\mathfrak{Y}_n^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\right\|+\\ \left\|\mathfrak{Y}_h^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_h^q(\mathscr{b}_i)\right\|+\\ \left\|\left(\mathfrak{Y}_m^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\right)-\left(\mathfrak{Y}_n^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\right)\right\|\end{array}\right)$.
Pinar and Boran [72]	$d_{PB}(\mathfrak{Y},\hat{\mathfrak{Y}}) = \left[\frac{1}{2k}\sum_{i=1}^{k}\left(\begin{array}{c}\left\|\begin{array}{c}(1-\tau)\left(\mathfrak{Y}_m^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\right)+\\ \tau\left(\sqrt[q]{1-\mathfrak{Y}_n^q(\mathscr{b}_i)}-\sqrt[q]{1-\mathfrak{Y}_n^q(\mathscr{b}_i)}\right)\end{array}\right\|^p+\\ \left\|\begin{array}{c}(1-\tau)\left(\mathfrak{Y}_n^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\right)+\\ \tau\left(\sqrt[q]{1-\mathfrak{Y}_m^q(\mathscr{b}_i)}-\sqrt[q]{1-\mathfrak{Y}(\mathscr{b}_i)}\right)\end{array}\right\|^p\end{array}\right)\right]^{1/p}$, where $p\geq 1$ is the L_p norm and $\tau=\frac{\left(\frac{1}{2}q^2+\frac{3}{2}q-\frac{1}{3}\right)}{(q^2+3q+1)}\in\left[\frac{1}{3},\frac{1}{2}\right]$ is a parameter of uncertainty.
Kamacı and Petchimuthu [76]	$d_{KP1}(\mathfrak{Y},\hat{\mathfrak{Y}}) = \frac{1}{k}\sum_{i=1}^{k}\left(\frac{\left\|\mathfrak{Y}_m^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\right\|+\left\|\mathfrak{Y}_n^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\right\|}{\max\{\mathfrak{Y}_m^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\}+\max\{\mathfrak{Y}_n^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\}}\right)$, $d_{KP2}(\mathfrak{Y},\hat{\mathfrak{Y}}) = \frac{1}{k}\sum_{i=1}^{k}\left(\frac{\begin{array}{c}\left\|\mathfrak{Y}_m^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\right\|+\left\|\mathfrak{Y}_n^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\right\|+\\ \left\|\mathfrak{Y}_h^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_h^q(\mathscr{b}_i)\right\|\end{array}}{\begin{array}{c}\max\{\mathfrak{Y}_m^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\}+\max\{\mathfrak{Y}_n^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\}+\\ \max\{\mathfrak{Y}_h^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_h^q(\mathscr{b}_i)\}\end{array}}\right)$, $d_{KP3}(\mathfrak{Y},\hat{\mathfrak{Y}}) = \frac{\sum_{i=1}^{k}\left(\begin{array}{c}\left\|\mathfrak{Y}_m^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\right\|+\left\|\mathfrak{Y}_n^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\right\|+\\ \left\|\mathfrak{Y}_h^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_h^q(\mathscr{b}_i)\right\|\end{array}\right)}{\sum_{i=1}^{k}\left(\begin{array}{c}\max\{\mathfrak{Y}_m^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\}+\max\{\mathfrak{Y}_n^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\}+\\ \max\{\mathfrak{Y}_h^q(\mathscr{b}_i),\hat{\mathfrak{Y}}_h^q(\mathscr{b}_i)\}\end{array}\right)}$.

Table 1. Continued.

Q-ROFDMs	**Schemes Distance Schemes**
Hussain et al. [77]	$d_{He}(\mathfrak{Y},\hat{\mathfrak{Y}})=\frac{1}{k}\sum_{i=1}^{k}\max\left\{\left|\mathfrak{Y}_m^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\right|,\left|\left(1-\mathfrak{Y}_n^q(\mathscr{b}_i)\right)-\left(1-\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\right)\right|\right\}.$
Ejegwa [78]	$d_E(\mathfrak{Y},\hat{\mathfrak{Y}})=\left[\frac{1}{3}\sum_{i=1}^{k}\varpi_i\left(\begin{array}{c}\left|\mathfrak{Y}_m^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\right|^p+\left|\mathfrak{Y}_n^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\right|^p+\\\left|\mathfrak{Y}_h^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_h^q(\mathscr{b}_i)\right|^p\end{array}\right)\right]^{1/p},$ where $p\geq 1$ ($p=1,2$) is the L_p norm.
Ali [79]	$d_A(\mathfrak{Y},\hat{\mathfrak{Y}})=\frac{1}{3\sqrt{k}}\sum_{i=1}^{k}\left(\begin{array}{c}\left|\mathfrak{Y}_m^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\right|+\left|\mathfrak{Y}_n^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\right|+\\\left|\mathfrak{Y}_h^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_h^q(\mathscr{b}_i)\right|\end{array}\right).$
Rani and Kumar [81]	$d_{RK}(\mathfrak{Y},\hat{\mathfrak{Y}})=\frac{1}{k}\sum_{i=1}^{k}\left(\frac{2\sin\left(\frac{\pi}{2}\max\left\{\left|\mathfrak{Y}_m^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\right|,\left|\mathfrak{Y}_n^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\right|\right\}\right)}{1+\sin\left(\frac{\pi}{2}\max\left\{\left|\mathfrak{Y}_m^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\right|,\left|\mathfrak{Y}_n^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\right|\right\}\right)}\right).$
Wang et al. [82]	$d_{We1}(\mathfrak{Y},\hat{\mathfrak{Y}})=\left(\frac{1}{2}\sum_{i=1}^{k}\left[\left(\begin{array}{c}\mathfrak{Y}_m^q(\mathscr{b}_i)\log\left(\frac{2\,\mathfrak{Y}_m^q(\mathscr{b}_i)}{\mathfrak{Y}_m^q(\mathscr{b}_i)+\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)}\right)+\\\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\log\left(\frac{2\,\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)}{\mathfrak{Y}_m^q(\mathscr{b}_i)+\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)}\right)\end{array}\right)+\left(\begin{array}{c}\mathfrak{Y}_n^q(\mathscr{b}_i)\log\left(\frac{2\,\mathfrak{Y}_n^q(\mathscr{b}_i)}{\mathfrak{Y}_n^q(\mathscr{b}_i)+\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)}\right)+\\\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\log\left(\frac{2\,\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)}{\mathfrak{Y}_n^q(\mathscr{b}_i)+\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)}\right)\end{array}\right)\right]\right)^{\frac{1}{2}},$ $d_{We2}(\mathfrak{Y},\hat{\mathfrak{Y}})=\left(\frac{1}{2}\sum_{i=1}^{k}\left[\left(\begin{array}{c}\mathfrak{Y}_m^q(\mathscr{b}_i)\log\left(\frac{2\,\mathfrak{Y}_m^q(\mathscr{b}_i)}{\mathfrak{Y}_m^q(\mathscr{b}_i)+\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)}\right)+\\\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\log\left(\frac{2\,\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)}{\mathfrak{Y}_m^q(\mathscr{b}_i)+\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)}\right)\end{array}\right)+\left(\begin{array}{c}\mathfrak{Y}_n^q(\mathscr{b}_i)\log\left(\frac{2\,\mathfrak{Y}_n^q(\mathscr{b}_i)}{\mathfrak{Y}_n^q(\mathscr{b}_i)+\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)}\right)+\\\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\log\left(\frac{2\,\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)}{\mathfrak{Y}_n^q(\mathscr{b}_i)+\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)}\right)\end{array}\right)+\left(\begin{array}{c}\mathfrak{Y}_h^q(\mathscr{b}_i)\log\left(\frac{2\,\mathfrak{Y}_h^q(\mathscr{b}_i)}{\mathfrak{Y}_h^q(\mathscr{b}_i)+\hat{\mathfrak{Y}}_h^q(\mathscr{b}_i)}\right)+\\\hat{\mathfrak{Y}}_h^q(\mathscr{b}_i)\log\left(\frac{2\,\hat{\mathfrak{Y}}_h^q(\mathscr{b}_i)}{\mathfrak{Y}_h^q(\mathscr{b}_i)+\hat{\mathfrak{Y}}_h^q(\mathscr{b}_i)}\right)\end{array}\right)\right]\right)^{\frac{1}{2}},$
Dutta et al. [83]	$d_{De}(\mathfrak{Y},\hat{\mathfrak{Y}})=\frac{1}{3k}\sum_{i=1}^{k}\left[\begin{array}{c}\left|\mathfrak{Y}_m^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\right|+\left|\mathfrak{Y}_n^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\right|+\\\left|\mathfrak{Y}_m^q(\mathscr{b}_i)\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)-\mathfrak{Y}_n^q(\mathscr{b}_i)\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\right|\end{array}\right].$
Suri et al. [84]	$d_{Se}(\mathfrak{Y},\hat{\mathfrak{Y}})=\frac{1}{2k\log 2}\sum_{i=1}^{k}\left[\begin{array}{c}\left(\mathfrak{Y}_m^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\right)\log\frac{1+\mathfrak{Y}_m^q(\mathscr{b}_i)}{1+\hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)}+\\\left(\mathfrak{Y}_n^q(\mathscr{b}_i)-\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\right)\log\frac{1+\mathfrak{Y}_n^q(\mathscr{b}_i)}{1+\hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)}\end{array}\right].$

The distance method developed by [70] is not structurally correct in the sense that it does not capture the unique properties of the Q-ROFSs. In addition, the method lacks precision. Among the distance methods in

[71], d_{PL10} and d_{PL11} are the only ones that are developed based on a 3-dimensional approach, where all the descriptive parameters of Q-ROFSs are captured to enhance precision. However, they did not consider the number of taxicab differences. Besides d_{PL10} and d_{PL11}, the rest of the distance methods are not based on 3-dimensional space, which will adversely influence their reliability. In addition, they made use of the extreme values, which may lead to the omission of vital information.

The distance method constructed in [72] does not consider all the descriptive parameters of Q-ROFSs, which will influence precision and reliability. Among the three distance methods presented [76], d_{KP2} and d_{KP3} are based on 3-dimensional space, where all the descriptive parameters of Q-ROFSs are captured, but exclude the non-determinacy grade, which is vital in resolving imprecision. In addition, d_{KP1}, d_{KP2}, and d_{KP3} are developed based on extreme values, which may lead to the exclusion of some information. The distance methods in [77-81] are based on 2-dimensional and extreme values, which may lead to the exclusion of some significant information. Wang et al. [82] and Suri et al. [84] developed distance methods based on logarithmic functions. While the methods [82] are based on a 3-dimensional approach to enhance reliability, the method in [84] excludes the non-determinacy grade, which is essential in resolving imprecision. However, all the logarithmic methods lack accuracy. Finally, the distance methods in [78], [79], [83] are well modeled and are based on a 3-dimensional approach to enhance reliable interpretation. However, the methods lack accuracy. The limitation of the existing Q-ROFDMs with regard to precision will be illustrated in the comparison section.

4 | New Q-Rung Orthopair Fuzzy Distance Measure

In this section, the new Q-ROFDM and its properties are presented. This new distance function modifies and extends the distance metric in [18] under IFSs to Q-ROFSs to boost accuracy and promote interpretation reliability in comparison to the existing Q-ROFDMs.

Definition 5. Let $\mathfrak{Y}$ and $\widehat{\mathfrak{Y}}$ be Q-ROFSs in $\mathfrak{D} = \{\ell_1, \ell, \ldots, \ell_k\}$ defined as follows:

$\mathfrak{Y} = \{(\ell_i, \mathfrak{Y}_m(\ell_i), \mathfrak{Y}_n(\ell_i)) | \ell_i \in \mathfrak{D}\}$ and $\widehat{\mathfrak{Y}} = \{(\ell_i, \widehat{\mathfrak{Y}}_m(\ell_i), \widehat{\mathfrak{Y}}_n(\ell_i)) | \ell_i \in \mathfrak{D}\}$, with the hesitancy functions as $\mathfrak{Y}_h(\ell_i) = \left(1 - \mathfrak{Y}_m^q(\ell_i) - \mathfrak{Y}_n^q(\ell_i)\right)^{\frac{1}{q}}$ and $\widehat{\mathfrak{Y}}_h(\ell_i) = \left(1 - \widehat{\mathfrak{Y}}_m^q(\ell_i) - \widehat{\mathfrak{Y}}_n^q(\ell_i)\right)^{\frac{1}{q}}$, respectively. Then, the new Q-ROFDM between $\mathfrak{Y}$ and $\widehat{\mathfrak{Y}}$ is given as *Eq. (2)*.

$$d_*(\mathfrak{Y}, \widehat{\mathfrak{Y}}) = \frac{1}{k}\sum_{i=1}^{k}\left(\begin{array}{c}\mathcal{L}_{\mathfrak{Y},\widehat{\mathfrak{Y}}}\dfrac{\left|\mathfrak{Y}_m^q(\ell_i) - \widehat{\mathfrak{Y}}_m^q(\ell_i)\right|^q}{1 + \mathfrak{Y}_m^q(\ell_i) + \widehat{\mathfrak{Y}}_m^q(\ell_i)} + \mathcal{M}_{\mathfrak{Y},\widehat{\mathfrak{Y}}}\dfrac{\left|\mathfrak{Y}_n^q(\ell_i) - \widehat{\mathfrak{Y}}_n^q(\ell_i)\right|^q}{1 + \mathfrak{Y}_n^q(\ell_i) + \widehat{\mathfrak{Y}}_n^q(\ell_i)} + \\ \mathcal{N}_{\mathfrak{Y},\widehat{\mathfrak{Y}}}\dfrac{\left|\mathfrak{Y}_h^q(\ell_i) - \widehat{\mathfrak{Y}}_h^q(\ell_i)\right|^q}{1 + \mathfrak{Y}_h^q(\ell_i) + \widehat{\mathfrak{Y}}_h^q(\ell_i)}\end{array}\right), \quad (2)$$

where

$$\mathcal{L}_{\mathfrak{Y},\widehat{\mathfrak{Y}}} = \frac{\sum_{i=1}^{k}\left(\mathfrak{Y}_m^q(\ell_i) + \widehat{\mathfrak{Y}}_m^q(\ell_i)\right)}{k}, \mathcal{M}_{\mathfrak{Y},\widehat{\mathfrak{Y}}} = \frac{\sum_{i=1}^{k}\left(\mathfrak{Y}_n^q(\ell_i) + \widehat{\mathfrak{Y}}_n^q(\ell_i)\right)}{k}, \text{ and } \mathcal{N}_{\mathfrak{Y},\widehat{\mathfrak{Y}}} = \frac{\sum_{j=1}^{k}\left(\mathfrak{Y}_h^q(\ell_i) + \widehat{\mathfrak{Y}}_h^q(\ell_i)\right)}{k}.$$

For $\mathcal{L}_{\mathfrak{Y},\widehat{\mathfrak{Y}}} \geq 0$, $\mathcal{M}_{\mathfrak{Y},\widehat{\mathfrak{Y}}} \geq 0$, $\mathcal{N}_{\mathfrak{Y},\widehat{\mathfrak{Y}}} \geq 0$, such that, $\mathcal{L}_{\mathfrak{Y},\widehat{\mathfrak{Y}}} + \mathcal{M}_{\mathfrak{Y},\widehat{\mathfrak{Y}}} + \mathcal{N}_{\mathfrak{Y},\widehat{\mathfrak{Y}}} = 2$. The parameters $\mathcal{L}_{\mathfrak{Y},\widehat{\mathfrak{Y}}}$, $\mathcal{M}_{\mathfrak{Y},\widehat{\mathfrak{Y}}}$ and $\mathcal{N}_{\mathfrak{Y},\widehat{\mathfrak{Y}}}$ are the tendency coefficients for the Q-ROF parameters, which epitomize the level of support, opposition, and neutrality. If $k = 1$, then *Eq. (1)* is simplified as *Eq. (3)*.

$$d_*(\mathfrak{Y}, \widehat{\mathfrak{Y}}) = \left.\begin{array}{c}\mathcal{L}_{\mathfrak{Y},\widehat{\mathfrak{Y}}}\dfrac{\left|\mathfrak{Y}_m^q(\ell) - \widehat{\mathfrak{Y}}_m^q(\ell)\right|^q}{1 + \mathfrak{Y}_m^q(\ell) + \widehat{\mathfrak{Y}}_m^q(\ell)} + \mathcal{M}_{\mathfrak{Y},\widehat{\mathfrak{Y}}}\dfrac{\left|\mathfrak{Y}_n^q(\ell) - \widehat{\mathfrak{Y}}_n^q(\ell)\right|^q}{1 + \mathfrak{Y}_n^q(\ell) + \widehat{\mathfrak{Y}}_n^q(\ell)} + \\ \mathcal{N}_{\mathfrak{Y},\widehat{\mathfrak{Y}}}\dfrac{\left|\mathfrak{Y}_h^q(\ell) - \widehat{\mathfrak{Y}}_h^q(\ell)\right|^q}{1 + \mathfrak{Y}_h^q(\ell) + \widehat{\mathfrak{Y}}_h^q(\ell)}\end{array}\right\}, \quad (3)$$

where $\mathcal{L}_{\mathfrak{Y},\hat{\mathfrak{Y}}} = \left(\mathfrak{Y}_m^q(\mathscr{b}) + \hat{\mathfrak{Y}}_m^q(\mathscr{b})\right)$, $\mathcal{M}_{\mathfrak{Y},\hat{\mathfrak{Y}}} = \left(\mathfrak{Y}_n^q(\mathscr{b}) + \hat{\mathfrak{Y}}_n^q(\mathscr{b})\right)$ and $\mathcal{N}_{\mathfrak{Y},\hat{\mathfrak{Y}}} = \left(\mathfrak{Y}_h^q(\mathscr{b}) + \hat{\mathfrak{Y}}_h^q(\mathscr{b})\right)$. Again, by factoring in the influence of element-weight in *Eq. (2)*, we obtain a weighted distance method in *Eq. (4)*.

$$d_*^{\varpi}(\mathfrak{Y},\hat{\mathfrak{Y}}) = \sum_{i=1}^{k} \varpi_i \left(\begin{array}{c} \mathcal{L}_{\mathfrak{Y},\hat{\mathfrak{Y}}} \dfrac{\left|\mathfrak{Y}_m^q(\mathscr{b}_i) - \hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)\right|^q}{1 + \mathfrak{Y}_m^q(\mathscr{b}_i) + \hat{\mathfrak{Y}}_m^q(\mathscr{b}_i)} + \mathcal{M}_{\mathfrak{Y},\hat{\mathfrak{Y}}} \dfrac{\left|\mathfrak{Y}_n^q(\mathscr{b}_i) - \hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)\right|^q}{1 + \mathfrak{Y}_n^q(\mathscr{b}_i) + \hat{\mathfrak{Y}}_n^q(\mathscr{b}_i)} + \\ \mathcal{N}_{\mathfrak{Y},\hat{\mathfrak{Y}}} \dfrac{\left|\mathfrak{Y}_h^q(\mathscr{b}_i) - \hat{\mathfrak{Y}}_h^q(\mathscr{b}_i)\right|^q}{1 + \mathfrak{Y}_h^q(\mathscr{b}_i) + \hat{\mathfrak{Y}}_h^q(\mathscr{b}_i)} \end{array} \right), \quad (4)$$

where ϖ_i is defined by *Eq. (1)*. Next, the properties of the new Q-ROFDM are described in terms of theorems as follows.

Theorem 1. Given three Q-ROFSs $\mathfrak{B}$, $\hat{\mathfrak{B}}$ and $\tilde{\mathfrak{B}}$ in $\mathfrak{D}$, then $d_*(\mathfrak{Y},\hat{\mathfrak{Y}})$ satisfies the properties in *Definition 3*.

Proof: We begin with the proof of I of *Definition 3*. It is certain *Eq. (2)* fulfills $d_*(\mathfrak{Y},\hat{\mathfrak{Y}}) \geq 0$. Thus, $d_*(\mathfrak{Y},\hat{\mathfrak{Y}}) \leq 1$ needs to be proved. Since $\hat{\mathfrak{Y}}_m(\mathscr{b}), \mathfrak{Y}_m(\mathscr{b}), \hat{\mathfrak{Y}}_n(\mathscr{b}), \mathfrak{Y}_n(\mathscr{b}), \hat{\mathfrak{Y}}_h(\mathscr{b}), \mathfrak{Y}_h(\mathscr{b}) \in [0,1]$, then we get

$$-1 \leq \mathfrak{Y}_m^q(\mathscr{b}) - \hat{\mathfrak{Y}}_m^q(\mathscr{b}) \leq 1.$$

$$-1 \leq \mathfrak{Y}_n^q(\mathscr{b}) - \hat{\mathfrak{Y}}_n^q(\mathscr{b}) \leq 1.$$

$$-1 \leq \mathfrak{Y}_h^q(\mathscr{b}) - \hat{\mathfrak{Y}}_h^q(\mathscr{b}) \leq 1.$$

Thus,

$$0 \leq \left|\mathfrak{Y}_m^q(\mathscr{b}) - \hat{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q \leq 1, 0 \leq \left|\mathfrak{Y}_n^q(\mathscr{b}) - \hat{\mathfrak{Y}}_n^q(\mathscr{b})\right|^q \leq 1, 0 \leq \left|\mathfrak{Y}_h^q(\mathscr{b}) - \hat{\mathfrak{Y}}_h^q(\mathscr{b})\right|^q \leq 1.$$

Hence,

$$0 \leq \left|\mathfrak{Y}_m^q(\mathscr{b}) - \hat{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q \leq \mathfrak{Y}_m^q(\mathscr{b}) + \hat{\mathfrak{Y}}_m^q(\mathscr{b}).$$

$$0 \leq \left|\mathfrak{Y}_n^q(\mathscr{b}) - \hat{\mathfrak{Y}}_n^q(\mathscr{b})\right|^q \leq \mathfrak{Y}_n^q(\mathscr{b}) + \hat{\mathfrak{Y}}_n^q(\mathscr{b}).$$

$$0 \leq \left|\mathfrak{Y}_h^q(\mathscr{b}) - \hat{\mathfrak{Y}}_h^q(\mathscr{b})\right|^q \leq \mathfrak{Y}_h^q(\mathscr{b}) + \hat{\mathfrak{Y}}_h^q(\mathscr{b}).$$

In addition,

$$0 \leq 2\left|\mathfrak{Y}_m^q(\mathscr{b}) - \hat{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q \leq 1 + \mathfrak{Y}_m^q(\mathscr{b}) + \hat{\mathfrak{Y}}_m^q(\mathscr{b}).$$

$$0 \leq 2\left|\mathfrak{Y}_n^q(\mathscr{b}) - \hat{\mathfrak{Y}}_n^q(\mathscr{b})\right|^q \leq 1 + \mathfrak{Y}_n^q(\mathscr{b}) + \hat{\mathfrak{Y}}_n^q(\mathscr{b}).$$

$$0 \leq 2\left|\mathfrak{Y}_h^q(\mathscr{b}) - \hat{\mathfrak{Y}}_h^q(\mathscr{b})\right|^q \leq 1 + \mathfrak{Y}_h^q(\mathscr{b}) + \hat{\mathfrak{Y}}_h^q(\mathscr{b}).$$

Resulting in the following:

$$0 \leq \frac{\left|\mathfrak{Y}_m^q(\mathscr{b}) - \hat{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q}{1 + \mathfrak{Y}_m^q(\mathscr{b}) + \hat{\mathfrak{Y}}_m^q(\mathscr{b})} \leq \frac{1}{2}, 0 \leq \frac{\left|\mathfrak{Y}_n^q(\mathscr{b}) - \hat{\mathfrak{Y}}_n^q(\mathscr{b})\right|^q}{1 + \mathfrak{Y}_n^q(\mathscr{b}) + \hat{\mathfrak{Y}}_n^q(\mathscr{b})} \leq \frac{1}{2}.$$

$$0 \leq \frac{\left|\mathfrak{Y}_h^q(\mathscr{b}) - \hat{\mathfrak{Y}}_h^q(\mathscr{b})\right|^q}{1 + \mathfrak{Y}_h^q(\mathscr{b}) + \hat{\mathfrak{Y}}_h^q(\mathscr{b})} \leq \frac{1}{2}.$$

Since $\mathcal{L}_{\mathfrak{Y},\hat{\mathfrak{Y}}} \geq 0$, $\mathcal{M}_{\mathfrak{Y},\hat{\mathfrak{Y}}} \geq 0, \mathcal{N}_{\mathfrak{Y},\hat{\mathfrak{Y}}} \geq 0$ and $\mathcal{L}_{\mathfrak{Y},\hat{\mathfrak{Y}}} + \mathcal{M}_{\mathfrak{Y},\hat{\mathfrak{Y}}} + \mathcal{N}_{\mathfrak{Y},\hat{\mathfrak{Y}}} = 2$, then

$$\mathcal{L}_{\mathfrak{Y},\hat{\mathfrak{Y}}} \frac{\left|\mathfrak{Y}_m^q(\mathscr{b}) - \hat{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q}{1 + \mathfrak{Y}_m^q(\mathscr{b}) + \hat{\mathfrak{Y}}_m^q(\mathscr{b})} + \mathcal{M}_{\mathfrak{Y},\hat{\mathfrak{Y}}} \frac{\left|\mathfrak{Y}_n^q(\mathscr{b}) - \hat{\mathfrak{Y}}_n^q(\mathscr{b})\right|^q}{1 + \mathfrak{Y}_n^q(\mathscr{b}) + \hat{\mathfrak{Y}}_n^q(\mathscr{b})} + \mathcal{N}_{\mathfrak{Y},\hat{\mathfrak{Y}}} \frac{\left|\mathfrak{Y}_h^q(\mathscr{b}) - \hat{\mathfrak{Y}}_h^q(\mathscr{b})\right|^q}{1 + \mathfrak{Y}_h^q(\mathscr{b}) + \hat{\mathfrak{Y}}_h^q(\mathscr{b})}$$

$$\leq \frac{1}{2}\mathcal{L}_{\mathfrak{Y},\hat{\mathfrak{Y}}} + \frac{1}{2}\mathcal{M}_{\mathfrak{Y},\hat{\mathfrak{Y}}} + \frac{1}{2}\mathcal{N}_{\mathfrak{Y},\hat{\mathfrak{Y}}} = \frac{1}{2}\left(\mathcal{L}_{\mathfrak{Y},\hat{\mathfrak{Y}}} + \mathcal{M}_{\mathfrak{Y},\hat{\mathfrak{Y}}} + \mathcal{N}_{\mathfrak{Y},\hat{\mathfrak{Y}}}\right) = 1.$$

Therefore,

$$d_*(\mathfrak{Y},\hat{\mathfrak{Y}}) = \mathcal{L}_{\mathfrak{Y},\hat{\mathfrak{Y}}}\frac{\left|\mathfrak{Y}_m^q(\mathscr{b}) - \hat{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q}{1 + \mathfrak{Y}_m^q(\mathscr{b}) + \hat{\mathfrak{Y}}_m^q(\mathscr{b})} + \mathcal{M}_{\mathfrak{Y},\hat{\mathfrak{Y}}}\frac{\left|\mathfrak{Y}_n^q(\mathscr{b}) - \hat{\mathfrak{Y}}_n^q(\mathscr{b})\right|^q}{1 + \mathfrak{Y}_n^q(\mathscr{b}) + \hat{\mathfrak{Y}}_n^q(\mathscr{b})} + \mathcal{N}_{\mathfrak{Y},\hat{\mathfrak{Y}}}\frac{\left|\mathfrak{Y}_h^q(\mathscr{b}) - \hat{\mathfrak{Y}}_h^q(\mathscr{b})\right|^q}{1 + \mathfrak{Y}_h^q(\mathscr{b}) + \hat{\mathfrak{Y}}_h^q(\mathscr{b})} \leq 1.$$

Hence, $0 \leq d_*(\mathfrak{Y},\hat{\mathfrak{Y}}) \leq 1$ is established. Next, we proof. Suppose $\mathfrak{Y} = \hat{\mathfrak{Y}}$ and $\mathcal{L}_{\mathfrak{Y},\hat{\mathfrak{Y}}} > 0, \mathcal{M}_{\mathfrak{Y},\hat{\mathfrak{Y}}} > 0, \mathcal{N}_{\mathfrak{Y},\hat{\mathfrak{Y}}} > 0$, it is certain that *Eq. (2)* holds for all $\mathcal{L}_{\mathfrak{Y},\hat{\mathfrak{Y}}}$, $\mathcal{M}_{\mathfrak{Y},\hat{\mathfrak{Y}}}$ and $\mathcal{N}_{\mathfrak{Y},\hat{\mathfrak{Y}}}$, and

$$d_*(\mathfrak{Y},\hat{\mathfrak{Y}}) = \mathcal{L}_{\mathfrak{Y},\hat{\mathfrak{Y}}}\frac{\left|\mathfrak{Y}_m^q(\mathscr{b}) - \hat{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q}{1 + \mathfrak{Y}_m^q(\mathscr{b}) + \hat{\mathfrak{Y}}_m^q(\mathscr{b})} + \mathcal{M}_{\mathfrak{Y},\hat{\mathfrak{Y}}}\frac{\left|\mathfrak{Y}_n^q(\mathscr{b}) - \hat{\mathfrak{Y}}_n^q(\mathscr{b})\right|^q}{1 + \mathfrak{Y}_n^q(\mathscr{b}) + \hat{\mathfrak{Y}}_n^q(\mathscr{b})} + \mathcal{N}_{\mathfrak{Y},\hat{\mathfrak{Y}}}\frac{\left|\mathfrak{Y}_h^q(\mathscr{b}) - \hat{\mathfrak{Y}}_h^q(\mathscr{b})\right|^q}{1 + \mathfrak{Y}_h^q(\mathscr{b}) + \hat{\mathfrak{Y}}_h^q(\mathscr{b})} = 0.$$

Also, if $\mathcal{L}_{\mathfrak{Y},\hat{\mathfrak{Y}}} \geq 0$, $\mathcal{M}_{\mathfrak{Y},\hat{\mathfrak{Y}}} \geq 0$, and $\mathcal{N}_{\mathfrak{Y},\hat{\mathfrak{Y}}} \geq 0$, we get

$$\mathcal{L}_{\mathfrak{Y},\hat{\mathfrak{Y}}}\frac{\left|\mathfrak{Y}_m^q(\mathscr{b}) - \hat{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q}{1 + \mathfrak{Y}_m^q(\mathscr{b}) + \hat{\mathfrak{Y}}_m^q(\mathscr{b})} \geq 0, \mathcal{M}_{\mathfrak{Y},\hat{\mathfrak{Y}}}\frac{\left|\mathfrak{Y}_n^q(\mathscr{b}) - \hat{\mathfrak{Y}}_n^q(\mathscr{b})\right|^q}{1 + \mathfrak{Y}_n^q(\mathscr{b}) + \hat{\mathfrak{Y}}_n^q(\mathscr{b})} \geq 0,$$

$$\mathcal{N}_{\mathfrak{Y},\hat{\mathfrak{Y}}}\frac{\left|\mathfrak{Y}_h^q(\mathscr{b}) - \hat{\mathfrak{Y}}_h^q(\mathscr{b})\right|^q}{1 + \mathfrak{Y}_h^q(\mathscr{b}) + \hat{\mathfrak{Y}}_h^q(\mathscr{b})} \geq 0,$$

Which shows that $d_*(\mathfrak{Y},\hat{\mathfrak{Y}}) = 0$. Conversely, for $d_*(\mathfrak{Y},\hat{\mathfrak{Y}}) = 0$, we get

$$\mathcal{L}_{\mathfrak{Y},\hat{\mathfrak{Y}}}\frac{\left|\mathfrak{Y}_m^q(\mathscr{b}) - \hat{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q}{1 + \mathfrak{Y}_m^q(\mathscr{b}) + \hat{\mathfrak{Y}}_m^q(\mathscr{b})} \geq 0, \mathcal{M}_{\mathfrak{Y},\hat{\mathfrak{Y}}}\frac{\left|\mathfrak{Y}_n^q(\mathscr{b}) - \hat{\mathfrak{Y}}_n^q(\mathscr{b})\right|^q}{1 + \mathfrak{Y}_n^q(\mathscr{b}) + \hat{\mathfrak{Y}}_n^q(\mathscr{b})} \geq 0,$$

$$\mathcal{N}_{\mathfrak{Y},\hat{\mathfrak{Y}}}\frac{\left|\mathfrak{Y}_h^q(\mathscr{b}) - \hat{\mathfrak{Y}}_h^q(\mathscr{b})\right|^q}{1 + \mathfrak{Y}_h^q(\mathscr{b}) + \hat{\mathfrak{Y}}_h^q(\mathscr{b})} \geq 0.$$

Therefore, if $\mathcal{L}_{\mathfrak{Y},\hat{\mathfrak{Y}}} > 0$, $\mathcal{M}_{\mathfrak{Y},\hat{\mathfrak{Y}}} > 0$, $\mathcal{N}_{\mathfrak{Y},\hat{\mathfrak{Y}}} > 0$, then $\mathfrak{Y}_m^q(\mathscr{b}) = \hat{\mathfrak{Y}}_m^q(\mathscr{b})$, $\mathfrak{Y}_n^q(\mathscr{b}) = \hat{\mathfrak{Y}}_n^q(\mathscr{b})$ and $\mathfrak{Y}_h^q(\mathscr{b}) = \hat{\mathfrak{Y}}_h^q(\mathscr{b})$. Thus, $\mathfrak{Y} = \hat{\mathfrak{Y}}$. Now, we verify III. This is easy to see since

$$d_*(\mathfrak{Y},\hat{\mathfrak{Y}}) = \mathcal{L}_{\mathfrak{Y},\hat{\mathfrak{Y}}}\frac{\left|\mathfrak{Y}_m^q(\mathscr{b}) - \hat{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q}{1 + \mathfrak{Y}_m^q(\mathscr{b}) + \hat{\mathfrak{Y}}_m^q(\mathscr{b})} + \mathcal{M}_{\mathfrak{Y},\hat{\mathfrak{Y}}}\frac{\left|\mathfrak{Y}_n^q(\mathscr{b}) - \hat{\mathfrak{Y}}_n^q(\mathscr{b})\right|^q}{1 + \mathfrak{Y}_n^q(\mathscr{b}) + \hat{\mathfrak{Y}}_n^q(\mathscr{b})}$$

$$+\mathcal{N}_{\mathfrak{Y},\hat{\mathfrak{Y}}}\frac{\left|\mathfrak{Y}_h^q(\mathscr{b}) - \hat{\mathfrak{Y}}_h^q(\mathscr{b})\right|^q}{1 + \mathfrak{Y}_h^q(\mathscr{b}) + \hat{\mathfrak{Y}}_h^q(\mathscr{b})} = \mathcal{L}_{\hat{\mathfrak{Y}},\mathfrak{Y}}\frac{\left|\hat{\mathfrak{Y}}_m^q(\mathscr{b}) - \mathfrak{Y}_m^q(\mathscr{b})\right|^q}{1 + \mathfrak{Y}_m^q(\mathscr{b}) + \hat{\mathfrak{Y}}_m^q(\mathscr{b})} + \mathcal{M}_{\hat{\mathfrak{Y}},\mathfrak{Y}}\frac{\left|\hat{\mathfrak{Y}}_n^q(\mathscr{b}) - \mathfrak{Y}_n^q(\mathscr{b})\right|^q}{1 + \mathfrak{Y}_n^q(\mathscr{b}) + \hat{\mathfrak{Y}}_n^q(\mathscr{b})}$$

$$+\mathcal{N}_{\hat{\mathfrak{Y}},\mathfrak{Y}}\frac{\left|\hat{\mathfrak{Y}}_h^q(\mathscr{b}) - \mathfrak{Y}_h^q(\mathscr{b})\right|^q}{1 + \mathfrak{Y}_h^q(\mathscr{b}) + \hat{\mathfrak{Y}}_h^q(\mathscr{b})}.$$

Thus, $d_*(\mathfrak{Y},\hat{\mathfrak{Y}}) = d_*(\hat{\mathfrak{Y}},\mathfrak{Y})$ as required. Finally, $d_*(\mathfrak{Y},\hat{\mathfrak{Y}}) + d_*(\hat{\mathfrak{Y}},\tilde{\mathfrak{Y}}) \geq d_*(\mathfrak{Y},\tilde{\mathfrak{Y}})$ is verified. To establish this, we need first to show that *Eq. (5)* holds.

$$\mathcal{L}_{\mathfrak{Y},\widehat{\mathfrak{Y}}}\frac{\left|\mathfrak{Y}_m^q(\mathscr{b})-\widehat{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q}{1+\mathfrak{Y}_m^q(\mathscr{b})+\widehat{\mathfrak{Y}}_m^q(\mathscr{b})}+\mathcal{L}_{\widetilde{\mathfrak{Y}},\widehat{\mathfrak{Y}}}\frac{\left|\widehat{\mathfrak{Y}}_m^q(\mathscr{b})-\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q}{1+\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})+\widehat{\mathfrak{Y}}_m^q(\mathscr{b})}\geq\mathcal{L}_{\mathfrak{Y},\widetilde{\mathfrak{Y}}}\frac{\left|\mathfrak{Y}_m^q(\mathscr{b})-\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q}{1+\mathfrak{Y}_m^q(\mathscr{b})+\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})}. \tag{5}$$

In fact, *Eq. (5)* holds if $\mathcal{L}_{\mathfrak{Y},\widehat{\mathfrak{Y}}}=\mathcal{L}_{\widetilde{\mathfrak{Y}},\widehat{\mathfrak{Y}}}=\mathcal{L}_{\mathfrak{Y},\widetilde{\mathfrak{Y}}}=0$. By the principle of the Boolean ring, $\left|\widehat{\mathfrak{Y}}_m^q(\mathscr{b})-\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q=\left|\widehat{\mathfrak{Y}}_m^q(\mathscr{b})-\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right|$, $\left|\mathfrak{Y}_m^q(\mathscr{b})-\widehat{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q=\left|\mathfrak{Y}_m^q(\mathscr{b})-\widehat{\mathfrak{Y}}_m^q(\mathscr{b})\right|$, and $\left|\mathfrak{Y}_m^q(\mathscr{b})-\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q=\left|\mathfrak{Y}_m^q(\mathscr{b})-\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right|$. Then, *Eq. (5)* becomes *Eq. (6)*.

$$\mathcal{L}_{\mathfrak{Y},\widehat{\mathfrak{Y}}}\frac{\left|\mathfrak{Y}_m^q(\mathscr{b})-\widehat{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q}{1+\mathfrak{Y}_m^q(\mathscr{b})+\widehat{\mathfrak{Y}}_m^q(\mathscr{b})}+\mathcal{L}_{\widetilde{\mathfrak{Y}},\widehat{\mathfrak{Y}}}\frac{\left|\widehat{\mathfrak{Y}}_m^q(\mathscr{b})-\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q}{1+\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})+\widehat{\mathfrak{Y}}_m^q(\mathscr{b})}\geq\mathcal{L}_{\mathfrak{Y},\widetilde{\mathfrak{Y}}}\frac{\left|\mathfrak{Y}_m^q(\mathscr{b})-\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q}{1+\mathfrak{Y}_m^q(\mathscr{b})+\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})}. \tag{6}$$

If $\mathcal{L}_{\mathfrak{Y},\widehat{\mathfrak{Y}}}=\mathcal{L}_{\mathfrak{Y},\widehat{\mathfrak{Y}}}=\mathcal{L}_{\mathfrak{Y},\widehat{\mathfrak{Y}}}>0$, then *Eq. (6)* is simplified as

$$\frac{\left|\mathfrak{Y}_m^q(\mathscr{b})-\widehat{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q}{1+\mathfrak{Y}_m^q(\mathscr{b})+\widehat{\mathfrak{Y}}_m^q(\mathscr{b})}+\frac{\left|\widehat{\mathfrak{Y}}_m^q(\mathscr{b})-\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q}{1+\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})+\widehat{\mathfrak{Y}}_m^q(\mathscr{b})}\geq\frac{\left|\mathfrak{Y}_m^q(\mathscr{b})-\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q}{1+\mathfrak{Y}_m^q(\mathscr{b})+\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})}.$$

Next, we verify whether the immediate inequality is valid in the following cases:

$$\mathfrak{Y}_m^q(\mathscr{b})\leq\widehat{\mathfrak{Y}}_m^q(\mathscr{b})\leq\widetilde{\mathfrak{Y}}_m^q(\mathscr{b}), \tag{a}$$

$$\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\leq\widehat{\mathfrak{Y}}_m^q(\mathscr{b})\leq\mathfrak{Y}_m^q(\mathscr{b}), \tag{b}$$

$$\widehat{\mathfrak{Y}}_m^q(\mathscr{b})\geq\max\{\mathfrak{Y}_m^q(\mathscr{b}),\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\}, \tag{c}$$

$$\widehat{\mathfrak{Y}}_m^q(\mathscr{b})\leq\min\{\mathfrak{Y}_m^q(\mathscr{b}),\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\}. \tag{d}$$

Under case (a), *Eq. (7)* is equivalent to:

$$\frac{\left|\widehat{\mathfrak{Y}}_m^q(\mathscr{b})-\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q}{1+\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})+\widehat{\mathfrak{Y}}_m^q(\mathscr{b})}+\frac{\left|\mathfrak{Y}_m^q(\mathscr{b})-\widehat{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q}{1+\mathfrak{Y}_m^q(\mathscr{b})+\widehat{\mathfrak{Y}}_m^q(\mathscr{b})}-\frac{\left|\mathfrak{Y}_m^q(\mathscr{b})-\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q}{1+\mathfrak{Y}_m^q(\mathscr{b})+\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})}\geq 0. \tag{7}$$

In fact, since $\widehat{\mathfrak{Y}}_m^q(\mathscr{b})\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\geq\mathfrak{Y}_m^q(\mathscr{b})\widehat{\mathfrak{Y}}_m^q(\mathscr{b})$, and $\mathfrak{Y}_m^q(\mathscr{b})\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\geq\mathfrak{Y}_m^q(\mathscr{b})\widehat{\mathfrak{Y}}_m^q(\mathscr{b})$, then

$$\frac{\left|\mathfrak{Y}_m^q(\mathscr{b})-\widehat{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q}{1+\mathfrak{Y}_m^q(\mathscr{b})+\widehat{\mathfrak{Y}}_m^q(\mathscr{b})}+\frac{\left|\widehat{\mathfrak{Y}}_m^q(\mathscr{b})-\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q}{1+\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})+\widehat{\mathfrak{Y}}_m^q(\mathscr{b})}-\frac{\left|\mathfrak{Y}_m^q(\mathscr{b})-\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right|^q}{1+\mathfrak{Y}_m^q(\mathscr{b})+\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})}$$

$$=\frac{\widehat{\mathfrak{Y}}_m^q(\mathscr{b})\mathfrak{Y}_m^q(\mathscr{b})-\widehat{\mathfrak{Y}}_m^q(\mathscr{b})\mathfrak{Y}_m^q(\mathscr{b})+\widehat{\mathfrak{Y}}_m^q(\mathscr{b})\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})-\widehat{\mathfrak{Y}}_m^q(\mathscr{b})\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})}{\left(1+\mathfrak{Y}_m^q(\mathscr{b})+\widehat{\mathfrak{Y}}_m^q(\mathscr{b})\right)\left(1+\widehat{\mathfrak{Y}}_m^q(\mathscr{b})+\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right)\left(1+\mathfrak{Y}_m^q(\mathscr{b})+\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right)}$$

$$+\frac{\mathfrak{Y}_m^q(\mathscr{b})\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})-\mathfrak{Y}_m^q(\mathscr{b})\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})}{\left(1+\mathfrak{Y}_m^q(\mathscr{b})+\widehat{\mathfrak{Y}}_m^q(\mathscr{b})\right)\left(1+\widehat{\mathfrak{Y}}_m^q(\mathscr{b})+\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right)\left(1+\mathfrak{Y}_m^q(\mathscr{b})+\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right)}$$

$$=\frac{\widehat{\mathfrak{Y}}_m^q(\mathscr{b})\mathfrak{Y}_m^q(\mathscr{b})\left[\mathfrak{Y}_m^q(\mathscr{b})-\widehat{\mathfrak{Y}}_m^q(\mathscr{b})\right]+\widehat{\mathfrak{Y}}_m^q(\mathscr{b})\mathfrak{Y}_m^q(\mathscr{b})\left[\widehat{\mathfrak{Y}}_m^q(\mathscr{b})-\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right]}{\left(1+\mathfrak{Y}_m^q(\mathscr{b})+\widehat{\mathfrak{Y}}_m^q(\mathscr{b})\right)\left(1+\widehat{\mathfrak{Y}}_m^q(\mathscr{b})+\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right)\left(1+\mathfrak{Y}_m^q(\mathscr{b})+\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right)}$$

$$+\frac{\mathfrak{Y}_m^q(\mathscr{b})\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\left[\mathfrak{Y}_m^q(\mathscr{b})-\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right]}{\left(1+\mathfrak{Y}_m^q(\mathscr{b})+\widehat{\mathfrak{Y}}_m^q(\mathscr{b})\right)\left(1+\widehat{\mathfrak{Y}}_m^q(\mathscr{b})+\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right)\left(1+\mathfrak{Y}_m^q(\mathscr{b})+\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right)}$$

$$\geq\frac{\widehat{\mathfrak{Y}}_m^q(\mathscr{b})\mathfrak{Y}_m^q(\mathscr{b})\left[\widehat{\mathfrak{Y}}_m^q(\mathscr{b})-\mathfrak{Y}_m^q(\mathscr{b})+\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})-\widehat{\mathfrak{Y}}_m^q(\mathscr{b})+\mathfrak{Y}_m^q(\mathscr{b})-\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right]}{\left(1+\mathfrak{Y}_m^q(\mathscr{b})+\widehat{\mathfrak{Y}}_m^q(\mathscr{b})\right)\left(1+\widehat{\mathfrak{Y}}_m^q(\mathscr{b})+\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right)\left(1+\mathfrak{Y}_m^q(\mathscr{b})+\widetilde{\mathfrak{Y}}_m^q(\mathscr{b})\right)}=0.$$

Hence, *Eq. (6)* is satisfied under this case (a). Similarly, *Eq. (6)* is satisfied under case (b). In case (c), where $\widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) \geq \max\{\mathfrak{Y}_{m}^{q}(\mathscr{b}), \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})\}$, we have two situations as follows: We have,

$$1 + \mathfrak{Y}_{m}^{q}(\mathscr{b}) + \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) \geq 1 + \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) + \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b}).$$

And

$$1 + \mathfrak{Y}_{m}^{q}(\mathscr{b}) + \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) \geq 1 + \mathfrak{Y}_{m}^{q}(\mathscr{b}) + \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) \text{ if } \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) \leq \mathfrak{Y}_{m}^{q}(\mathscr{b}) \leq \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}).$$

Thus,

$$\frac{\left|\widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) - \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})\right|}{1 + \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) + \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})} + \frac{\left|\mathfrak{Y}_{m}^{q}(\mathscr{b}) - \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b})\right|}{1 + \mathfrak{Y}_{m}^{q}(\mathscr{b}) + \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b})} - \frac{\left|\mathfrak{Y}_{m}^{q}(\mathscr{b}) - \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})\right|}{1 + \mathfrak{Y}_{m}^{q}(\mathscr{b}) + \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})}$$

$$= \frac{\widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) - \mathfrak{Y}_{m}^{q}(\mathscr{b})}{1 + \mathfrak{Y}_{m}^{q}(\mathscr{b}) + \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b})} + \frac{\widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) - \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})}{1 + \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) + \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})} - \frac{\mathfrak{Y}_{m}^{q}(\mathscr{b}) - \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})}{1 + \mathfrak{Y}_{m}^{q}(\mathscr{b}) + \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})}$$

$$= \frac{\widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) - \mathfrak{Y}_{m}^{q}(\mathscr{b})}{1 + \mathfrak{Y}_{m}^{q}(\mathscr{b}) + \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b})} + \frac{\widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) - \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})}{1 + \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) + \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})} + \frac{\widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) - \mathfrak{Y}_{m}^{q}(\mathscr{b})}{1 + \mathfrak{Y}_{m}^{q}(\mathscr{b}) + \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})}$$

$$\geq \frac{\widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) - \mathfrak{Y}_{m}^{q}(\mathscr{b}) + \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) - \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) - \mathfrak{Y}_{m}^{q}(\mathscr{b}) + \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})}{1 + \mathfrak{Y}_{m}^{q}(\mathscr{b}) + \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b})}$$

$$= \frac{2\left(\widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) - \mathfrak{Y}_{m}^{q}(\mathscr{b})\right)}{1 + \mathfrak{Y}_{m}^{q}(\mathscr{b}) + \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b})} \geq 0.$$

We get

$$1 + \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) + \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) \geq 1 + \mathfrak{Y}_{m}^{q}(\mathscr{b}) + \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b}).$$

And

$$1 + \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) + \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) \geq 1 + \mathfrak{Y}_{m}^{q}(\mathscr{b}) + \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) \text{ if } \mathfrak{Y}_{m}^{q}(\mathscr{b}) \leq \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) \leq \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}).$$

Thus,

$$\frac{\left|\widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) - \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})\right|}{1 + \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) + \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})} + \frac{\left|\mathfrak{Y}_{m}^{q}(\mathscr{b}) - \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b})\right|}{1 + \mathfrak{Y}_{m}^{q}(\mathscr{b}) + \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b})} - \frac{\left|\mathfrak{Y}_{m}^{q}(\mathscr{b}) - \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})\right|}{1 + \mathfrak{Y}_{m}^{q}(\mathscr{b}) + \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})}$$

$$= \frac{\widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) - \mathfrak{Y}_{m}^{q}(\mathscr{b})}{1 + \mathfrak{Y}_{m}^{q}(\mathscr{b}) + \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b})} + \frac{\widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) - \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})}{1 + \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) + \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})} - \frac{\widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) - \mathfrak{Y}_{m}^{q}(\mathscr{b})}{1 + \mathfrak{Y}_{m}^{q}(\mathscr{b}) + \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})}$$

$$= \frac{\widehat{\mathfrak{Y}}_{m}^{q}(\mathfrak{d}) - \mathfrak{Y}_{m}^{q}(\mathfrak{d})}{1 + \mathfrak{Y}_{m}^{q}(\mathscr{b}) + \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b})} + \frac{\widehat{\mathfrak{Y}}_{m}^{q}(\mathfrak{d}) - \widetilde{\mathfrak{Y}}_{m}^{q}(\mathfrak{d})}{1 + \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) + \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})} + \frac{\mathfrak{Y}_{m}^{q}(\mathfrak{d}) - \widetilde{\mathfrak{Y}}_{m}^{q}(\mathfrak{d})}{1 + \mathfrak{Y}_{m}^{q}(\mathscr{b}) + \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})}$$

$$\geq \frac{\widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) - \mathfrak{Y}_{m}^{q}(\mathscr{b}) + \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) - \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) + \mathfrak{Y}_{m}^{q}(\mathscr{b}) - \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})}{1 + \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) + \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})}$$

$$= \frac{2\left(\widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) - \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})\right)}{1 + \widehat{\mathfrak{Y}}_{m}^{q}(\mathscr{b}) + \widetilde{\mathfrak{Y}}_{m}^{q}(\mathscr{b})} \geq 0.$$

Hence, *Eq. (6)* is established under case (c), and similarly, *Eq. (6)* holds for case (d). In a similar way, if $\mathcal{M}_{\mathfrak{Y},\widehat{\mathfrak{Y}}} = \mathcal{M}_{\widetilde{\mathfrak{Y}},\widehat{\mathfrak{Y}}} = \mathcal{M}_{\mathfrak{Y},\widetilde{\mathfrak{Y}}} > 0$ and $\mathcal{N}_{\mathfrak{Y},\widehat{\mathfrak{Y}}} = \mathcal{N}_{\widetilde{\mathfrak{Y}},\widehat{\mathfrak{Y}}} = \mathcal{N}_{\mathfrak{Y},\widetilde{\mathfrak{Y}}} > 0$, then, we get *Eq. (7)* and *Eq. (8)*, respectively:

$$\mathcal{L}_{\mathfrak{Y},\widehat{\mathfrak{Y}}}\frac{\left|\mathfrak{Y}_{n}^{q}(\mathscr{b})-\widehat{\mathfrak{Y}}_{n}^{q}(\mathscr{b})\right|^{q}}{1+\mathfrak{Y}_{n}^{q}(\mathscr{b})+\widehat{\mathfrak{Y}}_{n}^{q}(\mathscr{b})}+\mathcal{L}_{\widehat{\mathfrak{Y}},\widetilde{\mathfrak{Y}}}\frac{\left|\widehat{\mathfrak{Y}}_{n}^{q}(\mathscr{b})-\widetilde{\mathfrak{Y}}_{n}^{q}(\mathscr{b})\right|^{q}}{1+\widetilde{\mathfrak{Y}}_{n}^{q}(\mathscr{b})+\widehat{\mathfrak{Y}}_{n}^{q}(\mathscr{b})}\geq\mathcal{L}_{\mathfrak{Y},\widetilde{\mathfrak{Y}}}\frac{\left|\mathfrak{Y}_{n}^{q}(\mathscr{b})-\widetilde{\mathfrak{Y}}_{n}^{q}(\mathscr{b})\right|^{q}}{1+\mathfrak{Y}_{n}^{q}(\mathscr{b})+\widetilde{\mathfrak{Y}}_{n}^{q}(\mathscr{b})}, \tag{7}$$

$$\mathcal{L}_{\mathfrak{Y},\widehat{\mathfrak{Y}}}\frac{\left|\mathfrak{Y}_{h}^{q}(\mathscr{b})-\widehat{\mathfrak{Y}}_{h}^{q}(\mathscr{b})\right|^{q}}{1+\mathfrak{Y}_{h}^{q}(\mathscr{b})+\widehat{\mathfrak{Y}}_{h}^{q}(\mathscr{b})}+\mathcal{L}_{\widehat{\mathfrak{Y}},\widetilde{\mathfrak{Y}}}\frac{\left|\widehat{\mathfrak{Y}}_{h}^{q}(\mathscr{b})-\widetilde{\mathfrak{Y}}_{h}^{q}(\mathscr{b})\right|^{q}}{1+\widetilde{\mathfrak{Y}}_{h}^{q}(\mathscr{b})+\widehat{\mathfrak{Y}}_{h}^{q}(\mathscr{b})}\geq\mathcal{L}_{\mathfrak{Y},\widetilde{\mathfrak{Y}}}\frac{\left|\mathfrak{Y}_{h}^{q}(\mathscr{b})-\widetilde{\mathfrak{Y}}_{h}^{q}(\mathscr{b})\right|^{q}}{1+\mathfrak{Y}_{h}^{q}(\mathscr{b})+\widetilde{\mathfrak{Y}}_{h}^{q}(\mathscr{b})}. \tag{8}$$

Using *Eqs. (6)-(8)*, $d_{*}(\widehat{\mathfrak{Y}},\widetilde{\mathfrak{Y}})+d_{*}(\mathfrak{Y},\widehat{\mathfrak{Y}})\geq d_{*}(\mathfrak{Y},\widetilde{\mathfrak{Y}})$ is satisfied.

Theorem 2. Given three Q-ROFSs $\mathfrak{Y}$, $\widehat{\mathfrak{Y}}$ and $\widetilde{\mathfrak{Y}}$ in $\mathfrak{D}$, the weighted Q-ROFDM, $d_{*}^{\varpi}(\mathfrak{Y},\widehat{\mathfrak{Y}})$ fulfills the conditions in *Definition 3*.

Proof: We omit the proof because it is akin to *Theorem 1*.

4.1 | Numerical Verifications

Some numerical illustrations of Q-ROFSs are given to show the appropriateness of the novel Q-ROFDM over the existing Q-ROFDMs. First, we want to verify whether the new Q-ROFDM satisfies the distance metric conditions numerically.

Example 1. Given three Q-ROFSs $\mathfrak{Y}$ and $\widehat{\mathfrak{Y}}$, and $\widetilde{\mathfrak{Y}}$ in $\mathfrak{D}=\{\mathscr{b}_1,\mathscr{b}_2,\mathscr{b}_3\}$, where

$\widehat{\mathfrak{Y}}=\{\langle\mathscr{b}_1,0.6,0.4\rangle,\langle\mathscr{b}_2,0.9,0.5\rangle,\langle\mathscr{b}_3,0.4,0.6\rangle\}.$

$\mathfrak{Y}=\{\langle\mathscr{b}_1,0.6,0.4\rangle,\langle\mathscr{b}_2,0.9,0.5\rangle,\langle\mathscr{b}_3,0.4,0.6\rangle\}.$

$\widetilde{\mathfrak{Y}}=\{\langle\mathscr{b}_1,0.6,0.6\rangle,\langle\mathscr{b}_2,0.5,0.8\rangle,\langle\mathscr{b}_3,0.8,0.9\rangle\}.$

If $q=4$, based on the new Q-ROFDM, the distance between $\mathfrak{B},\widehat{\mathfrak{B}}$ and $\widetilde{\mathfrak{B}}$ are computed as follows:

$d_{*}(\mathfrak{Y},\widehat{\mathfrak{Y}})=0,\quad d_{*}(\widehat{\mathfrak{Y}},\mathfrak{Y})=0.$

$d_{*}(\widehat{\mathfrak{Y}},\widetilde{\mathfrak{Y}})=0.0269,\quad d_{*}(\widetilde{\mathfrak{Y}},\widehat{\mathfrak{Y}})=0.0269.$

$d_{*}(\mathfrak{Y},\widetilde{\mathfrak{Y}})=0.0269,\quad d_{*}(\widetilde{\mathfrak{Y}},\mathfrak{Y})=0.0269.$

We can verify that the new Q-ROFDM satisfies the first three properties of a distance metric: $0\leq d_{*}(\mathfrak{Y},\widehat{\mathfrak{Y}})\leq 1$, $d(\mathfrak{Y},\widehat{\mathfrak{Y}})=0$ iff $\mathfrak{Y}=\widehat{\mathfrak{Y}}$, and $d(\mathfrak{Y},\widehat{\mathfrak{Y}})=d(\widehat{\mathfrak{Y}},\mathfrak{Y})$. In fact, these conditions hold for all values of q.

Example 2. Given three Q-ROFSs $\mathfrak{Y}$, $\widehat{\mathfrak{Y}}$, and $\widetilde{\mathfrak{Y}}$ in $\mathfrak{D}=\{\mathscr{b}_1,\mathscr{b}_2,\mathscr{b}_3\}$, where

$\widehat{\mathfrak{Y}}=\{\langle\mathscr{b}_1,0.54,0.35\rangle,\langle\mathscr{b}_2,0.8,0.56\rangle,\langle\mathscr{b}_3,0.45,0.56\rangle\}.$

$\mathfrak{Y}=\{\langle\mathscr{b}_1,0.6,0.4\rangle,\langle\mathscr{b}_2,0.8,0.5\rangle,\langle\mathscr{b}_3,0.4,0.6\rangle\}.$

$\widetilde{\mathfrak{Y}}=\{\langle\mathscr{b}_1,0.7,0.5\rangle,\langle\mathscr{b}_2,0.65,0.4\rangle,\langle\mathscr{b}_3,0.8,0.6\rangle\}.$

If $q=4$, based on the new Q-ROFDM, the distance between the Q-ROFSs $\mathfrak{Y}$, $\widehat{\mathfrak{Y}}$ and $\widetilde{\mathfrak{Y}}$ are computed as follows:

$d_{*}(\mathfrak{Y},\widetilde{\mathfrak{Y}})=0.0081,\quad d_{*}(\mathfrak{Y},\widehat{\mathfrak{Y}})=2.6\times10^{-6},\ d_{*}(\widehat{\mathfrak{Y}},\widetilde{\mathfrak{Y}})=0.0095.$

These distance values are represented by *Fig. 1* for quick understanding. From the distance values and *Fig. 1*, we see that, $d_{*}(\widehat{\mathfrak{Y}},\widetilde{\mathfrak{Y}})+d_{*}(\mathfrak{Y},\widehat{\mathfrak{Y}})=0.0095>d_{*}(\mathfrak{Y},\widetilde{\mathfrak{Y}})=0.0081$, Where $d_{*}(\mathfrak{Y},\widehat{\mathfrak{Y}})=A$, $d_{*}(\widehat{\mathfrak{Y}},\widetilde{\mathfrak{Y}})=B$, and $d_{*}(\mathfrak{Y},\widetilde{\mathfrak{Y}})=C$. Hence, the property $d_{*}(\mathfrak{Y},\widetilde{\mathfrak{Y}})\leq d_{*}(\widehat{\mathfrak{Y}},\widetilde{\mathfrak{Y}})+d_{*}(\mathfrak{Y},\widehat{\mathfrak{Y}})$ is verified. In fact, the Triangle inequality holds for all values of q. Next, we show the value of the new Q-ROFDM over the current Q-ROFDMs as follows.

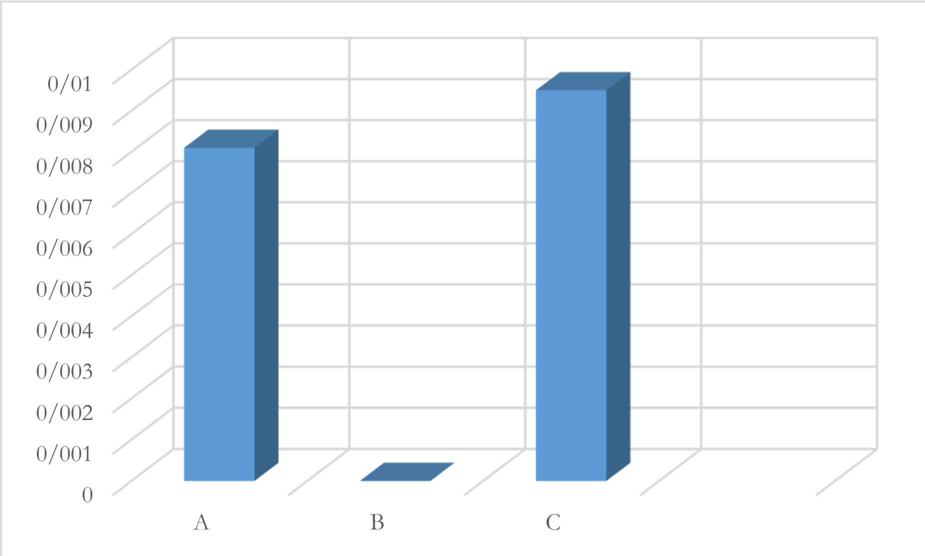

Fig. 1. Distance representation.

Example 3. Given two Q-ROFSs $\mathfrak{B}$ and $\widehat{\mathfrak{B}}$ in $\mathfrak{D} = \{\mathscr{b}_1, \mathscr{b}_2\}$ in two cases. To compare the effectiveness of the new Q-ROFDMs with the current Q-ROFDMs, two instances of Q-ROFSs are employed for $\mathrm{q} = 4, \mathrm{p} = 1$, and the comparison results are presented in *Table 2*. By observation, the Q-ROFSs in both cases are quite similar to each other. This experiment is to test the discriminating power of the new Q-ROFDM over the existing Q-ROFDMs. With the sharp similarity between the Q-ROFSs, the Q-ROFDMs are expected to yield very minimal distance values.

From *Table 2*, we notice that d_{D} [70] lacks the discriminating power, yields inappropriate and unreasonable results, and fails the distance axioms. The results from d_{PL7} [71], d_{PL9} [71], and d_{RK} [81] are negative, inappropriate, unreasonable, and fail the distance axioms. The results from d_{PL4}, d_{PL5}, d_{PL6}, d_{PL8} [71], d_{KP1} [76], and d_{Se} [84] are unreasonable because they do not capture the very close similarity between the considered Q-ROFSs. Although the distance approaches; d_{PL1}, d_{PL2}, d_{PL3}, d_{PL10}, d_{PL11} [71], d_{PB} [72], d_{KP2}, d_{KP3} [76], d_{He} [77], d_{E} [78], d_{A} [79], d_{We1} [82], d_{We2} [82], and d_{De} [83] yield results that are appropriate, reasonable, and fulfilling distance axioms, their rate of accuracy is not optimal. On the contrary, the new Q-ROFDM yields results that are appropriate, reasonable, fulfilling distance axioms, and the most precise of all the considered Q-ROFDMs.

Table 2. Comparative distance values.

Q-ROFDMs	Case I $\mathfrak{Y} = \langle \mathscr{b}_1, 0.2, 0.4\rangle$, $\langle \mathscr{b}_2, 0.7, 0.4\rangle$ $\widehat{\mathfrak{Y}} = \langle \mathscr{b}_1, 0.3, 0.45\rangle$, $\langle \mathscr{b}_2, 0.67, 0.3\rangle$	Case II $\mathfrak{Y} = \langle \mathscr{b}_1, 0.46, 0.4\rangle$, $\langle \mathscr{b}_2, 0.7, 0.4\rangle$ $\widehat{\mathfrak{Y}} = \langle \mathscr{b}_1, 0.5, 0.4\rangle$, $\langle \mathscr{b}_2, 0.6, 0.5\rangle$	A_1	A_2	A_3
d_*	0.0000036	0.0000228	✓	✓	✓
d_{D} [70]	1.0000000	1.0000000	✗	✗	✗
d_{PL1} [71]	0.0269975	0.0641127	✓	✓	✓
d_{PL2} [71]	0.0512289	0.1136523	✓	✓	✓
d_{PL3} [71]	0.0608851	0.1432375	✓	✓	✓
d_{PL4} [71]	0.3378523	0.4212939	✓	✗	✓
d_{PL5} [71]	0.6689262	0.7106470	✓	✗	✓
d_{PL6} [71]	0.7972525	0.7841626	✓	✗	✓
d_{PL7} [71]	-3.9795359	-3.2207082	✗	✗	✗
d_{PL8} [71]	0.5945049	0.5683252	✓	✗	✓
d_{PL9} [71]	-8.9590718	-7.4414165	✗	✗	✗

Table 2. Continued.

Q-ROFDMs	Case I $\mathfrak{Y} = \langle \ell_1, 0.2, 0.4 \rangle, \langle \ell_2, 0.7, 0.4 \rangle$ $\hat{\mathfrak{Y}} = \langle \ell_1, 0.3, 0.45 \rangle, \langle \ell_2, 0.67, 0.3 \rangle$	Case II $\mathfrak{Y} = \langle \ell_1, 0.46, 0.4 \rangle, \langle \ell_2, 0.7, 0.4 \rangle$ $\hat{\mathfrak{Y}} = \langle \ell_1, 0.5, 0.4 \rangle, \langle \ell_2, 0.6, 0.5 \rangle$	A_1	A_2	A_3
d_{PL10} [71]	0.0389975	0.0641127	✓	✓	✓
d_{PL11} [71]	0.0232481	0.052697	✓	✓	✓
d_{PB} [72]	0.0356364	0.0305455	✓	✓	✓
d_{KP1} [76]	0.1238778	0.2113200	✓	×	✓
d_{KP2} [76]	0.0375338	0.0602499	✓	✓	✓
d_{KP3} [76]	0.0750676	0.1204999	✓	✓	✓
d_{He} [77]	0.0269975	0.0641127	✓	✓	✓
d_{E} [78]	0.0259983	0.0427418	✓	✓	✓
d_{A} [79]	0.0367672	0.0604461	✓	✓	✓
d_{RK} [81]	0.3967586	-0.9883594	×	×	×
d_{We1} [82]	0.0729383	0.1144795	✓	✓	✓
d_{We2} [82]	0.0768005	0.1184342	✓	✓	✓
d_{De} [83]	0.0052683	0.0245604	✓	✓	✓
d_{Se} [84]	0.6651757	0.7531083	✓	×	✓

A_1: Appropriate outcomes A_2: Reasonable outcomes A_3: Fulfilling distance axioms

5 | Optimization of Energy Source Selection Process

The optimization of energy source selection is a complex DMP, and it is of environmental and industrial importance. With the increasing demand for sustainable energy solutions, selecting the most suitable energy source has become a crucial task. However, the inherent uncertainties and ambiguities associated with evaluating and prioritizing various renewable energy sources can be handled by introducing fuzziness into the DMP. Criteria like feasibility, risk of failure, and reliability, amongst others, must be considered, but their relative importance and interdependencies are often uncertain and subjective. This fuzziness complicates the optimization process, requiring innovative approaches to navigate the complexities in order to arrive at an optimal solution. Hence, we apply the new Q-ROFDM in the optimization of the energy source selection process based on MCGDM and MADM to achieve an efficient decision.

Linguistic variables

A Linguistic Variable (LV) is the use of numerical values to represent a linguistic term to enhance data collection based on a knowledge-based system. The LVs are: Very Very Low (VVL), Very Low (VL), Low (L), Moderately Low (ML), Extremely Low (EL), Fair (F), Moderately High (MH), Very Very High (VVH), Very High (VH), High (H), and Extremely High (EH). The LVs and their related numerical values are presented in *Table 3*.

Table 3. Linguistic terms and Q-ROFNs.

LVs	Q-ROFNs
EL	(0,1)
VVL	(0.05,0.9)
VL	(0.1,0.75)
L	(0.25,0.6)
ML	(0.4,0.5)
F	(0.5,0.4)
MH	(0.6,0.3)
H	(0.7,0.2)
VH	(0.8,0.15)
VVH	(0.9,0.1)
EH	(1,0)

Description of the criteria

The selection criteria for the energy sources are described as follows:

I. Feasibility ($\mathfrak{E}_1$): This criterion assesses the safety of the energy generation technology source implementation, by considering the number of effectively tested applications for the energy source.

II. Risk of failure ($\mathfrak{E}_2$): The risk criterion describes the security of the supply of an energy, itemizing the number of failures witnessed in the energy source.

III. Reliability ($\mathfrak{E}_3$): This is the reliability, the consistent quality of the electricity production of an energy resource.

IV. Local technical know-how ($\mathfrak{E}_4$): This criterion compares the energy source alternatives using the ability of local operators to provide necessary operating support for setting up and maintenance.

V. Pollutant emission ($\mathfrak{E}_5$): This criterion evaluates the pollution caused by the energy source alternatives. It takes into account the costs related to the waste treatment.

VI. Land requirements ($\mathfrak{E}_6$): This criterion assesses the energy source alternatives with regard to their areas of land requirement for the power installation.

VII. Waste disposal requirements ($\mathfrak{E}_7$): This criterion evaluates the level of wastes generated by the energy source alternatives, taking into account the environmental impact caused by the wastes.

VIII. Compatibility with national energy policy objectives ($\mathfrak{E}_8$): This criterion measures the energy source alternatives by their compatibility with the energy polices of a nation, where the government inquires whether the energy source alternatives align with the nation's energy goals and policies.

IX. Social acceptance ($\mathfrak{E}_9$): This criterion evaluates how the energy source alternatives are perceived by societies and communities, by evaluating any resistance to the energy sources.

X. Job creation ($\mathfrak{E}_{10}$): This criterion evaluates the energy source alternatives with regard to their direct and/or indirect job opportunities potentials, thereby affecting the overall economy of a nation.

XI. Energy tariff ($\mathfrak{E}_{11}$): This criterion discusses the rate at which energy is sold to consumers, which is directly related to the costs of generation, transmission, and distribution. This criterion assesses the costs associated with energy consumption, which varies based on the consumer's usage pattern, energy source type, and the time of day the consumption happens.

XII. Cost for generation, transmission, and distribution ($\mathfrak{E}_{12}$): The generation cost involves the cost insured for the generation of electric power, and the cost insured from the point of generation to the point where it is ready for distribution is referred to as the transmission cost. The distribution cost is the expenses incurred for transporting the electricity from the power plants to the consumers, and it includes infrastructure investment, maintenance costs, and operational costs. This criterion assesses the cost insured before power is ready to be consumed. The cost for generating, transmitting, and distributing energy varies based on the energy source type, location, market conditions, and infrastructure efficiency.

XIII. Energy storage capacity ($\mathfrak{E}_{13}$): This criterion refers to the amount of energy that the energy source alternatives have the capacity to store and provide when necessary. The criterion is primarily measured in either Kilowatt-Hours (kWh) or Megawatt-Hours (MWh).

XIV. Eco-friendly ($\mathfrak{E}_{14}$): This criterion measures how the energy source alternatives better the environment and climate conditions or worsen the environment and climate conditions over time.

XV. Security and health hazard ($\mathfrak{E}_{15}$): This criterion measures the security and health challenge the energy source alternative can pose to people in case of an accident.

XVI. Description of the energy source alternatives: The energy source alternatives considered for the selection analysis are based on renewable energy sources, fossil energy sources/natural gas, and nuclear energy sources. *Tables 4* and *5* clearly explained the energy source alternatives.

To determine the most suitable energy source for use based on expert knowledge, three energy source experts (I, II, and III) were approached to give their expert opinions on the seven energy source alternatives, and their opinions in terms of LVs are presented in *Table 6*.

Table 4. Energy source alternatives I.

Energy Source Alternatives	Type	Source(s)	Processes	Environmental Impacts	Advantages
Hydropower (E_1)	Renewable energy	Water source, water reservoir, dams	water source, turbine, generator, transmission	Disruption of the ecosystem via damming, fish migration, and alteration of natural water flows	Clean energy, cheap energy source, energy independence
Coal power (E_2)	Fossil fuel	coal	Coal mining, coal pulverization, combustion, steam generation, turbine and generator, cooling, transmission	Emission of carbon dioxide, habitat destruction, soil erosion, and water contamination	Cheap energy source, energy independence
Natural gas power (E_3)	Fossil fuel	Natural gas	Drilling and fracking, combustion in a gas turbine, turbine and generator, heat recovery, cooling, transmission	Emission of methane (A potent greenhouse gas), water pollution, and air pollution due to nitrogen oxides and sulphur dioxide	Emit less greenhouse gas than coal power, a cheap energy source, and energy independence
Wind power (E_4)	Renewable energy	Wind blow	Wind turbine, energy conversion, transmission	Noise pollution	No greenhouse gas emission, cleaner energy source, energy independence, etc.

Table 5. Energy source alternatives II.

Energy Source Alternatives	Type	Source(s)	Processes	Environmental Impacts	Advantages
Solar power (E_5)	Renewable energy [Photovoltaic Systems (PVS) and Solar Thermal Systems (STS)]	Sunlight	PVS: Solar panel, electricity generation, inverter, mounting structure, transmission STS: Collector, heat transfer fluid, electricity generation, transmission	Mining of silicon affects the environment, old panel wastes	No greenhouse gas emission, cleaner energy source, energy independence, job creation, etc.

Table 5. Continued.

Energy Source Alternatives	Type	Source(s)	Processes	Environmental Impacts	Advantages
Nuclear power (E_6)	Nuclear energy	Uranium	Nuclear fission, heat generation, turbine and generator, cooling, transmission	Poisonous nuclear waste deposits, air pollution, the environment, and high safety concerns	Reliable and efficient energy source, job creation, etc.
Biomass power (E_7)	Renewable energy	Wood waste, agricultural residues, animal wastes, algae, food waste, municipal solid wastes	Combustion, gasification, anaerobic digestion, fermentation, trans-esterification, electricity generation, and transmission	Desertification, air pollution, land pollution	Circular economy, waste management, energy independence, cleaner energy, job creation, etc.

Table 6. LVs from energy source experts.

Energy Sources and Criteria	Hydro Power (E_1)	Coal Power (E_2)	Natural Gas Power (E_3)	Wind Power (E_4)	Solar Power (E_5)	Nuclear Power (E_6)	Biomass Power (E_7)
$\mathfrak{E}_1$	VH, VH, H	H,VH,H	VH,VVH,VH	H,H,MH	VH,VH,VVH	VH,VVH,VVH	H,MH,VH
$\mathfrak{E}_2$	VH, VH, H	VH,H,MH	MH,F,F	VH,VH,H	L,L,ML	ML,L,L	MH,H,F
$\mathfrak{E}_3$	VH, VH, H	H,MH,VH	MH,VH,VH	MH,F,F	VH,VVH,VH	VH,VVH,VVH	VL,L,VL
$\mathfrak{E}_4$	H,MH,VH	H,VH,H	VH,H,VH	L,ML,ML	VVH,VH,VVH	L,VL,VVL	H,MH,H
$\mathfrak{E}_5$	MH,F,F	VH,VVH,VH	VH,VH,H	VL,VVL,VL	VVL,VL,EL	VH,VVH,VH	H,MH,H
$\mathfrak{E}_6$	MH,H,MH	MH,MH,MH	MH,MH,F	VVH,VH,VH	VVH,VH,H	MH,H,F	ML,L,F
$\mathfrak{E}_7$	F,MH,ML	H,MH,MH	ML,L,ML	VL,L,VVL	F,ML,F	VH,H,VH	VH,VVH,VH
$\mathfrak{E}_8$	VH,VH,VVH	ML,F,L	VH,VVH,VH	VH,VH,VVH	VVH,VVH,VH	F,ML,L	VH,VVH,VVH
$\mathfrak{E}_9$	VH,VH,VH	H,VH,MH	VH,VH,VVH	VVH,VH,VH	VVH,VVH,VVH	VH,VH,VH	F,ML,L
$\mathfrak{E}_{10}$	H,H,MH	VH,H,VH	H,H,H	MH,F,MH	VH,VH,VH	H,MH,MH	H,VH,H
$\mathfrak{E}_{11}$	VH,H,H	F,F,ML	VH,H,H	F,ML,ML	MH,MH,F	H,H,VH	ML,ML,F
$\mathfrak{E}_{12}$	H,H,H	H,H,MH	H,H,VH	MH,F,F	ML,L,ML	VH,VH,VH	F,F,ML
$\mathfrak{E}_{13}$	MH,MH,F	H,H,H	H,H,H	H,VH,H	VH,H,VH	VH,VH,H	H,MH,H
$\mathfrak{E}_{14}$	F,F,MH	L,L,VL	MH, MH,F	VVH, VVH,EH	VVH, EH,VVH	L, VL, VVL	VH,H,VH
$\mathfrak{E}_{15}$	ML,L,L	H,H,MH	L,L,ML	VVL,VVL,EL	EL,EL,VVL	VVH,EH,VH	ML,L,ML

The three LVs under each of the energy sources are from the three energy source experts. The LVs were converted to Q-ROFNs based on the information in *Table 3* to get *Tables 7-9*. To determine the most suitable renewable energy source for adoption based on the data collected from the three renewable energy source experts, the techniques of MCGDM and MADM are deployed.

5.1 | Multi Criteria Group Decision Making Algorithm

Consider an MCGDM problem with "p" alternatives $E_i = \{E_1, E_2, \ldots, E_p\}$, "r" criteria $\mathfrak{E}_j = \{\mathfrak{E}_1, \mathfrak{E}_2, \ldots, \mathfrak{E}_r\}$, and l experts $k = \{e_1, e_2, \ldots, e_l\}$ were requested to evaluate the alternatives with the weight vectors $\varpi = \{\varpi_1, \varpi_2, \ldots, \varpi_r\}$, satisfying $\varpi_j \in [0,1]$ and $\sum_{j=1}^{r} \varpi_j = 1$. Using the previous information, the MCGDM algorithm is presented as follows:

Step 1. Obtain the Q-ROF decision matrices using the data given by the l experts as follows:

$$
\begin{array}{cc}
 & \begin{array}{ccccc} \mathfrak{E}_1 & \ldots & \mathfrak{E}_j & \ldots & \mathfrak{E}_r, \end{array} \\
Q^{(k)} = \begin{array}{c} E_1 \\ \vdots \\ E_i \\ \vdots \\ E_p \end{array} &
\begin{pmatrix}
\langle E_{m_{11}}^{(k)}, E_{n_{11}}^{(k)} \rangle & \ldots & \langle E_{m_{1j}}^{(k)}, E_{n_{1j}}^{(k)} \rangle & \ldots & \langle E_{m_{1r}}^{(k)}, E_{n_{1r}}^{(k)} \rangle \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
\langle E_{m_{i1}}^{(k)}, E_{n_{i1}}^{(k)} \rangle & \ldots & \langle E_{m_{ij}}^{(k)}, E_{n_{ij}}^{(k)} \rangle & \ldots & \langle E_{m_{ir}}^{(k)}, E_{n_{ir}}^{(k)} \rangle \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
\langle E_{m_{p1}}^{(k)}, E_{n_{p1}}^{(k)} \rangle & \ldots & \langle E_{m_{pj}}^{(k)}, E_{n_{pj}}^{(k)} \rangle & \ldots & \langle E_{m_{pr}}^{(k)}, E_{n_{pr}}^{(k)} \rangle
\end{pmatrix},
\end{array} \tag{9}
$$

where $Q^{(k)} = \left[Q_{ij}^{(k)}\right]_{p\times q} = \left[m_{ij}^{(k)}, n_{ij}^{(k)}\right]_{p\times r}$ and $k = 1,2,\ldots,l$.

Step 2. Find the benefit criteria and cost criterion for the criteria, where the cost criterion is the least criterion with the least membership grade, and the other criteria are the benefit criteria.

Step 3. Use the criteria in *Step 2* to normalize the Q-ROF decision matrices $\widetilde{Q}^{(k)}$ via *Eq. (10)*.

$$
\widetilde{Q}^{(k)} = \begin{cases} Q_{ij}^{(k)}, & \text{for benefit criteria,} \\ Q_{ij}^{(k)^c}, & \text{for cost criteria.} \end{cases} \tag{10}
$$

Step 4. Take the average of $Q^{(k)}$ by the number of the experts and then find the Q-ROF decision matrix average Q-ROFMA using *Eq. (11)*.

$$
\begin{array}{cc}
 & \begin{array}{ccccc} \mathfrak{E}_1 & \ldots & \mathfrak{E}_j & \ldots & \mathfrak{E}_q, \end{array} \\
\left[\widehat{Q}_{ij}\right]_{p\times q} = \left[\widehat{m}_{ij}, \widehat{n}_{ij}\right]_{p\times r} = \begin{array}{c} E_1 \\ \vdots \\ E_i \\ \vdots \\ E_p \end{array} &
\begin{pmatrix}
\langle \widehat{m}_{11}, \widehat{n}_{11} \rangle & \ldots & \langle \widehat{m}_{1j}, \widehat{n}_{1j} \rangle & \ldots & \langle \widehat{m}_{1q}, \widehat{n}_{1q} \rangle \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
\langle \widehat{m}_{i1}, \widehat{n}_{i1} \rangle & \ldots & \langle m_{ij}^{(k)}, n_{ij}^{(k)} \rangle & \ldots & \langle \widehat{m}_{iq}, \widehat{n}_{iq} \rangle \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
\langle \widehat{m}_{p1}, \widehat{n}_{p1} \rangle & \ldots & \langle m_{pj}^{(k)}, n_{pj}^{(k)} \rangle & \ldots & \langle \widehat{m}_{pq}, \widehat{n}_{pq} \rangle
\end{pmatrix},
\end{array} \tag{11}
$$

where $\langle \widehat{m}_{ij}, \widehat{n}_{ij} \rangle = \langle \sqrt[q]{1 - \prod_{k=1}^{l}\left(1 - \left(m_{Q_k}\right)^q\right)^{\varpi_j}},\ \prod_{k=1}^{l}\left(n_{Q_k}\right)^{\varpi_j} \rangle$.

Step 5. Find the ideal alternative, $E^+ = \{E_1^+, E_2^+, \ldots, E_p^+\}$, where E_i^+ is determined based on *Eq. (12)*.

$$
E_i^+ = \max_{1\le i\le p} \widehat{Q}_{ij}. \tag{12}
$$

Step 6. Compute the distance $d_*(E_i, E^+)$ between the ideal alternative E^+ and each of the alternatives, E_i, using the discussed distance methods.

Step 7. Arrange the alternatives E_i according to the outcomes obtained in *Step 6*.

Step 8. Decide the best energy source alternative based on the highest ordering rank. The algorithm of the MCGDM technique is represented in *Fig. 2*.

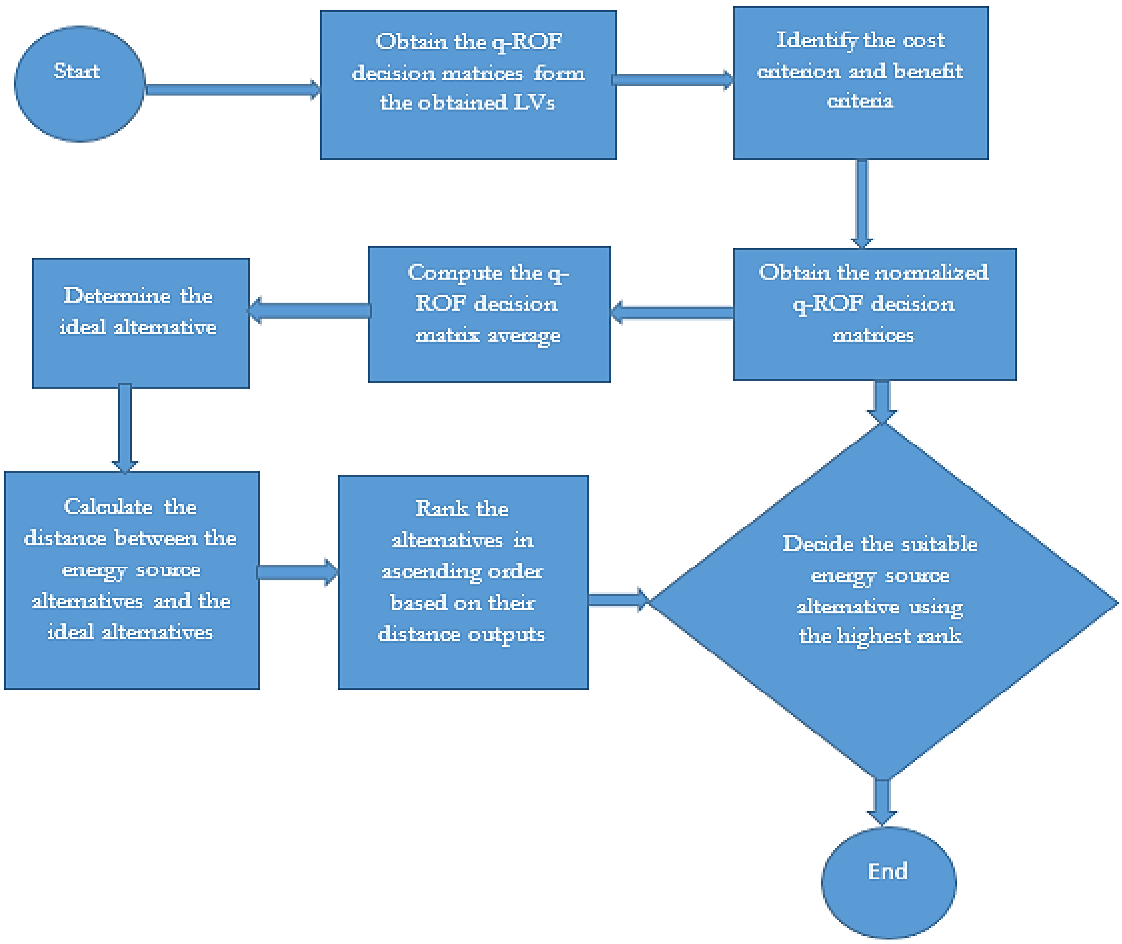

Fig. 2. Flowchart of MCGDM.

5.1.1 | Implementation of the multi criteria group decision making algorithm

Now, we use the MCGDM's algorithm to obtain a suitable decision. Using *Step 1*, we obtain the Q-ROF decision matrices as presented in *Tables 7-9*.

Table 7. Q-ROFNs from energy source expert I.

Energy Sources and Criteria	Hydro Power (E_1)	Coal Power (E_2)	Natural Gas Power (E_3)	Wind Power (E_4)	Solar Power (E_5)	Nuclear Power (E_6)	Biomass Power (E_7)
$\mathfrak{E}_1$	$\langle 0.80, 0.15 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.70, 0.20 \rangle$
$\mathfrak{E}_2$	$\langle 0.80, 0.15 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.25, 0.60 \rangle$	$\langle 0.40, 0.50 \rangle$	$\langle 0.60, 0.30 \rangle$
$\mathfrak{E}_3$	$\langle 0.80, 0.15 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.10, 0.75 \rangle$
$\mathfrak{E}_4$	$\langle 0.70, 0.20 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.25, 0.60 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.25, 0.60 \rangle$	$\langle 0.70, 0.20 \rangle$
$\mathfrak{E}_5$	$\langle 0.60, 0.30 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.10, 0.75 \rangle$	$\langle 0.05, 0.90 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.70, 0.20 \rangle$
$\mathfrak{E}_6$	$\langle 0.60, 0.30 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.40, 0.50 \rangle$

Table 7. Continued.

Energy Sources and Criteria	Hydro Power $(\mathbf{E_1})$	Coal Power $(\mathbf{E_2})$	Natural Gas Power $(\mathbf{E_3})$	Wind Power $(\mathbf{E_4})$	Solar Power $(\mathbf{E_5})$	Nuclear Power $(\mathbf{E_6})$	Biomass Power $(\mathbf{E_7})$
$\mathfrak{E}_7$	$\langle 0.50, 0.40\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.40, 0.50\rangle$	$\langle 0.10, 0.75\rangle$	$\langle 0.50, 0.40\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.80, 0.15\rangle$
$\mathfrak{E}_8$	$\langle 0.80, 0.15\rangle$	$\langle 0.50, 0.40\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.90, 0.10\rangle$	$\langle 0.50, 0.40\rangle$	$\langle 0.80, 0.15\rangle$
$\mathfrak{E}_9$	$\langle 0.80, 0.15\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.90, 0.10\rangle$	$\langle 0.90, 0.10\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.50, 0.40\rangle$
$\mathfrak{E}_{10}$	$\langle 0.70, 0.20\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.60, 0.30\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.70, 0.20\rangle$
$\mathfrak{E}_{11}$	$\langle 0.80, 0.15\rangle$	$\langle 0.50, 0.40\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.50, 0.40\rangle$	$\langle 0.60, 0.30\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.40, 0.50\rangle$
$\mathfrak{E}_{12}$	$\langle 0.70, 0.20\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.60, 0.30\rangle$	$\langle 0.40, 0.50\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.50, 0.40\rangle$
$\mathfrak{E}_{13}$	$\langle 0.60, 0.30\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.70, 0.20\rangle$
$\mathfrak{E}_{14}$	$\langle 0.50, 0.40\rangle$	$\langle 0.25, 0.60\rangle$	$\langle 0.60, 0.30\rangle$	$\langle 0.90, 0.10\rangle$	$\langle 0.90, 0.10\rangle$	$\langle 0.25, 0.60\rangle$	$\langle 0.80, 0.15\rangle$
$\mathfrak{E}_{15}$	$\langle 0.40, 0.50\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.25, 0.60\rangle$	$\langle 0.05, 0.90\rangle$	$\langle 0.00, 1.00\rangle$	$\langle 0.90, 0.10\rangle$	$\langle 0.40, 0.50\rangle$

Table 8. Q-ROFNs from energy source expert II.

Energy Sources and Criteria	Hydro Power $(\mathbf{E_1})$	Coal Power $(\mathbf{E_2})$	Natural Gas Power $(\mathbf{E_3})$	Wind Power $(\mathbf{E_4})$	Solar Power $(\mathbf{E_5})$	Nuclear Power $(\mathbf{E_6})$	Biomass Power $(\mathbf{E_7})$
$\mathfrak{E}_1$	$\langle 0.80, 0.15\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.90, 0.10\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.90, 0.10\rangle$	$\langle 0.60, 0.30\rangle$
$\mathfrak{E}_2$	$\langle 0.80, 0.15\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.50, 0.40\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.25, 0.60\rangle$	$\langle 0.25, 0.60\rangle$	$\langle 0.70, 0.20\rangle$
$\mathfrak{E}_3$	$\langle 0.80, 0.15\rangle$	$\langle 0.60, 0.30\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.50, 0.40\rangle$	$\langle 0.90, 0.10\rangle$	$\langle 0.90, 0.10\rangle$	$\langle 0.25, 0.60\rangle$
$\mathfrak{E}_4$	$\langle 0.60, 0.30\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.40, 0.50\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.10, 0.75\rangle$	$\langle 0.60, 0.30\rangle$
$\mathfrak{E}_5$	$\langle 0.50, 0.40\rangle$	$\langle 0.90, 0.10\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.05, 0.90\rangle$	$\langle 0.10, 0.75\rangle$	$\langle 0.90, 0.10\rangle$	$\langle 0.60, 0.30\rangle$
$\mathfrak{E}_6$	$\langle 0.70, 0.20\rangle$	$\langle 0.60, 0.30\rangle$	$\langle 0.60, 0.30\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.25, 0.60\rangle$
$\mathfrak{E}_7$	$\langle 0.60, 0.30\rangle$	$\langle 0.60, 0.30\rangle$	$\langle 0.25, 0.60\rangle$	$\langle 0.25, 0.60\rangle$	$\langle 0.40, 0.50\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.90, 0.10\rangle$
$\mathfrak{E}_8$	$\langle 0.80, 0.15\rangle$	$\langle 0.50, 0.40\rangle$	$\langle 0.90, 0.10\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.90, 0.10\rangle$	$\langle 0.40, 0.50\rangle$	$\langle 0.90, 0.10\rangle$
$\mathfrak{E}_9$	$\langle 0.80, 0.15\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.90, 0.10\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.40, 0.50\rangle$
$\mathfrak{E}_{10}$	$\langle 0.70, 0.20\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.50, 0.40\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.60, 0.30\rangle$	$\langle 0.80, 0.15\rangle$
$\mathfrak{E}_{11}$	$\langle 0.70, 0.20\rangle$	$\langle 0.50, 0.40\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.40, 0.50\rangle$	$\langle 0.60, 0.30\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.40, 0.50\rangle$
$\mathfrak{E}_{12}$	$\langle 0.70, 0.20\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.50, 0.40\rangle$	$\langle 0.25, 0.60\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.50, 0.40\rangle$
$\mathfrak{E}_{13}$	$\langle 0.60, 0.30\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.80, 0.15\rangle$	$\langle 0.60, 0.30\rangle$
$\mathfrak{E}_{14}$	$\langle 0.50, 0.40\rangle$	$\langle 0.25, 0.60\rangle$	$\langle 0.60, 0.30\rangle$	$\langle 0.90, 0.10\rangle$	$\langle 1.00, 0.00\rangle$	$\langle 0.10, 0.75\rangle$	$\langle 0.70, 0.20\rangle$
$\mathfrak{E}_{15}$	$\langle 0.25, 0.60\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.25, 0.60\rangle$	$\langle 0.05, 0.90\rangle$	$\langle 0.00, 1.00\rangle$	$\langle 1.00, 0.00\rangle$	$\langle 0.25, 0.60\rangle$

Table 9. Q-ROFNs from energy source expert III.

Energy Sources and Criteria	Hydro Power (E_1)	Coal Power (E_2)	Natural Gas Power (E_3)	Wind Power (E_4)	Solar Power (E_5)	Nuclear Power (E_6)	Biomass Power (E_7)
$\mathfrak{E}_1$	$\langle 0.70, 0.20 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.80, 0.15 \rangle$
$\mathfrak{E}_2$	$\langle 0.70, 0.20 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.50, 0.40 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.40, 0.50 \rangle$	$\langle 0.25, 0.60 \rangle$	$\langle 0.50, 0.40 \rangle$
$\mathfrak{E}_3$	$\langle 0.70, 0.20 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.50, 0.40 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.10, 0.75 \rangle$
$\mathfrak{E}_4$	$\langle 0.80, 0.15 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.40, 0.50 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.05, 0.90 \rangle$	$\langle 0.70, 0.20 \rangle$
$\mathfrak{E}_5$	$\langle 0.50, 0.40 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.10, 0.75 \rangle$	$\langle 0.00, 1.00 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.70, 0.20 \rangle$
$\mathfrak{E}_6$	$\langle 0.60, 0.30 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.50, 0.40 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.50, 0.40 \rangle$	$\langle 0.50, 0.40 \rangle$
$\mathfrak{E}_7$	$\langle 0.40, 0.50 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.40, 0.50 \rangle$	$\langle 0.05, 0.90 \rangle$	$\langle 0.50, 0.40 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.80, 0.15 \rangle$
$\mathfrak{E}_8$	$\langle 0.90, 0.10 \rangle$	$\langle 0.25, 0.60 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.25, 0.60 \rangle$	$\langle 0.90, 0.10 \rangle$
$\mathfrak{E}_9$	$\langle 0.80, 0.15 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.25, 0.60 \rangle$
$\mathfrak{E}_{10}$	$\langle 0.60, 0.30 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.70, 0.20 \rangle$
$\mathfrak{E}_{11}$	$\langle 0.70, 0.20 \rangle$	$\langle 0.40, 0.50 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.40, 0.50 \rangle$	$\langle 0.50, 0.40 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.50, 0.40 \rangle$
$\mathfrak{E}_{12}$	$\langle 0.70, 0.20 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.50, 0.40 \rangle$	$\langle 0.40, 0.50 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.40, 0.50 \rangle$
$\mathfrak{E}_{13}$	$\langle 0.50, 0.40 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.70, 0.20 \rangle$
$\mathfrak{E}_{14}$	$\langle 0.60, 0.30 \rangle$	$\langle 0.10, 0.75 \rangle$	$\langle 0.50, 0.40 \rangle$	$\langle 1.00, 0.00 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.05, 0.90 \rangle$	$\langle 0.80, 0.15 \rangle$
$\mathfrak{E}_{15}$	$\langle 0.25, 0.60 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.40, 0.50 \rangle$	$\langle 0.00, 1.00 \rangle$	$\langle 0.05, 0.90 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.40, 0.50 \rangle$

By *Step 2*, the cost criterion is the security and health hazard, represented by $\mathfrak{E}_{15}$.Then, $\mathfrak{E}_{15}$ is used to get the normalized Q-ROF decision matrices $\widetilde{Q}^{(k)}$ by *Step 3*, as presented in *Tables 10-12*.

Table 10. Normalized Q-ROF decision matrix from energy source expert I.

Energy Sources and Criteria	Hydro Power (E_1)	Coal Power (E_2)	Natural Gas Power (E_3)	Wind Power (E_4)	Solar Power (E_5)	Nuclear Power (E_6)	Biomass Power (E_7)
$\mathfrak{E}_1$	$\langle 0.80, 0.15 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.70, 0.20 \rangle$
$\mathfrak{E}_2$	$\langle 0.80, 0.15 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.25, 0.60 \rangle$	$\langle 0.40, 0.50 \rangle$	$\langle 0.60, 0.30 \rangle$
$\mathfrak{E}_3$	$\langle 0.80, 0.15 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.10, 0.75 \rangle$
$\mathfrak{E}_4$	$\langle 0.70, 0.20 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.25, 0.60 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.25, 0.60 \rangle$	$\langle 0.70, 0.20 \rangle$
$\mathfrak{E}_5$	$\langle 0.60, 0.30 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.10, 0.75 \rangle$	$\langle 0.05, 0.90 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.70, 0.20 \rangle$
$\mathfrak{E}_6$	$\langle 0.60, 0.30 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.40, 0.50 \rangle$
$\mathfrak{E}_7$	$\langle 0.50, 0.40 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.40, 0.50 \rangle$	$\langle 0.10, 0.75 \rangle$	$\langle 0.50, 0.40 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.80, 0.15 \rangle$
$\mathfrak{E}_8$	$\langle 0.80, 0.15 \rangle$	$\langle 0.50, 0.40 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.50, 0.40 \rangle$	$\langle 0.80, 0.15 \rangle$
$\mathfrak{E}_9$	$\langle 0.80, 0.15 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.50, 0.40 \rangle$
$\mathfrak{E}_{10}$	$\langle 0.70, 0.20 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.70, 0.20 \rangle$

Table 10. Continued.

Energy Sources and Criteria	Hydro Power (E_1)	Coal Power (E_2)	Natural Gas Power (E_3)	Wind Power (E_4)	Solar Power (E_5)	Nuclear Power (E_6)	Biomass Power (E_7)
$\mathfrak{E}_{11}$	$\langle 0.80, 0.15 \rangle$	$\langle 0.50, 0.40 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.50, 0.40 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.40, 0.50 \rangle$
$\mathfrak{E}_{12}$	$\langle 0.70, 0.20 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.40, 0.50 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.50, 0.40 \rangle$
$\mathfrak{E}_{13}$	$\langle 0.60, 0.30 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.70, 0.20 \rangle$
$\mathfrak{E}_{14}$	$\langle 0.50, 0.40 \rangle$	$\langle 0.25, 0.60 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.25, 0.60 \rangle$	$\langle 0.80, 0.15 \rangle$
$\mathfrak{E}_{15}$	$\langle 0.50, 0.40 \rangle$	$\langle 0.20, 0.70 \rangle$	$\langle 0.60, 0.25 \rangle$	$\langle 0.90, 0.05 \rangle$	$\langle 1.00, 0.00 \rangle$	$\langle 0.10, 0.90 \rangle$	$\langle 0.50, 0.40 \rangle$

Table 11. Normalized Q-ROF decision matrix from energy source expert II.

Energy Sources and Criteria	Hydro Power (E_1)	Coal Power (E_2)	Natural Gas Power (E_3)	Wind Power (E_4)	Solar Power (E_5)	Nuclear Power (E_6)	Biomass Power (E_7)
$\mathfrak{E}_1$	$\langle 0.80, 0.15 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.60, 0.30 \rangle$
$\mathfrak{E}_2$	$\langle 0.80, 0.15 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.50, 0.40 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.25, 0.60 \rangle$	$\langle 0.25, 0.60 \rangle$	$\langle 0.70, 0.20 \rangle$
$\mathfrak{E}_3$	$\langle 0.80, 0.15 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.50, 0.40 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.25, 0.60 \rangle$
$\mathfrak{E}_4$	$\langle 0.60, 0.30 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.40, 0.50 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.10, 0.75 \rangle$	$\langle 0.60, 0.30 \rangle$
$\mathfrak{E}_5$	$\langle 0.50, 0.40 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.05, 0.90 \rangle$	$\langle 0.10, 0.75 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.60, 0.30 \rangle$
$\mathfrak{E}_6$	$\langle 0.70, 0.20 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.25, 0.60 \rangle$
$\mathfrak{E}_7$	$\langle 0.60, 0.30 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.25, 0.60 \rangle$	$\langle 0.25, 0.60 \rangle$	$\langle 0.40, 0.50 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.90, 0.10 \rangle$
$\mathfrak{E}_8$	$\langle 0.80, 0.15 \rangle$	$\langle 0.50, 0.40 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.40, 0.50 \rangle$	$\langle 0.90, 0.10 \rangle$
$\mathfrak{E}_9$	$\langle 0.80, 0.15 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.40, 0.50 \rangle$
$\mathfrak{E}_{10}$	$\langle 0.70, 0.20 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.50, 0.40 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.80, 0.15 \rangle$
$\mathfrak{E}_{11}$	$\langle 0.70, 0.20 \rangle$	$\langle 0.50, 0.40 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.40, 0.50 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.40, 0.50 \rangle$
$\mathfrak{E}_{12}$	$\langle 0.70, 0.20 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.50, 0.40 \rangle$	$\langle 0.25, 0.60 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.50, 0.40 \rangle$
$\mathfrak{E}_{13}$	$\langle 0.60, 0.30 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.70, 0.20 \rangle$	$\langle 0.80, 0.15 \rangle$	$\langle 0.60, 0.30 \rangle$
$\mathfrak{E}_{14}$	$\langle 0.50, 0.40 \rangle$	$\langle 0.25, 0.60 \rangle$	$\langle 0.60, 0.30 \rangle$	$\langle 0.90, 0.10 \rangle$	$\langle 1.00, 0.00 \rangle$	$\langle 0.10, 0.75 \rangle$	$\langle 0.70, 0.20 \rangle$
$\mathfrak{E}_{15}$	$\langle 0.60, 0.25 \rangle$	$\langle 0.20, 0.70 \rangle$	$\langle 0.60, 0.25 \rangle$	$\langle 0.90, 0.05 \rangle$	$\langle 1.00, 0.00 \rangle$	$\langle 0.00, 1.00 \rangle$	$\langle 0.60, 0.25 \rangle$

Table 12. Normalized Q-ROF decision matrix from energy source expert III.

Energy Sources and Criteria	Hydro power (E_1)	Coal Power (E_2)	Natural Gas Power (E_3)	Wind Power (E_4)	Solar Power (E_5)	Nuclear Power (E_6)	Biomass Power (E_7)
$\mathfrak{E}_1$	⟨0.70, 0.20⟩	⟨0.70, 0.20⟩	⟨0.80, 0.15⟩	⟨0.60, 0.30⟩	⟨0.90, 0.10⟩	⟨0.90, 0.10⟩	⟨0.80, 0.15⟩
$\mathfrak{E}_2$	⟨0.70, 0.20⟩	⟨0.60, 0.30⟩	⟨0.50, 0.40⟩	⟨0.70, 0.20⟩	⟨0.40, 0.50⟩	⟨0.25, 0.60⟩	⟨0.50, 0.40⟩
$\mathfrak{E}_3$	⟨0.70, 0.20⟩	⟨0.80, 0.15⟩	⟨0.80, 0.15⟩	⟨0.50, 0.40⟩	⟨0.80, 0.15⟩	⟨0.90, 0.10⟩	⟨0.10, 0.75⟩
$\mathfrak{E}_4$	⟨0.80, 0.15⟩	⟨0.70, 0.20⟩	⟨0.80, 0.15⟩	⟨0.40, 0.50⟩	⟨0.90, 0.10⟩	⟨0.05, 0.90⟩	⟨0.70, 0.20⟩
$\mathfrak{E}_5$	⟨0.50, 0.40⟩	⟨0.80, 0.15⟩	⟨0.70, 0.20⟩	⟨0.10, 0.75⟩	⟨0.00, 1.00⟩	⟨0.80, 0.15⟩	⟨0.70, 0.20⟩
$\mathfrak{E}_6$	⟨0.60, 0.30⟩	⟨0.60, 0.30⟩	⟨0.50, 0.40⟩	⟨0.80, 0.15⟩	⟨0.70, 0.20⟩	⟨0.50, 0.40⟩	⟨0.50, 0.40⟩
$\mathfrak{E}_7$	⟨0.40, 0.50⟩	⟨0.60, 0.30⟩	⟨0.40, 0.50⟩	⟨0.05, 0.90⟩	⟨0.50, 0.40⟩	⟨0.80, 0.15⟩	⟨0.80, 0.15⟩
$\mathfrak{E}_8$	⟨0.90, 0.10⟩	⟨0.25, 0.60⟩	⟨0.80, 0.15⟩	⟨0.90, 0.10⟩	⟨0.80, 0.15⟩	⟨0.25, 0.60⟩	⟨0.90, 0.10⟩
$\mathfrak{E}_9$	⟨0.80, 0.15⟩	⟨0.60, 0.30⟩	⟨0.90, 0.10⟩	⟨0.80, 0.15⟩	⟨0.90, 0.10⟩	⟨0.80, 0.15⟩	⟨0.25, 0.60⟩
$\mathfrak{E}_{10}$	⟨0.60, 0.30⟩	⟨0.80, 0.15⟩	⟨0.70, 0.20⟩	⟨0.60, 0.30⟩	⟨0.80, 0.15⟩	⟨0.60, 0.30⟩	⟨0.70, 0.20⟩
$\mathfrak{E}_{11}$	⟨0.70, 0.20⟩	⟨0.40, 0.50⟩	⟨0.70, 0.20⟩	⟨0.40, 0.50⟩	⟨0.50, 0.40⟩	⟨0.80, 0.15⟩	⟨0.50, 0.40⟩
$\mathfrak{E}_{12}$	⟨0.70, 0.20⟩	⟨0.60, 0.30⟩	⟨0.80, 0.15⟩	⟨0.50, 0.40⟩	⟨0.40, 0.50⟩	⟨0.80, 0.15⟩	⟨0.40, 0.50⟩
$\mathfrak{E}_{13}$	⟨0.50, 0.40⟩	⟨0.70, 0.20⟩	⟨0.70, 0.20⟩	⟨0.70, 0.20⟩	⟨0.80, 0.15⟩	⟨0.70, 0.20⟩	⟨0.70, 0.20⟩
$\mathfrak{E}_{14}$	⟨0.60, 0.30⟩	⟨0.10, 0.75⟩	⟨0.50, 0.40⟩	⟨1.00, 0.00⟩	⟨0.90, 0.10⟩	⟨0.05, 0.90⟩	⟨0.80, 0.15⟩
$\mathfrak{E}_{15}$	⟨0.60, 0.25⟩	⟨0.30, 0.60⟩	⟨0.50, 0.40⟩	⟨1.00, 0.00⟩	⟨0.90, 0.05⟩	⟨0.15, 0.80⟩	⟨0.50, 0.40⟩

By *Step 4*, we compute the weight-criteria for $q = 4, 5, \ldots, 8$, which are presented in *Table 13*.

Table 13. Computed weights.

Weights	$q = 4$	$q = 5$	$q = 6$	$q = 7$	$q = 8$
w_1	0.0778	0.0759	0.0746	0.0733	0.0721
w_2	0.0574	0.0582	0.0585	0.0590	0.0595
w_3	0.0700	0.0693	0.0688	0.0683	0.0679
w_4	0.0644	0.0639	0.0636	0.0635	0.0634
w_5	0.0652	0.0647	0.0648	0.0649	0.0649
w_6	0.0617	0.0621	0.0621	0.0622	0.0624
w_7	0.0580	0.0590	0.0597	0.0604	0.0611
w_8	0.0781	0.0777	0.0775	0.0769	0.0760
w_9	0.0792	0.0781	0.0773	0.0764	0.0753
w_{10}	0.0660	0.0650	0.0640	0.0633	0.0630
w_{11}	0.0569	0.0579	0.0582	0.0587	0.0592
w_{12}	0.0595	0.0600	0.0601	0.0603	0.0606
w_{13}	0.0662	0.0650	0.0639	0.0631	0.0627
w_{14}	0.0706	0.0720	0.0735	0.0746	0.0753
w_{15}	0.0690	0.0711	0.0733	0.0751	0.0765

The Q-ROFMA for $q = 4$, is presented in *Table 14*. In addition, the alternative deal is also obtained and presented in *Table 14* by *Step 5*.

Table 14. Q-ROFMA and the ideal alternative based on MCGDM approach.

Energy Sources	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E^+
$\mathfrak{E}_1$	$\langle 0.7763, 0.1635\rangle$	$\langle 0.7480, 0.1783\rangle$	$\langle 0.8510, 0.1275\rangle$	$\langle 0.6757, 0.2259\rangle$	$\langle 0.8400, 0.1328\rangle$	$\langle 0.8785, 0.1129\rangle$	$\langle 0.7763, 0.1635\rangle$	$\langle 0.8785, 0.1129\rangle$
$\mathfrak{E}_2$	$\langle 0.7373, 0.1835\rangle$	$\langle 0.7183, 0.2072\rangle$	$\langle 0.5370, 0.3669\rangle$	$\langle 0.7763, 0.1635\rangle$	$\langle 0.3198, 0.5681\rangle$	$\langle 0.3198, 0.5681\rangle$	$\langle 0.7303, 0.2013\rangle$	$\langle 0.7763, 0.1635\rangle$
$\mathfrak{E}_3$	$\langle 0.7763, 0.1635\rangle$	$\langle 0.7114, 0.2158\rangle$	$\langle 0.7617, 0.1847\rangle$	$\langle 0.5370, 0.3669\rangle$	$\langle 0.8510, 0.5078\rangle$	$\langle 0.8785, 0.1129\rangle$	$\langle 0.7453, 0.2431\rangle$	$\langle 0.8785, 0.1129\rangle$
$\mathfrak{E}_4$	$\langle 0.7114, 0.2158\rangle$	$\langle 0.7480, 0.1783\rangle$	$\langle 0.7674, 0.1683\rangle$	$\langle 0.3720, 0.5281\rangle$	$\langle 0.8701, 0.2347\rangle$	$\langle 0.1922, 0.7409\rangle$	$\langle 0.6757, 0.2259\rangle$	$\langle 0.8701, 0.2347\rangle$
$\mathfrak{E}_5$	$\langle 0.5370, 0.3669\rangle$	$\langle 0.8510, 0.1275\rangle$	$\langle 0.7763, 0.1635\rangle$	$\langle 0.1273, 0.8067\rangle$	$\langle 0.1177, 0.8636\rangle$	$\langle 0.8510, 0.1275\rangle$	$\langle 0.6176, 0.2896\rangle$	$\langle 0.8510, 0.1275\rangle$
$\mathfrak{E}_6$	$\langle 0.6473, 0.2551\rangle$	$\langle 0.6000, 0.3000\rangle$	$\langle 0.5759, 0.3270\rangle$	$\langle 0.8400, 0.1328\rangle$	$\langle 0.8229, 0.1448\rangle$	$\langle 0.6295, 0.2781\rangle$	$\langle 0.6068, 0.3096\rangle$	$\langle 0.8400, 0.1328\rangle$
$\mathfrak{E}_7$	$\langle 0.5307, 0.3812\rangle$	$\langle 0.6367, 0.2656\rangle$	$\langle 0.3610, 0.5378\rangle$	$\langle 0.2044, 0.7245\rangle$	$\langle 0.4679, 0.4373\rangle$	$\langle 0.7674, 0.1683\rangle$	$\langle 0.6703, 0.2734\rangle$	$\langle 0.7674, 0.1683\rangle$
$\mathfrak{E}_8$	$\langle 0.8400, 0.1328\rangle$	$\langle 0.4613, 0.4517\rangle$	$\langle 0.8510, 0.1275\rangle$	$\langle 0.8400, 0.1328\rangle$	$\langle 0.8785, 0.1129\rangle$	$\langle 0.4177, 0.4939\rangle$	$\langle 0.8000, 0.1500\rangle$	$\langle 0.8785, 0.1129\rangle$
$\mathfrak{E}_9$	$\langle 0.8000, 0.1500\rangle$	$\langle 0.7303, 0.2013\rangle$	$\langle 0.8400, 0.1328\rangle$	$\langle 0.8400, 0.1328\rangle$	$\langle 0.9000, 0.1000\rangle$	$\langle 0.8000, 0.1500\rangle$	$\langle 0.7531, 0.2013\rangle$	$\langle 0.9000, 0.1000\rangle$
$\mathfrak{E}_{10}$	$\langle 0.6757, 0.2259\rangle$	$\langle 0.7674, 0.1683\rangle$	$\langle 0.7000, 0.2000\rangle$	$\langle 0.5670, 0.3366\rangle$	$\langle 0.8000, 0.1500\rangle$	$\langle 0.6367, 0.2656\rangle$	$\langle 0.7000, 0.2000\rangle$	$\langle 0.8000, 0.1500\rangle$
$\mathfrak{E}_{11}$	$\langle 0.7373, 0.1835\rangle$	$\langle 0.5000, 0.4000\rangle$	$\langle 0.7373, 0.1835\rangle$	$\langle 0.4380, 0.4676\rangle$	$\langle 0.5759, 0.3270\rangle$	$\langle 0.7373, 0.1835\rangle$	$\langle 0.7140, 0.2347\rangle$	$\langle 0.7373, 0.1835\rangle$
$\mathfrak{E}_{12}$	$\langle 0.7000, 0.2000\rangle$	$\langle 0.6757, 0.2259\rangle$	$\langle 0.7373, 0.1835\rangle$	$\langle 0.5370, 0.3669\rangle$	$\langle 0.3610, 0.5378\rangle$	$\langle 0.8000, 0.1500\rangle$	$\langle 0.6608, 0.2462\rangle$	$\langle 0.8000, 0.1500\rangle$
$\mathfrak{E}_{13}$	$\langle 0.5759, 0.3270\rangle$	$\langle 0.7000, 0.2000\rangle$	$\langle 0.7000, 0.2000\rangle$	$\langle 0.7480, 0.1783\rangle$	$\langle 0.7674, 0.1683\rangle$	$\langle 0.7763, 0.1635\rangle$	$\langle 0.6367, 0.2656\rangle$	$\langle 0.7763, 0.1683\rangle$
$\mathfrak{E}_{14}$	$\langle 0.5370, 0.3669\rangle$	$\langle 0.2293, 0.6415\rangle$	$\langle 0.5759, 0.3270\rangle$	$\langle 1.0000, 0.0000\rangle$	$1.0000, 0.0000\rangle$	$\langle 0.1922, 0.7489\rangle$	$\langle 0.6549, 0.2980\rangle$	$\langle 1.0000, 0.0000\rangle$
$\mathfrak{E}_{15}$	$\langle 0.5759, 0.2879\rangle$	$\langle 0.2442, 0.6684\rangle$	$\langle 0.5759, 0.2879\rangle$	$\langle 1.0000, 0.0000\rangle$	$1.0000, 0.0000\rangle$	$\langle 0.1161, 0.9061\rangle$	$\langle 0.5370, 0.3474\rangle$	$\langle 1.0000, 0.0000\rangle$

By *Step 6*, the distances between each E_i and E^+ are obtained using the new Q-ROFDM as follows:

$d_*(E_1, E^+) = 0.0908$, $d_*(E_2, E^+) = 0.0980$, $d_*(E_3, E^+) = 0.0834$, $d_*(E_4, E^+) = 0.0161$, $d_*(E_5, E^+) = 0.0063$, $d_*(E_6, E^+) = 0.0764$, $d_*(E_7, E^+) = 0.0743$.

Using the results obtained in *Step 6*, we obtain the following ordering; $E_5 \prec E_4 \prec E_7 \prec E_6 \prec E_3 \prec E_1 \prec E_2$. Using the ordering, the MCGDM approach ranks solar energy, E_5 as the optimal choice for a renewable energy source. This choice tallies with the fact that solar energy is more feasible, has less risk of failure, is reliable, has minimal waste disposal, provides job creation, has lower energy tariffs, is eco-friendly, and has no health hazards.

5.1.2 | Sensitivity analysis based on multi criteria group decision making

Next, we examine the sway of the computed weights and the levels of the Q-ROFSs on the distance outputs and the interpretation of the results based on the new method. We juxtapose the impact of the distance values with and without the computed weights on the MCGDM approach. The distance indexes are presented in *Table 15*, which are represented in *Fig. 3*.

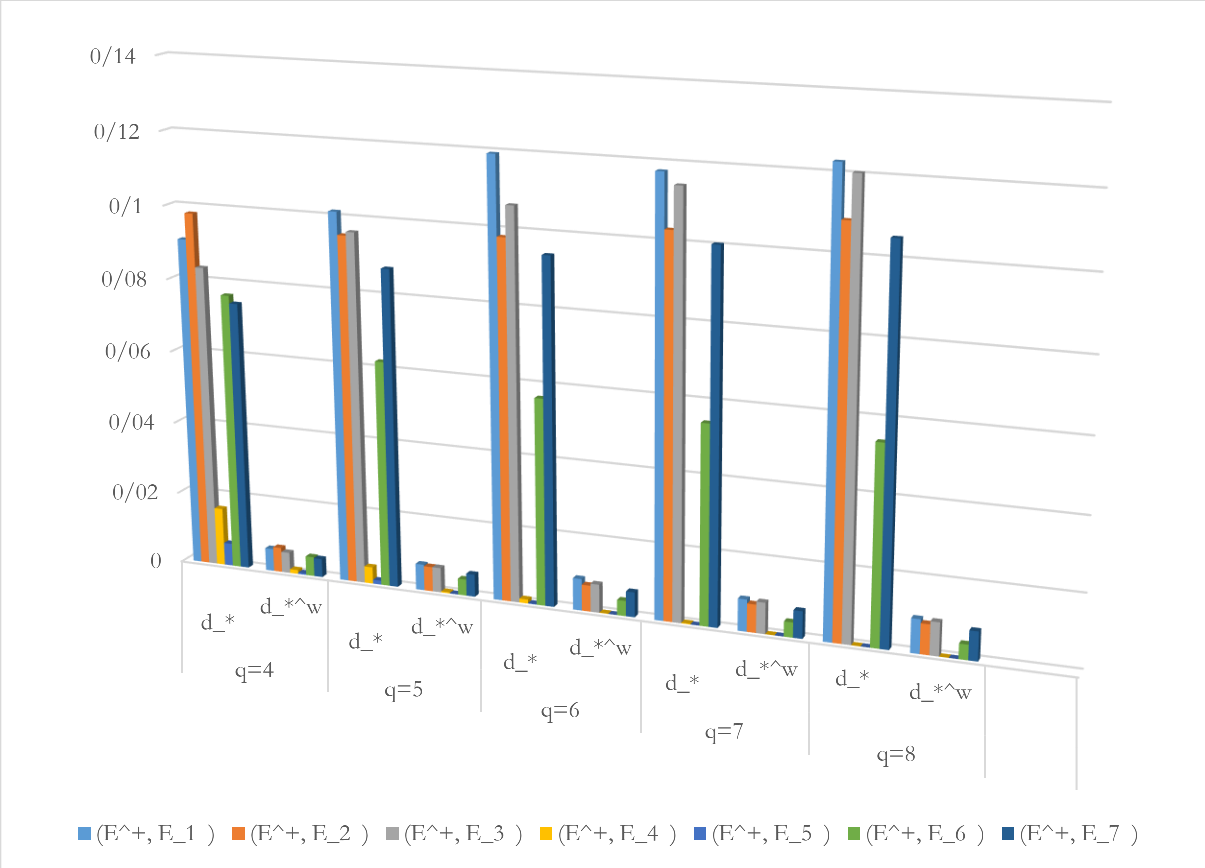

Fig. 3. Sensitivity analysis based on MCGDM.

Table 15. Sensitivity analysis for distance indexes.

Q-ROFDM		(E^+, E_1)	(E^+, E_2)	(E^+, E_3)	(E^+, E_4)	(E^+, E_5)	(E^+, E_6)	(E^+, E_7)
q = 4	d_*	0.0907911	0.0980064	0.0834140	0.0161271	0.0063267	0.0763890	0.0743397
	d_*^{ϖ}	0.0063179	0.0069118	0.0058038	0.0010913	0.0004055	0.0054181	0.0051559
q = 5	d_*	0.1014645	0.0953238	0.0963380	0.0046453	0.0011041	0.0622176	0.0873801
	d_*^{ϖ}	0.0072514	0.0068585	0.0068843	0.0003164	0.0000720	0.0045263	0.0062337
q = 6	d_*	0.1193929	0.0981470	0.1065014	0.0011177	0.0001545	0.0566286	0.0943683
	d_*^{ϖ}	0.0087620	0.0072310	0.0078130	0.0000765	0.0000103	0.0042296	0.0069190
q = 7	d_*	0.1177607	0.1032568	0.1145380	0.0002147	0.0000174	0.0545340	0.1004280
	d_*^{ϖ}	0.0088119	0.0077515	0.0085713	0.0000147	0.0000012	0.0041521	0.0075190
q = 8	d_*	0.1229157	0.1088507	0.1204719	0.0000329	0.0000016	0.0542012	0.1053623
	d_*^{ϖ}	0.0093259	0.0082792	0.0091417	0.0000023	0.0000001	0.0041762	0.0080058

Table 15 and *Fig. 3* show the effect of weights on the precision of the distance metrics. We observe that the distance metrics built with the weight of elements produce more efficient and precise distance values in comparison with the distance metrics built without the weights of elements. However, both distance metrics give the same ordering as, $E_5 \prec E_4 \prec E_6 \prec E_7 \prec E_2 \prec E_3 \prec E_1$. Thus, to attain accurate distance outputs, it is better to consider the weights of elements in a distance metric. In addition, as q increases, the distance values based on the weights of elements yield better results, while the precision of the distance value without the weight-element decreases.

5.1.3 | Comparative multi criteria group decision making analysis

To investigate the effectiveness of the novel Q-ROFDM, we compare its interpretation ability with the existing Q-ROFDMs for $q = 8$, using the energy source data based on MCGDM. Now, we compute the distance values between the energy sources and the ideal alternative, and get the ordering in *Table 16*.

Table 16. Comparative ordering based on MCGDM.

Q-ROFDMs	Ordering	Decision
d_*	$E_5 \prec E_4 \prec E_6 \prec E_7 \prec E_2 \prec E_3 \prec E_1$	E_5
d_D [70]	$E_4 \prec E_5 \prec E_7 \prec E_3 \prec E_1 \prec E_2 \prec E_6$	E_4
d_{PL1} [71]	$E_5 \prec E_4 \prec E_6 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{PL2} [71]	$E_5 \prec E_4 \prec E_6 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{PL3} [71]	$E_5 \prec E_4 \prec E_6 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{PL4} [71]	$E_5 \prec E_4 \prec E_6 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{PL5} [71]	$E_5 \prec E_4 \prec E_6 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{PL6} [71]	$E_5 \prec E_4 \prec E_6 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{PL7} [71]	$E_5 \prec E_4 \prec E_6 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{PL8} [71]	$E_5 \prec E_4 \prec E_6 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{PL9} [71]	$E_5 \prec E_4 \prec E_6 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{PL10} [71]	$E_5 \prec E_4 \prec E_6 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{PL11} [71]	$E_5 \prec E_4 \prec E_6 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{PB} [72]	$E_5 \prec E_4 \prec E_3 \prec E_7 \prec E_1 \prec E_2 \prec E_6$	E_5
d_{KP1} [76]	$E_5 \prec E_4 \prec E_3 \prec E_6 \prec E_7 \prec E_1 \prec E_2$	E_5
d_{KP2} [76]	$E_5 \prec E_4 \prec E_6 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{KP3} [76]	$E_5 \prec E_4 \prec E_6 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{He} [77]	$E_5 \prec E_4 \prec E_6 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_E [78]	$E_5 \prec E_4 \prec E_6 \prec E_3 \prec E_7 \prec E_1 \prec E_2$	E_5
d_A [79]	$E_5 \prec E_4 \prec E_6 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{RK} [81]	$E_1 \prec E_7 \prec E_5 \prec E_2 \prec E_6 \prec E_4 \prec E_3$	E_1
d_{De} [83]	$E_5 \prec E_4 \prec E_3 \prec E_1 \prec E_7 \prec E_2 \prec E_6$	E_5
d_{Se} [84]	$E_5 \prec E_4 \prec E_3 \prec E_6 \prec E_1 \prec E_7 \prec E_2$	E_5

The information in *Table 16* presents a similar ordering between each of the methods, with each method presenting solar power (E_5) as the optimal renewable energy source based on the evaluated criteria. Notwithstanding, the method d_D presents wind power (E_4) as the suitable energy source and d_{RK} [81] shows that hydropower (E_1) is a suitable energy source. The interpretation discrepancies of d_D [70] and d_{RK} is because d_D is not constructed using the q^{th} property of Q-ROFSs and d_{RK} excludes the hesitation factor of Q-ROFSs.

5.2 | Multiple Attribute Decision Making's Algorithm

Consider a MADM problem with "p" alternatives $i = \{E_1, E_1, \dots, E_p\}$, "r" criteria $j = \{\mathfrak{E}_1, \mathfrak{E}_2, \dots, \mathfrak{E}_r\}$ and l expert $k = \{e_1, e_1, \dots, e_l\}$ are invited to assess the alternatives. The MADM algorithm is as follows:

Step 1. Formulate the Q-ROF decision matrices based on the data provided by the l experts, which is *Eq. (13)*.

$$\begin{array}{cccccc} & \mathfrak{E}_1 & \cdots & \mathfrak{E}_j & \cdots & \mathfrak{E}_r, \end{array}$$

$$E^{(k)} = \begin{matrix} E_1 \\ \vdots \\ E_i \\ \vdots \\ E_p \end{matrix} \begin{pmatrix} \langle E_{m_{11}}^{(k)}, E_{n_{11}}^{(k)} \rangle & \cdots & \langle E_{m_{1j}}^{(k)}, E_{n_{1j}}^{(k)} \rangle & \cdots & \langle E_{m_{1r}}^{(k)}, E_{n_{1r}}^{(k)} \rangle \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \langle E_{m_{i1}}^{(k)}, E_{n_{i1}}^{(k)} \rangle & \cdots & \langle E_{m_{ij}}^{(k)}, E_{n_{ij}}^{(k)} \rangle & \cdots & \langle E_{m_{ir}}^{(k)}, E_{n_{ir}}^{(k)} \rangle \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \langle E_{m_{p1}}^{(k)}, E_{n_{p1}}^{(k)} \rangle & \cdots & \langle E_{m_{pj}}^{(k)}, E_{n_{pj}}^{(k)} \rangle & \cdots & \langle E_{m_{pr}}^{(k)}, E_{n_{pr}}^{(k)} \rangle \end{pmatrix}, \tag{13}$$

where $E^{(k)} = \left[E_{ij}^{(k)}\right]_{p\times q} = \left[E_{m_{ij}}^{(k)}, E_{n_{ij}}^{(k)}\right]_{p\times r}$ for $k = 1,2,\dots,l$, E_i indicates the energy source alternatives $(i = 1, 2, \dots, p)$ and $\mathfrak{E}_j$ denotes criteria $(j = 1, 2, \dots, r)$.

Step 2. Obtain the Q-ROFMA given as *Eq. (14)*.

$$\begin{array}{ccccc} \mathfrak{E}_1 & \cdots & \mathfrak{E}_j & \cdots & \mathfrak{E}_r, \end{array} \tag{14}$$

$$\bar{E} = \begin{matrix} E_1 \\ \vdots \\ E_i \\ \vdots \\ E_p \end{matrix} \begin{pmatrix} \langle \bar{E}_{m_{11}}, \bar{E}_{n_{11}} \rangle & \cdots & \langle \bar{E}_{m_{1j}}, \bar{E}_{n_{1j}} \rangle & \cdots & \langle \bar{E}_{m_{1r}}, \bar{E}_{n_{1r}} \rangle \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \langle \bar{E}_{m_{i1}}, \bar{E}_{n_{i1}} \rangle & \cdots & \langle \bar{E}_{m_{ij}}, \bar{E}_{n_{ij}} \rangle & \cdots & \langle \bar{E}_{m_{ir}}, \bar{E}_{n_{ir}} \rangle \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \langle \bar{E}_{m_{p1}}, \bar{E}_{n_{p1}} \rangle & \cdots & \langle \bar{E}_{m_{pj}}, \bar{E}_{n_{pj}} \rangle & \cdots & \langle \bar{E}_{m_{pr}}, \bar{E}_{n_{pr}} \rangle \end{pmatrix},$$

Where $\text{Q-ROFMA}\left(E_{ij}^{(1)}, E_{ij}^{(2)}, \ldots, E_{ij}^{(l)}\right) = \bar{E}$ and $\bar{E} = \left(\bar{E}_{ij}\right)_{p\times r} = \frac{\sum_{k=1}^{l} E_{ij}^{(k)}}{l}$.

Step 3. Compute the distance indexes between E_i $(i = 1, 2, \ldots, 7)$ and the perfect energy source alternative I defined as:

$$I = \left\{ \begin{matrix} (1.0, 0.0), (1.0, 0.0), (1.0, 0.0), (1.0, 0.0), (1.0, 0.0), (1.0, 0.0), (1.0, 0.0), \\ (1.0, 0.0), (1.0, 0.0), (1.0, 0.0), (1.0, 0.0), (1.0, 0.0), (1.0, 0.0), (1.0, 0.0), (1.0, 0.0) \end{matrix} \right\}.$$

Step 4. Find the minimum distance, denoted as $d^*(E_i, I)$ and defined as *Eq. (15)*.

$$d^*(E_i, I) = \min_{1\le j\le 10} \{d(E_i, I)\}. \quad \textbf{(15)}$$

Step 4. Compute the degree of confidence denoted as Π and defined as *Eq. (16)*.

$$\Pi = \sum_{i=1}^{p} |d(E_i, I) - d^*(E_i, I)|. \quad \textbf{(16)}$$

Step 5. The suitable energy source alternative is the one whose distance from I is the smallest. The algorithm of the MADM technique is presented in *Fig. 4*.

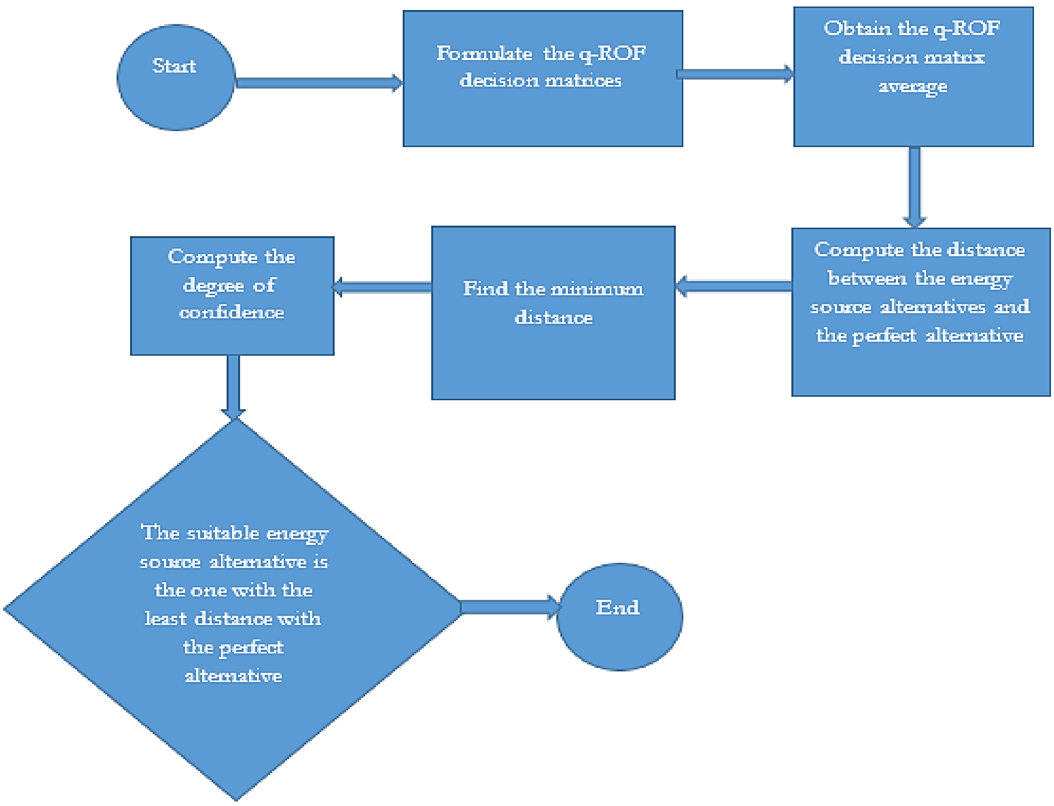

Fig. 4. Flowchart of MADM.

5.2.1 | Implementation of the multi-attribute decision making's algorithm

Using *Step 1*, we convert the LVs from the three experts to get the Q-ROF decision matrices. By *Step 2*, we take the average of the Q-ROF decision matrices and get the Q-ROFMA. The MCGDM algorithm has implemented these steps. Based on *Step 3*, the distance indexes between $\mathrm{E_i}$ $(\mathrm{i} = 1, 2, \dots 7)$ and I based on the new Q-ROFDM for $\mathrm{q} = 4$ are as follows:

$$d_*(\mathrm{E_1}, \mathrm{I}) = 0.4106, d_*(\mathrm{E_2}, \mathrm{I}) = 0.4188, d_*(\mathrm{E_3}, \mathrm{I}) = 0.3720, d_*(\mathrm{E_4}, \mathrm{I}) = 0.4300,$$
$$d_*(\mathrm{E_5}, \mathrm{I}) = 0.3400, d_*(\mathrm{E_6}, \mathrm{I}) = 0.3401, d_*(\mathrm{E_7}, \mathrm{I}) = 0.4934. \tag{13}$$

By *Step 4*, the minimum distance value is $d_*^*(\mathrm{E_i}, \mathrm{I}) = 0.3400$, and the degree of confidence is $\Pi = 0.4249$ using *Step 5*. From *Step 6*, the most efficient energy source alternative is $\mathrm{E_5}$, which represents solar power. The ranking of the energy source alternatives based on MADM is: $\mathrm{E_5} \prec \mathrm{E_6} \prec \mathrm{E_3} \prec \mathrm{E_1} \prec \mathrm{E_2} \prec \mathrm{E_4} \prec \mathrm{E_7}$.

5.2.2 | Sensitivity analysis based on multiple attribute decision making

Next, the impact of the computed weights (in *Table 7*) on the new method is investigated by comparing the results and interpretations of the method with and without the weights for $\mathrm{q} = 8$ using the MADM approach. The distance indexes are presented in *Table 17* and *Fig. 5*.

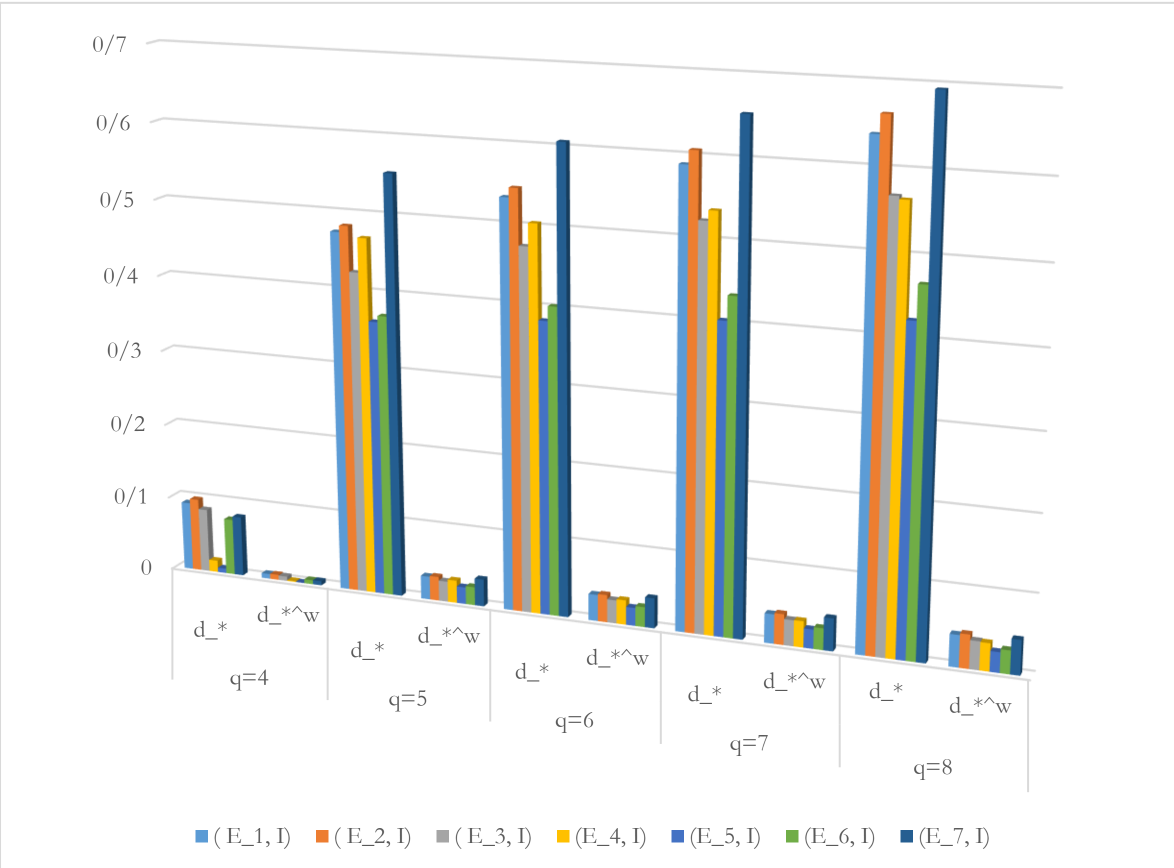

Fig. 5. Sensitivity analysis based on MADM.

Table 17. Sensitivity analysis for distance indexes based on MADM.

Q-ROFDM		(E_1, I)	(E_2, I)	(E_3, I)	(E_4, I)	(E_5, I)	(E_6, I)	(E_7, I)
$q = 4$	d_*	0.0921	0.0982	0.0855	0.0158	0.0061	0.0760	0.0809
	d_*^{ϖ}	0.0064	0.0069	0.0059	0.0011	0.0004	0.0054	0.0056
$q = 5$	d_*	0.4744	0.4830	0.4244	0.4691	0.3625	0.3708	0.5537
	d_*^{ϖ}	0.0311	0.0323	0.0274	0.0303	0.0226	0.0245	0.0365
$q = 6$	d_*	0.5342	0.5465	0.4751	0.5046	0.3836	0.4026	0.6068
	d_*^{ϖ}	0.0351	0.0365	0.0309	0.0326	0.0240	0.0266	0.0400
$q = 7$	d_*	0.5892	0.6070	0.5233	0.5363	0.4037	0.4349	0.6534
	d_*^{ϖ}	0.0388	0.0406	0.0342	0.0346	0.0253	0.0286	0.0431
$q = 8$	d_*	0.6390	0.6630	0.5688	0.5650	0.4234	0.4673	0.6942
	d_*^{ϖ}	0.0421	0.0444	0.0373	0.0365	0.0267	0.0307	0.0458

Table 17 and *Fig. 5* show the effect of weights on the precision of the distance metric. It is observed that the distance metrics built with weight-element produce more efficient distance values in comparison with the distance metrics built without the weight-element, and both distance metrics give the same ordering as, $E_5 \prec E_6 \prec E_3 \prec E_1 \prec E_2 \prec E_4 \prec E_7$. Thus, to attain accurate distance outputs, it is better to consider the weights of the elements in a distance metric.

5.2.3 | Comparative analysis based on multi-attribute decision making approach

To investigate the effectiveness of the new Q-ROFDM, we compare it with the existing Q-ROFDMs for $q = 8$, and $p = 2$, using the energy source alternatives data based on the MADM approach to show its effectiveness in terms of decision interpretation. Now, we compute the distance values between the energy source alternatives and the ideal alternative and get the ordering in *Table 18*.

Table 18, which presents the ordering of the energy source alternatives based on the new and existing Q-ROFDM via MADM, shows a similar ordering between each of the methods, with each method presenting E_5 as the optimal energy source alternative based on the evaluated criteria. However, the methods: d_D [70], d_{PB} [72], d_{RK} [81], d_{De} [83], and d_{Se} [84] yield dissimilar decisions because of their construction defectiveness.

Table 18. Comparative ordering based on MADM.

Q-ROFDMs	Ordering	Decision
d_*	$E_5 \prec E_6 \prec E_3 \prec E_1 \prec E_2 \prec E_4 \prec E_7$	E_5
d_D [70]	$E_6 \prec E_5 \prec E_3 \prec E_1 \prec E_2 \prec E_7 \prec E_4$	E_6
d_{PL1} [71]	$E_5 \prec E_6 \prec E_4 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{PL2} [71]	$E_5 \prec E_6 \prec E_4 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{PL3} [71]	$E_5 \prec E_6 \prec E_4 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{PL4} [71]	$E_5 \prec E_6 \prec E_4 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{PL5} [71]	$E_5 \prec E_6 \prec E_4 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{PL6} [71]	$E_5 \prec E_6 \prec E_4 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{PL7} [71]	$E_5 \prec E_6 \prec E_4 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{PL8} [71]	$E_5 \prec E_6 \prec E_4 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{PL9} [71]	$E_5 \prec E_6 \prec E_4 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{PL10} [71]	$E_5 \prec E_6 \prec E_4 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{PL11} [71]	$E_5 \prec E_6 \prec E_3 \prec E_4 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{PB} [72]	$E_3 \prec E_1 \prec E_2 \prec E_6 \prec E_5 \prec E_7 \prec E_4$	E_3
d_{KP1} [76]	$E_5 \prec E_6 \prec E_4 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{KP2} [76]	$E_5 \prec E_6 \prec E_4 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{KP3} [76]	$E_5 \prec E_6 \prec E_4 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{He} [77]	$E_5 \prec E_6 \prec E_4 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_E [78]	$E_5 \prec E_6 \prec E_4 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_A [79]	$E_5 \prec E_6 \prec E_4 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_5
d_{RK} [81]	$E_6 \prec E_2 \prec E_7 \prec E_1 \prec E_5 \prec E_3 \prec E_4$	E_6
d_{De} [83]	$E_6 \prec E_5 \prec E_3 \prec E_1 \prec E_7 \prec E_2 \prec E_4$	E_6
d_{Se} [84]	$E_6 \prec E_5 \prec E_4 \prec E_3 \prec E_1 \prec E_7 \prec E_2$	E_6

5 | Conclusion

Energy sources are very significant to the sustenance of life. Due to the damaging effects of some energy source alternatives on the ecosystem, in terms of global warming, it is expedient to select an efficient and eco-friendly energy source alternative. Oftentimes, the process of selection is encumbered with fuzzy and hesitation factors, which need to be curbed to enhance reliable selection. Based on this, this article has presented a novel Q-ROFDM, which is effective in making a reliable decision. The novel efficient Q-ROFDM was described to show superiority over some extant Q-ROFDMs.

In addition, the optimization of energy source selection was considered using the novel distance technique, which selected an efficient energy source alternative from among the several energy source alternatives considered using assorted energy criteria based on the decision-making approaches. In recap, the article constructed an innovative Q-ROFDM, described the innovative Q-ROFDM based on the distance metric conditions for the aim of validation, utilized the innovative Q-ROFDM to augment an effective selection of eco-friendly energy source alternatives based on MCGDM and MADM, and presented a comparative analysis of the innovative Q-ROFDM to buttress its advantages over the existing Q-ROFDMs.

The developed Q-ROFDM is appropriate, reliable, and can be adopted in other uncertainty fields. Although the novel Q-ROFDM is compelling in real-life applications, it cannot be helpful where the available data are spherical fuzzy sets, interval-valued Q-ROFSs, neutrosophic Q-ROFSs, and bipolar spherical fuzzy sets due to its construction limitations. However, the new Q-ROFDM can be extended to these environments with minimal modifications and used for group decision-making.

Authors' Contributions

Conceptualization, P.A.E., M.T.A., C.O.N.; Methodology M.T.A., P.A.E.; Software, P.A.E., M.T.A.; Validation, N.K., C.O.N., T.C.; Formal analysis and investigation, N.K., T.C.; Resources, N.K., T.C.; Writing-reviewing and editing, P.A.E., M.T.A., T.C., N.K., C.O.N.; Data maintenance, M.T.A. All authors have read and agreed to the published version of the manuscript.

Conflicts of Interest

The authors have no conflict of interest to disclose.

Data Availability

All data generated or analyzed during this study are included in this article.

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