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Novel Q-Rung orthopair fuzzy correlation measure based on spearman's correlation scheme with application in vehicle selection problem

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Abstract

Background Q-rung orthopair fuzzy sets (Q-ROFS) have been widely employed in decision-making problems due to their strong ability to handle uncertainty, indecision, and imprecision. Consequently, several q-rung orthopair fuzzy correlation measures (Q-ROFCM) have been developed and applied in various decision-making contexts. However, many existing correlation measures exhibit inherent limitations, which reduce their effectiveness in addressing practical, real-world problems.

Methods In this study, a novel q-rung orthopair fuzzy correlation coefficient (Q-ROFCC) based on Spearman's correlation scheme is proposed to overcome the shortcomings of existing approaches. The fundamental mathematical properties of the proposed correlation measure are rigorously analyzed to ensure compliance with the standard axioms of correlation coefficients. Furthermore, the proposed method is incorporated into a multi-attribute decision-making (MADM) framework.

Results The results demonstrate that the proposed Spearman-based Q-ROFCM technique is reliable, effective, and accurate when compared with existing methods. Its applicability is illustrated through a vehicle selection problem, where the most suitable alternative is identified based on optimal performance and user satisfaction. Comparative analysis confirms the superiority of the proposed approach over Pearson-based Q-ROFCM approaches.

Conclusions The proposed Q-ROFCM technique provides a robust and efficient alternative for solving MADM problems under uncertainty. Owing to its improved performance and practical applicability, the method is well suited for real-life decision-making scenarios.

Keywords Selection problem, Multi Attribute decision, Making approach, Q-Rung orthopair fuzzy correlation measure, Q-Rung orthopair fuzzy set

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1 Introduction

1.1 Background and motivation

Humans make exact decisions in ambiguous scenarios that arise in daily life. Zadeh [1] proposed the FS theory, which is described using membership degree (MD) defined within the range of 0 and 1 to address ambiguity in decision-making. However, contrary to what is suggested in fuzzy sets in real-life situations, 1 deducted from the MD is not the non-membership degree (NMD) every time, as suggested in FS. This prompted Atanassov to propose the idea of intuitionistic fuzzy sets (IFS), which addresses the hesitation margin (HM) that can exist in everyday situations [2]. As the sum of MD and NMD may not always be within the interval of 0 and 1, Atanassov [3] developed an IFS of type 2, also known as the Pythagorean fuzzy sets (PFS), to handle situations where the addition of the squares of MD and NMD is at most 1. Adding MD and NMD may not always fall within the range of 0 and 1.

Furthermore, adding the squares for MD and NMD may result in a value greater than 1 in specific real-world situations. This explains why PFS is not ideal for solving complex situations. Therefore, Fermatean fuzzy sets (FFS) were introduced by Senapati and Yager [4], where the sum of the third powers of MD and NMD is at most 1. The FFS cannot be used to reduce uncertainty when the sum of the third power of MD and NMD is greater than one. To permanently address the shortcomings noticed in IFS, PFS, and FFS, the term "q-rung orthopair fuzzy sets (Q-ROFS)" was introduced by Yager [5]. Due to the exceptional qualities of the outcomes of Q-ROFS in multi-criteria decision-making (MCDM), many scholars have conducted in-depth studies of Q-ROFS to tackle decision-making challenges. In [6], an MCDM problem was resolved using the Bonferroni mean operator and [7] discussed clustering analysis, pattern recognition, investment analysis, and selection problem based on distance-similarity operators. A q-rung orthopair fuzzy (Q-ROF) decision support scheme was presented in [8], and a process of medical diagnosis was discussed in [9] based on the Q-ROF composite relation. In [10], a renewable energy source problem was discussed in the Q-ROF setting, and the Q-ROF composite relation in [4] was modified to facilitate a reliable discussion of medical diagnostic analysis [11].

Several techniques for calculating the correlation coefficient have been developed for Q-ROFS, based on the Pearson correlation scheme. The research on the q-rung orthopair fuzzy correlation measure (Q-ROFCM) was initiated in [12], and two Q-ROFCM techniques were presented in [13] with applications to clustering analysis. In [14], some Q-ROFCM techniques were presented from a statistical viewpoint, and some improved Q-ROFCM methods were developed and used in clustering [15]. Similarly, a Q-ROFCM technique was presented and utilized to discuss decision-making [16]. In this work, we introduce an efficient Q-ROFCM method using Spearman's correlation coefficient and apply it to the selection process. The proposed Spearman correlation measure-based Q-ROFCM is more effective than existing Pearson correlation measure-based Q-ROFCM methods because it measures monotonic relationships, unlike the Pearson correlation measure-based methods, which merely measure linear relationships. Additionally, the Spearman correlation measure-based Q-ROFCM method is suitable for smaller sample sizes and does not require normality assumptions. On the contrary, the existing Pearson correlation measure-based Q-ROFCM techniques are unsuitable for small sample sizes and require normality assumptions. In MCDM problems, pattern recognition, and uncertainty modeling, the Spearman correlation measure-based Q-ROFCM technique is advantageous because:

- It emphasizes preference or alternatives, not numerical magnitude.
- It can translate real-world decision problems into a decision support system seamlessly, making it a reliable model for utilization in Q-ROF systems.

The Spearman correlation measure-based q-ROFCM technique is expedient since it is flexible, robust, and less assumption-dependent, making it extremely appropriate for evaluating nonlinear, uncertain, or fuzzy relations in MCDM.

Due to the itemized advantages of the Spearman's correlation scheme, the model is extended to the Q-ROF setting to measure the precise correlation between Q-ROFS and to enhance real-world application potentials in decision-making, selection problems, etc. To the best of our knowledge, q-rung orthopair fuzzy correlation coefficient (Q-ROFCC) approaches have been developed only based on Pearson's correlation model. Therefore, it is necessary to study Q-ROFCC based on Spearman's correlation model to explore the already established advantages of Spearman's correlation model in the Q-ROF setting.

Many practical applications of Q-ROFS have been explored based on q-ROFCM, but each one has some drawbacks, and none is constructed through Spearman's correlation scheme. In particular, the drawbacks of the existing techniques include:

i. Though the outcome of the method of Du [12] seems to produce comparable results, it cannot be reliable as it is based on a two-dimensional (2D) principle, which would lead to the omission of important information.

ii. The outcomes of the techniques in [13, 14] for the first level, i.e., $q = 1$ could yield a correlation coefficient value of 1 even when the Q-ROFS are not equal, in contrast to the q-ROFCM condition.
iii. The techniques in [13, 15] are also based on a 2D principle, i.e., by excluding the hesitation margins. Hence, their outcomes cannot be reliable.
iv. Although the technique in [16] has no defectiveness, its outcome lacks precision. In fact, all the methods lack precision.

In light of this, this work is motivated to develop a new technique for Q-ROFCM using Spearman's correlation scheme, which is employed to analyze the vehicle selection process for purchasing a suitable economic car for lower cadre civil servants. The contributions of the work are:

i. Analysis of the extant methods of Q-ROFCM and the development of a new technique of Q-ROFCM through Spearman's correlation scheme.
ii. Utilization of the new technique of Q-ROFCM to decide the most suitable car for lower cadre civil servants through the MADM scheme.
iii. The rationale for the construction of the new technique of Q-ROFCM is presented via comparison with the extant techniques of Q-ROFCM.

1.2 Review of literature

Purchasing a vehicle is one of the most significant ventures many people undertake in life. There are numerous factors to consider when selecting a vehicle that will provide optimal satisfaction. These factors include safety, low price, fuel efficiency, suitability for everyday use, driving comfort, high quality, good warranty/customer service, and design. According to popular opinion, safety and fuel efficiency are the top factors, with design being the least important. To achieve a reliable selection process for purchasing a used car, the uncertainties in the process must be well controlled. Raviv [17] examined the sequence of bidding in an open-outcry auction of used cars to assess the uncertain factors influencing auction outcomes, utilizing a dataset from public auctions of used cars in New Jersey. However, the study does not consider the selection uncertainties. In [18], the analysis of adverse selection in the used-car market was considered, and it was found that adverse selection probably affects quality in the secondhand market and trade volume. Nonetheless, the study does not discuss the effect of uncertainty in the selection process. Fuzzy set theory was initiated to address uncertainty [1]. Due to the high level of interest in the concept, it has been modified as IFS [2], PFS [3], FFS [4], and q-ROFS [5] with various applications in MCDM [19, 20], supplier selection [21], and decision support systems [22]. In fact, the Q-ROFS is a generalization of IFS, PFS, and FFS, in which the sum of NMD and MD cannot exceed one.

The theory of Q-ROFS has been applied to discuss many practical problems. Sarka and Biswas [6] presented a dual hesitant Q-ROF Dombi t-conorm and t-norm via mean operators for solving MCDM problems. Pen et al. [8] created a Q-ROF decision-making framework for integrating mobile edge caching scheme preferences. Krishankumar et al. [10] established renewable energy source selection problems using Q-ROF-based integrated decision-making methodology. Yin et al. [23] created a product operation on a Q-ROF graph. Further applications of Q-ROFS include Q-ROF MCDM [24–26], knowledge measures [27], graph theory [28], and others. Diverse correlation coefficient measures have been discussed in several uncertain settings to discuss the tasks in decision-making. Dumitrescu [29] initiated research on the fuzzy correlation coefficient, establishing a correlation coefficient method under fuzzy sets. In [30], students' time management and academic performance in high school were based on fuzzy correlation, and a similar application was discussed in [31]. The study of intuitionistic fuzzy correlation was initiated in [32], explored in [33, 34], and applied to discuss various practical problems, such as clustering [35] and pattern recognition [36]. Similarly, several studies have presented correlation coefficients under PFS in pattern recognition [37] and decision support systems [38–40] discussions, and correlation coefficients have been studied under FFS and applied in medical analysis [41], clustering, pattern analysis, and selection problems [21, 42], and decision-making [43, 44].

Furthermore, the concept of the correlation coefficient in the Q-ROFS setting has been investigated due to the reliability of Q-ROFS in resolving complex decision problems. Du [12] introduced a method of Q-ROFCM by lessening the restrictions on the grades of support for or against to estimate the statistical connections among Q-ROFS. The work was used to discuss clustering analysis and decision problems involving the appraisal of companies. Although the outcome from the method in [12] appears to produce comparable results, it cannot be considered reliable, as it is based on a 2D principle—it uses only membership and non-membership grades, which would lead to the omission of important information. Li et al. [13] developed two techniques for Q-ROFCM and discussed their applications in clustering analysis. In addition, the techniques in [13] were based on a 2D principle, i.e., by excluding the hesitation margins. Hence, their outcomes cannot be reliable. Singh and Ganie [14] studied Q-ROFCM through variance and covariance to create two approaches based on Pearson's scheme, which were used to discuss decision-making. However, the methods violate the conditions of Q-ROFCM in some cases and also lack precision. Bashir et al. [15] presented two Q-ROFCM techniques by modifying the methods in [45] under IFS and used them to analyze clusters. The outcomes from these techniques cannot be relied upon because they are based on a 2D principle. In [16], a novel methodology for Q-ROFCM was constructed to measure the resemblance between arbitrary Q-ROFS and discussed its application in pattern categorization and healthcare services. Although the technique in [16] has no defectiveness, its outcome lacks precision.

Given the limitations in the existing methodologies of Q-ROFCM, it is necessary to introduce a new Q-ROFCM method based on Spearman's scheme, which efficiently alleviates the aforementioned limitations to enable performance rating. From the reviewed methodologies, it is noticeable that none of the methods were constructed through Spearman's scheme. The methods were created in line with Pearson's model. Due to the benefits of Spearman's scheme over Pearson's scheme, we are motivated to develop a new technique of Q-ROFCM based on Spearman's scheme and demonstrate its application in the process of selecting a suitable car using the MADM approach.

The rest of the article is outlined as follows: Sect. 2 discusses the preliminaries of q-ROFSs; Sect. 3 presents the new technique of Q-ROFCM and its properties; Sect. 4 presents the existing techniques of Q-ROFCM and numerical illustrations; Sect. 5 discusses a suitable car selection process through the new method of Q-ROFCM; Sect. 6 presents a comparison to express the importance of the new Q-ROFCM technique over the extant Q-ROFCM methods via MADM; and Sect. 7 concludes the article and states some suggestions for further investigation.

2 Methods

In this section, the preliminaries on Q-ROFS are presented and some extant methods of Q-ROFCM are mentioned.

2.1 Preliminaries

Throughout this work, the universe of discourse is represented by$R = \{r_1, r_2, \ldots r_k\}$, where$k \in \mathbb{N}$.

Definition 1 [2]. The scheme of the form $Z = \left\{ \left(r_i, Z_\alpha(r_i), Z_\beta(r_i) \right) | r_i \in R \right\}$ for $i = 1,2,\ldots,k$, where $Z_\alpha : R \to [0,1]$ and $Z_\beta : R \to [0,1]$, such that $Z_\alpha(r_i) + Z_\beta(r_i) \leq 1$ for all $r_i \in R$ is called an IFS in R. The functions $Z_\alpha(r_i)$ and $Z_\beta(r_i)$ are MD and NMD of r_i to R. The HM is $Z_\gamma(r_i) = 1 - Z_\alpha(r_i) - Z_\beta(r_i)$, and it is the grade of indeterminacy of $r_i \in R$ and $Z_\gamma(r_i) \in [0,1]$.

Definition 2 [46]. The scheme of the form $\breve{Z} = \left\{ \left(r_i, \breve{Z}_\alpha(r_i), \breve{Z}_\beta(r_i) \right) | r_i \in R \right\}$ for $i = 1, 2, \ldots, k$, where $\breve{Z}_\alpha(r_i), \breve{Z}_\beta(r_i) \in [0, 1]$ stand for the MD and NMD for $r_i \in$ R, such that $0 \leq \breve{Z}^2_\alpha(r_i) + \breve{Z}^2_\beta(r_i) \leq 1$ is called a PFS in R. The grade of indeterminacy is given by $\breve{Z}_\gamma(r_i) = \left(1 - \breve{Z}^2_\alpha(r_i) - \breve{Z}^2_\beta(r_i) \right)^{\frac{1}{2}}$.

Definition 3 [4]. The scheme of the form $\overline{Z} = \left\{ \left(r_i, \overline{Z}_\alpha(r_i), \overline{Z}_\beta(r_i) \right) | r_i \in R \right\}$ for i = 1,2,...,k, where $\overline{Z}_\alpha(r_i) \in [0,1]$ and $\overline{Z}_\beta(r_i) \in [0,1]$ stand for MD and NMD for $r_i \in R$ such that $0 \leq \overline{Z}^3_\alpha(r_i) + \overline{Z}^3_\beta(r_i) \leq 1$ is called an FFS in R. Then, $\overline{Z}_\gamma(r_i) = \left(1 - \overline{Z}^3_\alpha(r_i) - \overline{Z}^3_\beta(r_i) \right)^{\frac{1}{3}}$ is identified as the grade of indeterminacy of $r_i \in R$.

Definition 4 [5]. The scheme of the form $\widetilde{Z} = \left\{ \left(r_i, \widetilde{Z}_\alpha(r_i), \widetilde{Z}_\beta(r_i) \right) | r_i \in R \right\}$, where $\widetilde{Z}_\alpha(r_i) \in [0,1]$ and $\widetilde{Z}_\beta(r_i) \in [0,1]$ represent MD and NMD for $r_i \in R$ and $0 \leq \widetilde{Z}^q_\alpha(r_i) + \widetilde{Z}^q_\beta(r_i) \leq 0$, $\mathfrak{p} \geq 1$ called a Q-ROFS in R. Then, $\widetilde{Z}_\gamma(r_i) = \left(1 - \widetilde{Z}^q_\alpha(r_i) - \widetilde{Z}^q_\beta(r_i) \right)^{\frac{1}{q}}$ is identified as the grade of indeterminacy of $r_i \in R$. For simplicity, $\left(\widetilde{Z}_\alpha(r_i), \widetilde{Z}_\beta(r_i) \right)$ is the Q-ROF number (Q-ROFN).

Definition 5 [5]. Supposing $\widetilde{Z}$, $\widetilde{Z}_1$ and $\widetilde{Z}_2$ are Q-ROFS in R, then the following hold:

i. Intersection; $\widetilde{Z}^c = \{ \left(r_i, \widetilde{Z}_\beta(r_i), \widetilde{Z}_\alpha(r_i) \right) | r_i \in R \}$,

ii. Sum; $\tilde{Z}_1 \oplus \tilde{Z}_2 = \left(\sqrt[q]{\tilde{Z}^q_{1\alpha}(r_i) + \tilde{Z}^q_{2\alpha}(r_i) - \tilde{Z}^q_{1\alpha}(r_i)\tilde{Z}^q_{2\alpha}(r_i)}, \tilde{Z}^q_{1\beta}(r_i)\tilde{Z}^q_{2\beta}(r_i) \right)$,

iii. Product;$\tilde{Z}_1 \otimes \tilde{Z}_2 = \left(\tilde{Z}^q_{1\alpha}(r_i)\tilde{Z}^q_{2\alpha}(r_i), \sqrt[q]{\tilde{Z}^q_{1\beta}(r_i) + \tilde{Z}^q_{2\beta}(r_i) - \tilde{Z}^q_{1\beta}(r_i)\tilde{Z}^q_{2\beta}(r_i)} \right)$,

iv. Intersection; $\widetilde{Z}_1 \cap \widetilde{Z}_2 = \{ \left(r_i, \min\left\{ \widetilde{Z}_{1\alpha}(r_i), \widetilde{Z}_{2\alpha}(r_i) \right\}, \max\{ \widetilde{Z}_{1\beta}(r_i), \widetilde{Z}_{2\beta}(r_i) \} \right) | r_i \in R \}$,

v. Union; $\widetilde{Z}_1 \cup \widetilde{Z}_2 = \left\{ \left(r_i, \max\left\{ \widetilde{Z}_{1\alpha}(r_i), \widetilde{Z}_{2\alpha}(r_i) \right\}, \min\{ \widetilde{Z}_{1\beta}(r_i), \widetilde{Z}_{2\beta}(r_i) \} \right) | r_i \in R \right\}$.

Definition 6 [16]. Suppose $\widetilde{Z}_1$ and $\widetilde{Z}_2$ are Q-ROFS in $R = \{r_1, r_2, \ldots, r_k\}$. Then, the Q-ROFCM between them designated by $\Phi_*(\widetilde{Z}_1, \widetilde{Z}_2)$ is a function, $\Phi : \tilde{Z}_1 \times \tilde{Z}_2 \to [0,1], [-1,1]$ which fulfills:

i. $\Phi\left(\widetilde{Z}_1, \widetilde{Z}_2\right) \in [0{,}1], [-1{,}1]$
ii. $\Phi(\widetilde{Z}_1, \widetilde{Z}_2) = \Phi(\widetilde{Z}_2, \widetilde{Z}_1)$,
iii. $\Phi(\widetilde{Z}_1, \widetilde{Z}_2) = 1$ iff $\widetilde{Z}_1 = \widetilde{Z}_2$.

The Q-ROFCM is a technique that appraises the strength of the relationship between $\widetilde{Z}_1$ and $\widetilde{Z}_2$. If $\Phi(\widetilde{Z}_1, \widetilde{Z}_2)$ or $\Phi(\widetilde{Z}_1, \widetilde{Z}_2)$ approach 1, then the connection between $\widetilde{Z}_1$ and $\widetilde{Z}_2$ is strong. Once more, if $\Phi(\widetilde{Z}_1, \widetilde{Z}_2)$ tends to 0 it means the connection between $\widetilde{Z}_1$ and $\widetilde{Z}_2$ is weak and $\Phi(\widetilde{Z}_1, \widetilde{Z}_2)$ tends to -1 means negative correlation. In addition, if $\Phi(\widetilde{Z}_1, \widetilde{Z}_2) = 1$, it means perfect correlation, and $\Phi(\widetilde{Z}_1, \widetilde{Z}_2) = 0$ means no association between the Q-ROFS. But $\Phi\left(\widetilde{Z}_1, \widetilde{Z}_2\right) = -1$ means a perfect negative correlation between the q-ROFSs.

2.2 Existing Q-Rung Orthopair fuzzy correlation measures

Suppose $\widetilde{Z}_1$ and $\widetilde{Z}_2$ are Q-ROFS in $R = \{r_1, r_2, \ldots, r_k\}$. Then, the Q-ROFCM method in [12] between the Q-ROFS is as follows:

$$\Phi_D\left(\widetilde{Z}_1, \widetilde{Z}_2\right) = \left(\frac{\left(\sum_{i=1}^{k} \widetilde{Z}_{1\alpha}^{q}(r_i)\widetilde{Z}_{2\alpha}^{q}(r_i) + \widetilde{Z}_{1\beta}^{q}(r_i)\widetilde{Z}_{2\beta}^{q}(r_i)\right)^2}{\sum_{i=1}^{K}\left(\widetilde{Z}_{1\alpha}^{2q}(r_i) + \widetilde{Z}_{1\beta}^{2q}(r_i)\right)\sum_{i=1}^{K}\left(\widetilde{Z}_{2\alpha}^{2q}(r_i) + \widetilde{Z}_{2\beta}^{2q}(r_i)\right)}\right)^{\frac{1}{2q}} \tag{1}$$

for $q \geq 1$.

The Q-ROFCM technique of Du [12] is deficient because it cannot measure the correlation coefficient of any Q-ROFS where the MD and NMD are zeros and alternating. For instance, to measure the correlation coefficient between $\widetilde{Z}_1 = ((0.5{,}0.0), (0.0, 0.6))$ and $\widetilde{Z}_2 = ((0.0{,}1.0), (1.0, 0.0))$ is impracticable with the technique, Φ_D because $\Phi_D\left(\widetilde{Z}_1, \widetilde{Z}_2\right) = 0$ for any value of q. In addition, the technique is based on a 2D approach, omitting the hesitation factor whose inclusion is vital for a reliable measure.

Singh and Ganie [14] introduced a method of Q-ROFCM based on Pearson's scheme as (2):

$$\Phi_{SG}\left(\widetilde{Z}_1, \widetilde{Z}_2\right) = \frac{1}{3}\left(\begin{array}{c}\frac{\sum_{i=1}^{k}\left(\widetilde{Z}_{1\alpha}^{q}(r_i) - \overline{\widetilde{Z}}_{1\alpha}^{q}\right)\left(\widetilde{Z}_{2\alpha}^{q}(r_i) - \overline{\widetilde{Z}}_{2\alpha}^{q}\right)}{\sqrt{\sum_{i=1}^{k}\left(\widetilde{z}_{1\alpha}^{q}(r_i) - \overline{\widetilde{Z}}_{1\alpha}^{q}\right)^2}\sqrt{\sum_{i=1}^{k}\left(\widetilde{Z}_{2\alpha}^{q}(r_i) - \overline{\widetilde{Z}}_{2\alpha}^{q}\right)^2}} + \frac{\sum_{i=1}^{k}\left(\widetilde{Z}_{1\beta}^{q}(r_i) - \overline{\widetilde{Z}}_{1\beta}^{q}\right)\left(\widetilde{Z}_{2\beta}^{q}(r_i) - \overline{\widetilde{Z}}_{2\beta}^{q}\right)}{\sqrt{\sum_{i=1}^{k}\left(\widetilde{Z}_{1\beta}^{q}(r_i) - \overline{\widetilde{Z}}_{1\beta}^{q}\right)^2}\sqrt{\sum_{i=1}^{k}\left(\widetilde{Z}_{2\beta}^{q}(r_i) - \overline{\widetilde{Z}}_{2\beta}^{q}\right)^2}} \\ + \frac{\sum_{i=1}^{k}\left(\widetilde{Z}_{1\gamma}^{q}(r_i) - \overline{\widetilde{Z}}_{1\gamma}^{q}\right)\left(\widetilde{Z}_{2\beta}^{q}(r_i) - \overline{\widetilde{Z}}_{2\beta}^{q}\right)}{\sqrt{\sum_{i=1}^{k}\left(\widetilde{Z}_{1\gamma}^{q}(r_i) - \overline{\widetilde{Z}}_{1\gamma}^{q}\right)^2}\sqrt{\sum_{i=1}^{k}\left(\widetilde{Z}_{2\gamma}^{q}(r_i) - \overline{\widetilde{Z}}_{2\gamma}^{q}\right)^2}}\end{array}\right) \tag{2}$$

for $q \geq 1$, where

$$\overline{\widetilde{Z}}_{1\alpha} = \sum_{i=1}^{k}\frac{\widetilde{Z}_{1\alpha}(r_i)}{k}, \overline{\widetilde{Z}}_{2\alpha} = \sum_{i=1}^{k}\frac{\widetilde{Z}_{2\alpha}(r_i)}{k}\overline{\widetilde{Z}}_{1\beta} = \sum_{i=1}^{k}\frac{\widetilde{Z}_{1\beta}(r_i)}{k}, \overline{\widetilde{Z}}_{2\beta} = \sum_{i=1}^{k}\frac{\widetilde{Z}_{2\beta}(r_i)}{k}, \overline{\widetilde{Z}}_{1\gamma} = \sum_{i=1}^{k}\frac{\widetilde{Z}_{1\gamma}(r_i)}{k}, \overline{\widetilde{Z}}_{2\gamma} = \sum_{i=1}^{k}\frac{\widetilde{Z}_{2\gamma}(r_i)}{k}$$

Again, the Q-ROFCM technique of Singh and Ganie [14] gives results that are not within the correlation coefficient range of values for some Q-ROFS. For example, for Q-ROFS, $\widetilde{Z}_1 = \left(\left(0.\bar{6}, 0.\bar{6}\right), \left(0.\bar{3}, 0.\bar{6}\right)\right)$ and $\widetilde{Z}_2 = \left(\left(0.\bar{6}, 0.\bar{3}\right), \left(0.\bar{6}, 0.\bar{6}\right)\right)$, we have $\Phi_{SG}\left(\widetilde{Z}_1, \widetilde{Z}_2\right) = \infty$ for any value of q. Certainly, the correlation value, ∞ is meaningless since $\infty \notin [0{,}1]$ or $\infty \notin [-1{,}1]$.

By adjusting the approaches in [32, 45], Bashir et al. [15] introduced the techniques of Q-ROFCM as (3) and (4):

$$\Phi_{BE1}\left(\widetilde{Z}_1,\widetilde{Z}_2\right)=\frac{\sum_{i=1}^{k}\left(\widetilde{Z}_{1\alpha}^{q}(r_i)\widetilde{Z}_{2\alpha}^{q}(r_i)+\widetilde{Z}_{1\beta}^{q}(r_i)\widetilde{Z}_{2\beta}^{q}(r_i)\right)}{\sqrt{\sum_{i=1}^{k}\left(\widetilde{Z}_{1\alpha}^{2q}(r_i)+\widetilde{Z}_{1\beta}^{2q}(r_i)\right)\sum_{i=1}^{k}\left(\widetilde{Z}_{2\alpha}^{2q}(r_i)+\widetilde{Z}_{2\beta}^{2q}(r_i)\right)}} \tag{3}$$

$$\Phi_{BE2}\left(\widetilde{Z}_1,\widetilde{Z}_2\right)=\frac{\sum_{i=1}^{k}\left(\widetilde{Z}_{1\alpha}^{q}(r_i)\widetilde{Z}_{2\alpha}^{q}(r_i)+\widetilde{Z}_{1\beta}^{q}(r_i)\widetilde{Z}_{2\beta}^{q}(r_i)\right)}{\max\left\{\sum_{i=1}^{k}\left(\widetilde{Z}_{1\alpha}^{2q}(r_i)+\widetilde{Z}_{1\beta}^{2q}(r_i)\right)\right\},\left\{\sum_{i=1}^{k}\left(\widetilde{Z}_{2\alpha}^{2q}(r_i)+\widetilde{Z}_{2\beta}^{2q}(r_i)\right)\right\}} \tag{4}$$

for $q \geq 1$.

Similar to the Q-ROFCM technique in [12], the techniques in [15] cannot compute the correlation coefficient of Q-ROFS with alternative parametric grades' values of zero. Using the same example for the case of Φ_D, we have $\Phi_{BE1}\left(\widetilde{Z}_1,\widetilde{Z}_2\right)=\Phi_{BE2}\left(\widetilde{Z}_1,\widetilde{Z}_2\right)=0$. In addition, the techniques are based on a 2D approach, omitting the hesitation factor whose inclusion is important for accuracy's sake.

In [13], another approach of Q-ROFCM was constructed:

$$\Phi_{LE}\left(\widetilde{Z}_1,\widetilde{Z}_2\right)=(1-\delta)\left(\frac{\sum_{i=1}^{k}\left(\widetilde{Z}_{1\alpha}^{q}(r_i)-\overline{\widetilde{Z}}_{1\alpha}\right)\left(\widetilde{Z}_{2\alpha}^{q}(r_i)-\overline{\widetilde{Z}}_{2\alpha}\right)}{\sqrt{\sum_{i=1}^{k}\left(\widetilde{Z}_{1\alpha}^{q}(r_i)-\overline{\widetilde{Z}}_{1\alpha}\right)^2}\sqrt{\sum_{i=1}^{k}\left(\widetilde{Z}_{2\alpha}^{q}(r_i)-\overline{\widetilde{Z}}_{2\alpha}\right)^2}}\right)$$

$$+\left(\frac{\sum_{i=1}^{k}\left(\widetilde{Z}_{1\beta}^{q}(r_i)-\overline{\widetilde{Z}}_{1\beta}\right)\left(\widetilde{Z}_{2\beta}^{q}(r_i)-\overline{\widetilde{Z}}_{2\beta}\right)}{\sqrt{\sum_{i=1}^{k}\left(\widetilde{Z}_{1\beta}^{q}(r_i)-\overline{\widetilde{Z}}_{1\beta}\right)^2}\sqrt{\sum_{i=1}^{k}\left(\widetilde{Z}_{2\beta}^{q}(r_i)-\overline{\widetilde{Z}}_{2\beta}\right)^2}}\right) \tag{5}$$

where $q \geq 1, \delta \in [0,1]$ and

$$\overline{\widetilde{Z}}_{1\alpha}=\sum_{i=1}^{k}\frac{\widetilde{Z}_{1\alpha}(r_i)}{k},\overline{\widetilde{Z}}_{2\alpha}=\sum_{i=1}^{k}\frac{\widetilde{Z}_{2\alpha}(r_i)}{k},\overline{\widetilde{Z}}_{1\beta}=\sum_{i=1}^{k}\frac{\widetilde{Z}_{1\beta}(r_i)}{k},\overline{\widetilde{Z}}_{2\beta}=\sum_{i=1}^{k}\frac{\widetilde{Z}_{2\beta}(r_i)}{k}$$

In like manner, the Q-ROFCM technique in [13] has a similar deficiency to the technique in [14], in the sense that it measures the correlation coefficient between the Q-ROFS, $\widetilde{Z}_1=\left((0.\overline{6},0.\overline{6}),(0.\overline{3},0.\overline{6})\right)$ and $\widetilde{Z}_2=\left((0.\overline{6},0.\overline{3}),(0.\overline{6},0.\overline{6})\right)$ as $\Phi_{LE}\left(\widetilde{Z}_1,\widetilde{Z}_2\right)=\infty$, which violates the condition of the correlation coefficient. In addition, the technique is based on 2D approach, omitting the hesitation factor whose inclusion is important.

Finally, in [16], a method of Q-ROFCM was constructed:

$$\Phi_{ES}\left(\widetilde{Z}_1,\widetilde{Z}_2\right)=\frac{1}{3k}\sum_{i=1}^{k}\left(\omega_i\left(1-\Delta_{\alpha_i}\right)+\psi_i\left(1-\Delta_{\beta_i}\right)\right.$$
$$\left.++\aleph_i\left(1-\Delta_{\gamma_i}\right)\right) \tag{6}$$

where $q \geq 1$ for

$$\omega_i=\frac{q-\Delta_{\alpha_i}-\Delta_{\alpha_{max}}}{q-\Delta_{\alpha_{min}}-\Delta_{\alpha_{max}}},\psi_i=\frac{q-\Delta_{\beta_i}-\Delta_{\beta_{max}}}{q-\Delta_{\beta_{min}}-\Delta_{\beta_{max}}},$$
$$\aleph_i=\frac{q-\Delta_{\gamma_i}-\Delta_{\gamma_{max}}}{q-\Delta_{\gamma_{min}}-\Delta_{\gamma_{max}}}$$

$$\Delta_{\alpha_{min}}=\min_{1\leq i\leq k}\left\{\left|\widetilde{Z}_{1\alpha}^{q}(r_i)-\widetilde{Z}_{2\alpha}^{q}(r_i)\right|\right\},$$
$$\Delta_{\beta_{min}}=\min_{1\leq i\leq k}\left\{\left|\widetilde{Z}_{1\beta}^{q}(r_i)-\widetilde{Z}_{2\beta}^{q}(r_i)\right|\right\}$$

$$\Delta_{\gamma_{min}}=\min_{1\leq i\leq k}\left\{\left|\widetilde{Z}_{1\gamma}^{q}(r_i)-\widetilde{Z}_{2\gamma}^{q}(r_i)\right|\right\},$$

$$\Delta_{\alpha_{max}}=\max_{1\leq i\leq k}\left\{\left|\widetilde{Z}_{1\alpha}^{q}(r_i)-\widetilde{Z}_{2\alpha}^{q}(r_i)\right|\right\},$$
$$\Delta_{\beta_{max}}=\max_{1\leq i\leq k}\left\{\left|\widetilde{Z}_{1\beta}^{q}(r_i)-\widetilde{Z}_{2\beta}^{q}(r_i)\right|\right\}$$

$$\Delta_{\gamma_{max}} = \max_{1 \le i \le k} \left\{ \left| \widetilde{Z}^q_{1\gamma}(r_i) - \widetilde{Z}^q_{2\gamma}(r_i) \right| \right\}$$

$$\Delta_\alpha = \left| \widetilde{Z}^q_{1\alpha}(r_i) - \widetilde{Z}^q_{2\alpha}(r_i) \right|, \Delta_\beta = \left| \widetilde{Z}^q_{1\beta}(r_i) - \widetilde{Z}^q_{2\beta}(r_i) \right|, \Delta_\gamma = \left| \widetilde{Z}^q_{1\gamma}(r_i) - \widetilde{Z}^q_{2\gamma}(r_i) \right|$$

The Q-ROFCM technique in [16] has no computational error, but its output lacks precision.

3 Results

To create a new method of Q-ROFCM with an improved outcome, we constructed a new approach of Q-ROFCM through Spearman's scheme. Let $\widetilde{Z}_1$ and $\widetilde{Z}_2$ be Q-ROFS in $R = \{r_1, r_2, \ldots, r_k\}$, then the new methodology is as follows:

$$\Phi\left(\widetilde{Z}_1, \widetilde{Z}_2\right) = \frac{1}{3}\left(\psi_\alpha\left(\widetilde{Z}_1, \widetilde{Z}_2\right) + \psi_\beta\left(\widetilde{Z}_1, \widetilde{Z}_2\right) + \psi_\gamma\left(\widetilde{Z}_1, \widetilde{Z}_2\right)\right) \tag{7}$$

where the Spearman's correlation ranks are represented by $\psi_\alpha\left(\widetilde{Z}_1, \widetilde{Z}_2\right), \psi_\beta\left(\widetilde{Z}_1, \widetilde{Z}_2\right)$ and$\psi_\gamma\left(\widetilde{Z}_1, \widetilde{Z}_2\right)$, and are defined as:

$$\left.\begin{aligned} \psi_\alpha\left(\widetilde{Z}_1, \widetilde{Z}_2\right) &= 1 - \frac{6\sum_{i=1}^k \left|\widetilde{Z}^q_{1\alpha}(r_i) - \widetilde{Z}^q_{2\alpha}(r_i)\right|^q}{(k+1)^3 - (k+1)} \\ \psi_\beta\left(\widetilde{Z}_1, \widetilde{Z}_2\right) &= 1 - \frac{6\sum_{i=1}^k \left|\widetilde{Z}^q_{1\beta}(r_i) - \widetilde{Z}^q_{2\beta}(r_i)\right|^q}{(k+1)^3 - (k+1)} \\ \psi_\gamma\left(\widetilde{Z}_1, \widetilde{Z}_2\right) &= 1 - \frac{6\sum_{i=1}^k \left|\widetilde{Z}^q_{1\gamma}(r_i) - \widetilde{Z}^q_{2\gamma}(r_i)\right|^q}{(k+1)^3 - (k+1)} \end{aligned}\right\} \tag{8}$$

The following theorems regarding the new Q-ROFCM are stated to buttress its mathematical validity. The theorems are as follows:

Theorem 1 *The new Q-ROFCM technique,* $\Phi\left(\widetilde{Z}_1, \widetilde{Z}_2\right)$ *between the Q-ROFS* $\widetilde{Z}_1$ *and* $\widetilde{Z}_2$ *in R is the same as*

$$\Phi\left(\widetilde{Z}_1, \widetilde{Z}_2\right) = 1 - \frac{6\sum_{i=1}^k \left(\left|\widetilde{Z}^q_{1\alpha}(r_i) - \widetilde{Z}^q_{2\alpha}(r_i)\right|^q + \left|\widetilde{Z}^q_{1\beta}(r_i) - \widetilde{Z}^q_{2\beta}(r_i)\right|^q + \left|\widetilde{Z}^q_{1\gamma}(r_i) - \widetilde{Z}^q_{2\gamma}(r_i)\right|^q \right)}{3\left((k+1)^3 - (k+1)\right)}$$

1 ***Proof***

The new Q-ROFCM is given as.

$$\Phi\left(\widetilde{Z}_1, \widetilde{Z}_2\right) = \frac{1}{3}\left[\begin{array}{l} \frac{3\left((k+1)^3 - (k+1)\right) - 6\sum_{i=1}^k \left|\widetilde{Z}^q_{1\alpha}(r_i) - \widetilde{Z}^q_{2\alpha}(r_i)\right|^q}{(k+1)^3 - (k+1)} \\ + \frac{(k+1)^3 - (k+1) - 6\sum_{i=1}^k \left|\widetilde{Z}^q_{1\beta}(r_i) - \widetilde{Z}^q_{2\beta}(r_i)\right|^q}{(k+1)^3 - (k+1)} \\ + \frac{(k+1)^3 - (k+1) - 6\sum_{i=1}^k \left|\widetilde{Z}^q_{1\gamma}(r_i) - \widetilde{Z}^q_{2\gamma}(r_i)\right|^q}{(k+1)^3 - (k+1)} \end{array}\right]$$

Then, we have

$$\Phi\left(\widetilde{Z}_1, \widetilde{Z}_2\right) = \frac{1}{3\left((k+1)^3 - (k+1)\right)}\left[3\left((k+1)^3 - (k+1)\right) - 6\sum_{i=1}^k \left(\begin{array}{c} \left|\widetilde{Z}^q_{1\alpha}(r_i) - \widetilde{Z}^q_{2\alpha}(r_i)\right|^q + \left|\widetilde{Z}^q_{1\beta}(r_i) - \widetilde{Z}^q_{2\beta}(r_i)\right|^q + \\ \left|\widetilde{Z}^q_{1\gamma}(r_i) - \widetilde{Z}^q_{2\gamma}(r_i)\right|^q \end{array}\right)\right]$$

$$= 1 - \frac{6\sum_{i=1}^k \left(\left|\widetilde{Z}^q_{1\alpha}(r_i) - \widetilde{Z}^q_{2\alpha}(r_i)\right|^q + \left|\widetilde{Z}^q_{1\beta}(r_i) - \widetilde{Z}^q_{2\beta}(r_i)\right|^q + \left|\widetilde{Z}^q_{1\gamma}(r_i) - \widetilde{Z}^q_{2\gamma}(r_i)\right|^q \right)}{3\left((k+1)^3 - (k+1)\right)},$$

hence the proof.

Theorem 2 *The new technique of Q-ROFCM satisfies the conditions of Q-ROFCM.*

Proof The Q-ROFCM's conditions between Q-ROFS, $\widetilde{Z}_1$ and $\widetilde{Z}_2$ in R are as follows: (i) $\Phi\left(\widetilde{Z}_1,\widetilde{Z}_2\right)=\Phi\left(\widetilde{Z}_2,\widetilde{Z}_1\right)$, (ii) $\Phi\left(\widetilde{Z}_1,\widetilde{Z}_2\right)=1$ iff$\widetilde{Z}_1=\widetilde{Z}_2$, (iii) $\left|\Phi\left(\widetilde{Z}_1,\widetilde{Z}_2\right)\right|\leq 1$.

Since we have,

$$\Phi\left(\widetilde{Z}_1,\widetilde{Z}_2\right)=1-\frac{6\sum_{i=1}^{k}\left(\left|\widetilde{Z}_{1\alpha}^q(r_i)-\widetilde{Z}_{2\alpha}^q(r_i)\right|^q+\left|\widetilde{Z}_{1\beta}^q(r_i)-\widetilde{Z}_{2\beta}^q(r_i)\right|^q+\left|\widetilde{Z}_{1\gamma}^q(r_i)-\widetilde{Z}_{2\gamma}^q(r_i)\right|^q\right)}{3\left((k+1)^3-(k+1)\right)}$$

Then,

$$\Phi\left(\widetilde{Z}_1,\widetilde{Z}_2\right)=\frac{3\left((k+1)^3-(k+1)\right)-6\sum_{i=1}^{k}\left(\left|\widetilde{Z}_{1\alpha}^q(r_i)-\widetilde{Z}_{2\alpha}^q(r_i)\right|^q+\left|\widetilde{Z}_{1\beta}^q(r_i)-\widetilde{Z}_{2\beta}^q(r_i)\right|^q+\left|\widetilde{Z}_{1\gamma}^q(r_i)-\widetilde{Z}_{2\gamma}^q(r_i)\right|^q\right)}{3\left((k+1)^3-(k+1)\right)}$$

$$=\frac{3(k+1)^3-(k+1)-6\sum_{i=1}^{k}\left(\left|\widetilde{Z}_{2\alpha}^q(r_i)-\widetilde{Z}_{1\alpha}^q(r_i)\right|^q+\left|\widetilde{Z}_{2\beta}^q(r_i)-\widetilde{Z}_{1\beta}^q(r_i)\right|^q+\left|\widetilde{Z}_{2\gamma}^q(r_i)-\widetilde{Z}_{1\gamma}^q(r_i)\right|^q\right)}{3\left((k+1)^3-(k+1)\right)}$$

$$=\Phi\left(\widetilde{Z}_1,\widetilde{Z}_2\right)$$

As a result , the proof of (i) is complete.

Next, suppose $\Phi\left(\widetilde{Z}_1,\widetilde{Z}_2\right)=1$. Then we have

$$\frac{6\sum_{i=1}^{k}\left(\left|\widetilde{Z}_{1\alpha}^q(r_i)-\widetilde{Z}_{2\alpha}^q(r_i)\right|^q+\left|\widetilde{Z}_{1\beta}^q(r_i)-\widetilde{Z}_{2\beta}^q(r_i)\right|^q+\left|\widetilde{Z}_{1\gamma}^q(r_i)-\widetilde{Z}_{2\gamma}^q(r_i)\right|^q\right)}{3\left((k+1)^3-(k+1)\right)}=0$$

That is,

$$6\sum_{i=1}^{k}\left(\left|\widetilde{Z}_{1\alpha}^q(r_i)-\widetilde{Z}_{2\alpha}^q(r_i)\right|^q+\left|\widetilde{Z}_{1\beta}^q(r_i)-\widetilde{Z}_{2\beta}^q(r_i)\right|^q+\left|\widetilde{Z}_{1\gamma}^q(r_i)-\widetilde{Z}_{2\gamma}^q(r_i)\right|^q\right)=0\Rightarrow$$

$$\left|\widetilde{Z}_{1\alpha}^q(r_i)-\widetilde{Z}_{2\alpha}^q(r_i)\right|^q=0,\left|\widetilde{Z}_{1\beta}^q(r_i)-\widetilde{Z}_{2\beta}^q(r_i)\right|^q=0,\left|\widetilde{Z}_{1\gamma}^q(r_i)-\widetilde{Z}_{2\gamma}^q(r_i)\right|^q=0$$

Thus, $\widetilde{Z}_{1\alpha}^q(r_i)=\widetilde{Z}_{2\alpha}^q(r_i)$, $\widetilde{Z}_{1\beta}^q(r_i)=\widetilde{Z}_{2\beta}^q(r_i)$ and $\widetilde{Z}_{1\gamma}^q(r_i)=\widetilde{Z}_{2\gamma}^q(r_i)$. Hence, $\widetilde{Z}_1=\widetilde{Z}_2$.

Conversely, assume $\widetilde{Z}_1=\widetilde{Z}_2$. Thus

$$\sum_{i=1}^{k}\left(\left|\widetilde{Z}_{1\alpha}^q(r_i)-\widetilde{Z}_{2\alpha}^q(r_i)\right|^q+\left|\widetilde{Z}_{1\beta}^q(r_i)-\widetilde{Z}_{2\beta}^q(r_i)\right|^{\mathfrak{p}}+\left|\widetilde{Z}_{1\gamma}^q(r_i)-\widetilde{Z}_{2\gamma}^q(r_i)\right|^q\right)=0$$

$\Phi\left(\widetilde{Z}_1, \widetilde{Z}_2\right) = 0$ as predicted, and so (ii) is verified.

To end with, $\left|\Phi\left(\widetilde{Z}_1, \widetilde{Z}_2\right)\right| \leq 1$ means $0 \leq \Phi\left(\widetilde{Z}_1, \widetilde{Z}_2\right) \leq 1$. Because

$$\sum_{i=1}^{k}\left(\left|\widetilde{Z}^q_{1\alpha}(r_i) - \widetilde{Z}^q_{2\alpha}(r_i)\right|^q + \left|\widetilde{Z}^q_{1\beta}(r_i) - \widetilde{Z}^q_{2\beta}(r_i)\right|^q + \left|\widetilde{Z}^q_{1\gamma}(r_i) - \widetilde{Z}^q_{2\gamma}(r_i)\right|^q\right) \geq 0$$

hence $\Phi\left(\widetilde{Z}_1, \widetilde{Z}_2\right) \geq 0$.

Now, assume that

$$6\sum_{i=1}^{k}\left|\widetilde{Z}^q_{1\alpha}(r_i) - \widetilde{Z}^q_{2\alpha}(r_i)\right|^q = Q_1, 6\sum_{i=1}^{k}\left|\widetilde{Z}^q_{1\beta}(r_i) - \widetilde{Z}^q_{2\beta}(r_i)\right|^q = Q_2, 6\sum_{i=1}^{k}\left|\widetilde{Z}^q_{1\gamma}(r_i) - \widetilde{Z}^q_{2\gamma}(r_i)\right|^q = Q_3$$

then we have

$$\Phi\left(\widetilde{Z}_1, \widetilde{Z}_2\right) = 1 - \frac{6\sum_{i=1}^{k}\left(\left|\widetilde{Z}^q_{1\alpha}(r_i) - \widetilde{Z}^q_{2\alpha}(r_i)\right|^q + \left|\widetilde{Z}^q_{1\beta}(r_i) - \widetilde{Z}^q_{2\beta}(r_i)\right|^q + \left|\widetilde{Z}^q_{1\gamma}(r_i) - \widetilde{Z}^q_{2\gamma}(r_i)\right|^q\right)}{3\left((k+1)^3 - (k+1)\right)}$$

$$\leq 1 - \frac{6\sum_{i=1}^{k}\left(\left|\widetilde{Z}^q_{1\alpha}(r_i) - \widetilde{Z}^q_{2\alpha}(r_i)\right|^q\right) + 6\sum_{i=1}^{k}\left(\left|\widetilde{Z}^q_{1\beta}(r_i) - \widetilde{Z}^q_{2\beta}(r_i)\right|^q\right) + 6\sum_{i=1}^{k}\left(\left|\widetilde{Z}^q_{1\gamma}(r_i) - \widetilde{Z}^q_{2\gamma}(r_i)\right|^q\right)}{3\left((k+1)^3 - (k+1)\right)}$$

$$= 1 - \frac{Q_1 + Q_2 + Q_3}{3\left((k+1)^3 - (k+1)\right)}$$

By take away 1 from both sides, we have

$$\Phi\left(\widetilde{Z}_1, \widetilde{Z}_2\right) - 1 = -\left(\frac{Q_1 + Q_2 + Q_3}{3\left((k+1)^3 - (k+1)\right)}\right) \leq 0$$

Hence, $\Phi\left(\widetilde{Z}_1, \widetilde{Z}_2\right) - 1 \leq 0 \Rightarrow \Phi\left(\widetilde{Z}_1, \widetilde{Z}_2\right) \leq 1$. Thus, $\Phi\left(\widetilde{Z}_1, \widetilde{Z}_2\right) \leq 1$, as necessary.

3.1 Numerical illustration of the new Q-Rung orthopair fuzzy correlation measure

The following example is used to implement the new Q-ROFCM in measuring the correlation coefficient between Q-ROFS.

Example 1 Let $\widetilde{Z}_1$ and $\widetilde{Z}_2$ be Q-ROFS in $R = \{r_1, r_2\}$ defined by $\widetilde{Z}_1 = \{(r_1, 0.4, 0.3), (r_2, 0.3, 0.2)\}$ and $\widetilde{Z}_2 = \{(r_1, 0.3, 0.2), (r_2, 0.2, 0.1)\}$. The correlation coefficient between the Q-ROFS is computed for each level of the Q-ROFS, i.e., for $q = 1, 2, 3, 4$.

First, we verify for $q = 1$,

$$\psi_\alpha\left(\widetilde{Z}_1, \widetilde{Z}_2\right) = 1 - \frac{6(|0.4 - 0.3| + |0.3 - 0.2|)}{(2+1)^3 - (2+1)} = 0.9500$$

$$\psi_\beta\left(\widetilde{Z}_1, \widetilde{Z}_2\right) = 1 - \frac{6(|0.3 - 0.2| + |0.2 - 0.1|)}{(2+1)^3 - (2+1)} = 0.9500$$

$$\psi_\gamma\left(\widetilde{Z}_1, \widetilde{Z}_2\right) = 1 - \frac{6(|0.3 - 0.5| + |0.5 - 0.7|)}{(2+1)^3 - (2+1)} = 1.1000$$

Table 1 Q-Rung Orthopair Fuzzy Correlation Measure Methods' Outcomes

q-ROFCMs	$q = 2$	$q = 3$	$q = 4$
Φ_D[12]	0.9890	0.9874	0.9880
Φ_{LE}[13]	0.1228	0.2160	0.2261
Φ_{SG}[14]	0.6224	0.8349	0.6316
Φ_{BE1}[15]	0.6198	0.4432	0.2714
Φ_{BE2}[15]	0.6905	0.3859	0.2958
Φ_{ES}[16]	0.9397	0.8072	0.9631
Novel Q-ROFCM	0.9979	0.9999	0.9999

$$\Rightarrow \Phi\left(\tilde{Z}_1, \tilde{Z}_2\right) = \frac{1}{3}(0.9500 + 0.9500 + 1.1000) = 1.0000$$

Next, for $q = 2$,

$$\psi_\alpha\left(\widetilde{Z}_1, \widetilde{Z}_2\right) = 1 - \frac{6\left(\left|0.4^2 - 0.3^2\right|^2 + \left|0.3^2 - 0.2^2\right|\right)^2}{(2+1)^3 - (2+1)} = 0.9992$$

$$\psi_\beta\left(\widetilde{Z}_1, \widetilde{Z}_2\right) = 1 - \frac{6\left(\left|0.3^2 - 0.2^2\right|^2 + \left|0.2^2 - 0.1^2\right|\right)^2}{(2+1)^3 - (2+1)} = 0.9997$$

Table 2 Linguistic Variables for the Vehicle Selection

Linguistic Variables	Q-ROFN
Extremely Good (EG)	(1.0, 0.0)
Very Good (VG)	(0.9, 0.1)
Good (G)	(0.7, 0.3)
Fair (F)	(0.5, 0.5)
Poor (P)	(0.3, 0.7)
Very Poor (VP)	(0.1, 0.9)
Extremely Poor (EP)	(0.0, 1.0)

Table 3 Linguistic Variables from Vehicle Maintenance Expert (VME) I

Car Brands	Main Cost	Fuel Efficiency	Safety Features	Resale Value	Purchase Cost	Reliability	Driving Comfort	Good Warranty
$\mathfrak{W}_1$	G	G	P	VG	F	VG	G	VG
$\mathfrak{W}_2$	EG	G	G	EG	P	G	EG	P
$\mathfrak{W}_3$	VP	F	VG	EP	VG	F	VP	G
$\mathfrak{W}_4$	F	F	G	G	G	G	EG	P
$\mathfrak{W}_5$	EG	EG	VP	F	EG	P	F	G
$\mathfrak{W}_6$	G	EP	F	VP	VG	G	G	VG
$\mathfrak{W}_7$	EG	G	F	F	EG	F	P	G

Table 4 Linguistic Variables from VME II

Car Brands	Main Cost	Fuel Efficiency	Safety Features	Resale Value	Purchase Cost	Reliability	Driving Comfort	Good Warranty
$\mathfrak{W}_1$	G	G	P	EG	VG	F	G	VG
$\mathfrak{W}_2$	EG	G	VG	G	VG	VG	G	EG
$\mathfrak{W}_3$	VP	F	VG	VG	G	EG	G	P
$\mathfrak{W}_4$	F	F	G	G	F	EG	F	G
$\mathfrak{W}_5$	EG	EG	VP	VP	EP	P	VG	P
$\mathfrak{W}_6$	G	EP	F	F	F	EG	EP	F
$\mathfrak{W}_7$	EG	G	F	EG	F	P	EP	P

Table 5 Linguistic Variables from VME III

Car Brands	Main Cost	Fuel Efficiency	Safety Features	Resale Value	Purchase Cost	Reliability	Driving Comfort	Good Warranty
$\mathfrak{W}_1$	VG	G	G	G	G	VG	G	EG
$\mathfrak{W}_2$	G	VG	G	G	VG	G	VG	G
$\mathfrak{W}_3$	EP	P	EG	EG	EG	EG	EP	F
$\mathfrak{W}_4$	EG	EG	G	G	F	G	F	VG
$\mathfrak{W}_5$	EG	VG	G	G	G	VG	G	EG
$\mathfrak{W}_6$	G	VG	G	G	P	VG	EP	P
$\mathfrak{W}_7$	P	P	VG	EP	EG	G	EG	EP

Table 6 Q-Rung Orthopair Fuzzy Numbers (Q-ROFN) from VME I

Car Brands	Main Cost	Fuel Efficiency	Safety Features	Resale Value	Purchase Cost	Reliability	Driving Comfort	Good Warranty
$\mathfrak{W}_1$	(0.7,0.3)	(0.7,0.3)	(0.3,0.7)	(0.9,0.1)	(0.5,0.5)	(0.9,0.1)	(0.7,0.3)	(0.9,0.1)
$\mathfrak{W}_2$	(1.0,0.0)	(0.7,0.3)	(0.7,0.3)	(1.0,0.0)	(0.3,0.7)	(0.7,0.3)	(1.0,0.0)	(0.3,0.7)
$\mathfrak{W}_3$	(0.1,0.9)	(0.5,0.5)	(0.9,0.1)	(0.0,1.0)	(0.9,0.1)	(0.5,0.5)	(0.1,0.9)	(0.7,0.3)
$\mathfrak{W}_4$	(0.5,0.5)	(0.5,0.5)	(0.7,0.3)	(0.7,0.3)	(0.7,0.3)	(0.7,0.3)	(1.0,0.0)	(0.3,0.7)
$\mathfrak{W}_5$	(1.0,0.0)	(1.0, 0.0)	(0.3,0.7)	(0.5,0.5)	(1.0, 0.0)	(0.3,0.7)	(0.5,0.5)	(0.7,0.3)
$\mathfrak{W}_6$	(0.7,0.3)	(0.0,1.0)	(0.5,0.5)	(0.1,0.9)	(0.9,0.1)	(0.7,0.3)	(0.7,0.3)	(0.9,0.1)
$\mathfrak{W}_7$	(1.0,0.0)	(0.7,0.3)	(0.5,0.5)	(0.5,0.5)	(1.0, 0.0)	(0.5,0.5)	(0.3,0.7)	(0.7,0.3)

Table 7 Q-ROFN from VME II

Car Brands	Main Cost	Fuel Efficiency	Safety Features	Resale Value	Purchase Cost	Reliability	Driving Comfort	Good Warranty
$\mathfrak{W}_1$	(0.7,0.3)	(0.7,0.3)	(0.3,0.7)	(1.0,0.0)	(0.9,0.1)	(0.5,0.5)	(0.7,0.3)	(0.9,0.1)
$\mathfrak{W}_2$	(1.0,0.0)	(0.7,0.3)	(0.9,0.1)	(0.7,0.3)	(0.9,0.1)	(0.9,0.1)	(0.7,0.3)	(1.0,0.0)
$\mathfrak{W}_3$	(0.1,0.9)	(0.5,0.5)	(0.9,0.1)	(0.9,0.1)	(0.7,0.3)	(1.0, 0.0)	(0.7,0.3)	(0.3,0.7)
$\mathfrak{W}_4$	(0.5,0.5)	(0.5,0.5)	(0.7,0.3)	(0.7,0.3)	(0.5,0.5)	(1.0, 0.0)	(0.5,0.5)	(0.7,0.3)
$\mathfrak{W}_5$	(1.0,0.0)	(1.0, 0.0)	(0.1,0.9)	(0.1,0.9)	(0,1)	(0.3,0.7)	(0.9,0.1)	(0.3,0.7)
$\mathfrak{W}_6$	(0.7,0.3)	(0.0,1.0)	(0.5,0.5)	(0.5,0.5)	(0.5,0.5)	(1.0, 0.0)	(0.0,1.0)	(0.5,0.5)
$\mathfrak{W}_7$	(1.0,0.0)	(0.7,0.3)	(0.5,0.5)	(1.0,0.0)	(0.5,0.5)	(0.3,0.7)	(0.0,1.0)	(0.3,0.7)

Table 8 Q-ROFN from VME III

Car Brands	Main Cost	Fuel Efficiency	Safety Features	Resale Value	Purchase Cost	Reliability	Driving Comfort	Good Warranty
$\mathfrak{W}_1$	(0.9,0.1)	(0.7,0.3)	(0.7,0.3)	(0.7,0.3)	(0.7,0.3)	(0.9,0.1)	(0.7,0.3)	(1.0,0.0)
$\mathfrak{W}_2$	(0.7,0.3)	(0.9,0.1)	(0.7,0.3)	(0.7,0.3)	(0.9,0.1)	(0.7,0.3)	(0.9,0.1)	(0.7,0.3)
$\mathfrak{W}_3$	(0.0,1.0)	(0.3,0.7)	(1.0,0.0)	(1.0,0.0)	(1.0,0.0)	(1.0,0.0)	(0.0,1.0)	(0.5,0.5)
$\mathfrak{W}_4$	(1.0,0.0)	(1.0,0.0)	(0.7,0.3)	(0.7,0.3)	(0.5,0.5)	(0.7,0.3)	(0.5,0.5)	(0.9,0.1)
$\mathfrak{W}_5$	(1.0,0.0)	(0.9,0.1)	(0.7,0.3)	(0.7,0.3)	(0.7,0.3)	(0.9,0.1)	(0.7,0.3)	(1.0,0.0)
$\mathfrak{W}_6$	(0.7,0.3)	(0.9,0.1)	(0.7,0.3)	(0.7,0.3)	(0.3,0.7)	(0.9,0.1)	(0.0,1.0)	(0.3,0.7)
$\mathfrak{W}_7$	(0.3,0.7)	(0.3,0.7)	(0.9,0.1)	(0.0,1.0)	(1.0,0.0)	(0.7,0.3)	(1.0,0.0)	(0.0,1.0)

$$\psi_\gamma\left(\widetilde{Z}_1,\widetilde{Z}_2\right) = 1 - \frac{6\left(\left|0.8660^2 - 0.9327^2\right|^2 + \left|0.9327^2 - 0.9746^2\right|\right)^2}{(2+1)^3 - (2+1)} = 0.9948$$

$$\Rightarrow \Phi\left(\tilde{Z}_1,\tilde{Z}_2\right) = \frac{1}{3}(0.9992 + 0.9997 + 0.9948) = 0.9979$$

For $q = 3$,

$$\psi_\alpha\left(\widetilde{Z}_1,\widetilde{Z}_2\right) = 1 - \frac{6\left(\left|0.4^3 - 0.3^3\right|^3 + \left|0.3^3 - 0.2^3\right|^3\right)}{(2+1)^3 - (2+1)} = 0.9999 \quad \psi_\beta\left(\widetilde{Z}_1,\widetilde{Z}_2\right) = 1 - \frac{6\left(\left|0.3^3 - 0.2^3\right|^3 + \left|0.2^3 - 0.1^3\right|^3\right)}{(2+1)^3 - (2+1)} = 0.9999$$

$$\psi_\gamma\left(\widetilde{Z}_1,\widetilde{Z}_2\right) = 1 - \frac{6\left(\left|0.9686^3 - 0.9881^3\right|^3 + \left|0.9881^3 - 0.9969^3\right|^3\right)}{(2+1)^3 - (2+1)} = 1.0000$$

Table 9 Average Q-ROFN from the VME

Cars	Main Cost	Fuel Efficiency	Safety Features	Resale Value	Purchase Cost	Reliability	Driving Comfort	Good Warranty
$\mathfrak{W}_1$	$\begin{pmatrix}0.7667\\0.2333\end{pmatrix}$	$\begin{pmatrix}0.7000\\0.3000\end{pmatrix}$	$\begin{pmatrix}0.4333\\0.5667\end{pmatrix}$	$\begin{pmatrix}0.8667\\0.1333\end{pmatrix}$	$\begin{pmatrix}0.7000\\0.3000\end{pmatrix}$	$\begin{pmatrix}0.7667\\0.2333\end{pmatrix}$	$\begin{pmatrix}0.7000\\0.3000\end{pmatrix}$	$\begin{pmatrix}0.9333\\0.0667\end{pmatrix}$
$\mathfrak{W}_2$	$\begin{pmatrix}0.9000\\0.1000\end{pmatrix}$	$\begin{pmatrix}0.7667\\0.2333\end{pmatrix}$	$\begin{pmatrix}0.7667\\0.2333\end{pmatrix}$	$\begin{pmatrix}0.8000\\0.2000\end{pmatrix}$	$\begin{pmatrix}0.7000\\0.3000\end{pmatrix}$	$\begin{pmatrix}0.7667\\0.2333\end{pmatrix}$	$\begin{pmatrix}0.8667\\0.1333\end{pmatrix}$	$\begin{pmatrix}0.6667\\0.3333\end{pmatrix}$
$\mathfrak{W}_3$	$\begin{pmatrix}0.0667\\0.9333\end{pmatrix}$	$\begin{pmatrix}0.4333\\0.5667\end{pmatrix}$	$\begin{pmatrix}0.9333\\0.0667\end{pmatrix}$	$\begin{pmatrix}0.6333\\0.3667\end{pmatrix}$	$\begin{pmatrix}0.8667\\0.1333\end{pmatrix}$	$\begin{pmatrix}0.8333\\0.1667\end{pmatrix}$	$\begin{pmatrix}0.2667\\0.7333\end{pmatrix}$	$\begin{pmatrix}0.5000\\0.5000\end{pmatrix}$
$\mathfrak{W}_4$	$\begin{pmatrix}0.6667\\0.3333\end{pmatrix}$	$\begin{pmatrix}0.6667\\0.3333\end{pmatrix}$	$\begin{pmatrix}0.7000\\0.3000\end{pmatrix}$	$\begin{pmatrix}0.6333\\0.3667\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$	$\begin{pmatrix}0.8000\\0.2000\end{pmatrix}$	$\begin{pmatrix}0.6667\\0.3333\end{pmatrix}$	$\begin{pmatrix}0.6333\\0.3667\end{pmatrix}$
$\mathfrak{W}_5$	$\begin{pmatrix}1.0000\\0.0000\end{pmatrix}$	$\begin{pmatrix}0.9667\\0.0333\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$	$\begin{pmatrix}0.4333\\0.5667\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$	$\begin{pmatrix}0.5000\\0.5000\end{pmatrix}$	$\begin{pmatrix}0.7000\\0.3000\end{pmatrix}$	$\begin{pmatrix}0.6667\\0.3333\end{pmatrix}$
$\mathfrak{W}_6$	$\begin{pmatrix}0.7000\\0.3000\end{pmatrix}$	$\begin{pmatrix}0.3333\\0.6667\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$	$\begin{pmatrix}0.4333\\0.5667\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$	$\begin{pmatrix}0.8667\\0.1333\end{pmatrix}$	$\begin{pmatrix}0.2333\\0.7667\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$
$\mathfrak{W}_7$	$\begin{pmatrix}0.7667\\0.2333\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$	$\begin{pmatrix}0.6333\\0.3667\end{pmatrix}$	$\begin{pmatrix}0.5000\\0.5000\end{pmatrix}$	$\begin{pmatrix}0.8333\\0.1667\end{pmatrix}$	$\begin{pmatrix}0.5000\\0.5000\end{pmatrix}$	$\begin{pmatrix}0.4333\\0.5667\end{pmatrix}$	$\begin{pmatrix}0.3333\\0.6667\end{pmatrix}$

$$\Rightarrow \Phi\left(\tilde{Z}_1, \tilde{Z}_2\right) = \frac{1}{3}(0.9999 + 0.9999 + 1.0000) = 0.9999$$

For $q = 4$

$$\psi_\alpha\left(\widetilde{Z}_1, \widetilde{Z}_2\right) = 1 - \frac{6\left(\left|0.4^4 - 0.3^4\right|^4 + \left|0.3^4 - 0.2^4\right|^4\right)}{(2+1)^3 - (2+1)} = 0.9999$$

$$\psi_\beta\left(\widetilde{Z}_1, \widetilde{Z}_2\right) = 1 - \frac{6\left(\left|0.3^4 - 0.2^4\right|^4 + \left|0.2^4 - 0.1^4\right|^4\right)}{(2+1)^3 - (2+1)} = 1.0000$$

$$\psi_\gamma\left(\widetilde{Z}_1, \widetilde{Z}_2\right) = 1 - \frac{6\left(\left|0.9914^4 - 0.9975^4\right|^4 + \left|0.9975^4 - 0.9995^4\right|^4\right)}{(2+1)^3 - (2+1)} = 0.9999$$

$$\Rightarrow \Phi\left(\widetilde{Z}_1, \widetilde{Z}_2\right) = = \frac{1}{3}(0.9999 + 1.0000 + 0.9999) = 0.9999$$

Hence, the new Q-ROFCM satisfies Theorems 1 and 2. For $q = 1$, $\Phi\left(\widetilde{Z}_1, \widetilde{Z}_2\right) = 1.0000$ although $\widetilde{Z}_1 \neq \widetilde{Z}_2$ shows

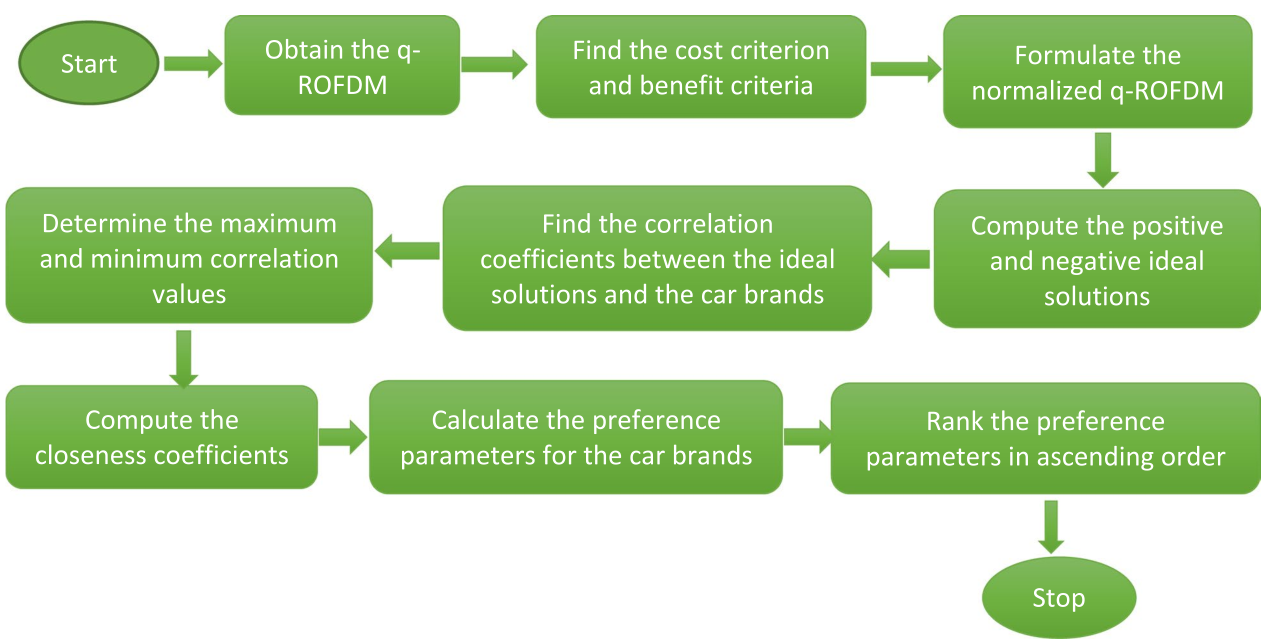

Fig. 1 Representation of the algorithm

the limitation of intuitionistic fuzzy set theory. As the level increases, the correlation values improve, which attests to the significance of the new Q-ROFCM. The studied Q-ROFS are quite related in corroboration with physical observation.

Now, the superiority of the new Q-ROFCM is demonstrated by comparing it with the existing Q-ROFCM methods to showcase the advantages of the new technique. By using the extant Q-ROFCM techniques to measure the correlation between the same Q-ROFS in Example 1, the outcomes in Table 1 are obtained.

In all, the results from the new Q-ROFCM technique give the most exact correlation between the Q-ROFS. The results for Q-ROFCM techniques in [12, 15] decrease as q increases, which again show the weakness of the techniques. The reliability of the correlation coefficient value increases as q increase.

4 Discussion

In this section, the application of the new Q-ROFCM approach is discussed in the vehicle selection process within the Nigeria setting. Choosing the right car/vehicle in Nigeria involves considering various factors such

Table 10 Normalized Q-Rung Orthopair Fuzzy Decision Matrix (Q-ROFDM)

Cars	Main Cost	Fuel Efficiency	Safety Features	Resale Value	Purchase Cost	Reliability	Driving Comfort	Good Warranty
$\mathfrak{W}_1$	$\begin{pmatrix}0.7667\\0.2333\end{pmatrix}$	$\begin{pmatrix}0.7000\\0.3000\end{pmatrix}$	$\begin{pmatrix}0.4333\\0.5667\end{pmatrix}$	$\begin{pmatrix}0.1333\\0.8667\end{pmatrix}$	$\begin{pmatrix}0.7000\\0.3000\end{pmatrix}$	$\begin{pmatrix}0.7667\\0.2333\end{pmatrix}$	$\begin{pmatrix}0.7000\\0.3000\end{pmatrix}$	$\begin{pmatrix}0.9333\\0.0667\end{pmatrix}$
$\mathfrak{W}_2$	$\begin{pmatrix}0.9000\\0.1000\end{pmatrix}$	$\begin{pmatrix}0.7667\\0.2333\end{pmatrix}$	$\begin{pmatrix}0.7667\\0.2333\end{pmatrix}$	$\begin{pmatrix}0.2000\\0.8000\end{pmatrix}$	$\begin{pmatrix}0.7000\\0.3000\end{pmatrix}$	$\begin{pmatrix}0.7667\\0.2333\end{pmatrix}$	$\begin{pmatrix}0.8667\\0.1333\end{pmatrix}$	$\begin{pmatrix}0.6667\\0.3333\end{pmatrix}$
$\mathfrak{W}_3$	$\begin{pmatrix}0.0667\\0.9333\end{pmatrix}$	$\begin{pmatrix}0.4333\\0.5667\end{pmatrix}$	$\begin{pmatrix}0.9333\\0.0667\end{pmatrix}$	$\begin{pmatrix}0.3667\\0.6333\end{pmatrix}$	$\begin{pmatrix}0.8667\\0.1333\end{pmatrix}$	$\begin{pmatrix}0.8333\\0.1667\end{pmatrix}$	$\begin{pmatrix}0.2667\\0.7333\end{pmatrix}$	$\begin{pmatrix}0.5000\\0.5000\end{pmatrix}$
$\mathfrak{W}_4$	$\begin{pmatrix}0.6667\\0.3333\end{pmatrix}$	$\begin{pmatrix}0.6667\\0.3333\end{pmatrix}$	$\begin{pmatrix}0.7000\\0.3000\end{pmatrix}$	$\begin{pmatrix}0.3667\\0.6333\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$	$\begin{pmatrix}0.8000\\0.2000\end{pmatrix}$	$\begin{pmatrix}0.6667\\0.3333\end{pmatrix}$	$\begin{pmatrix}0.6333\\0.3667\end{pmatrix}$
$\mathfrak{W}_5$	$\begin{pmatrix}1.0000\\0.0000\end{pmatrix}$	$\begin{pmatrix}0.9667\\0.0333\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$	$\begin{pmatrix}0.5000\\0.5000\end{pmatrix}$	$\begin{pmatrix}0.7000\\0.3000\end{pmatrix}$	$\begin{pmatrix}0.6667\\0.3333\end{pmatrix}$
$\mathfrak{W}_6$	$\begin{pmatrix}0.7000\\0.3000\end{pmatrix}$	$\begin{pmatrix}0.3333\\0.6667\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$	$\begin{pmatrix}0.8667\\0.1333\end{pmatrix}$	$\begin{pmatrix}0.2333\\0.7667\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$
$\mathfrak{W}_7$	$\begin{pmatrix}0.7667\\0.2333\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$	$\begin{pmatrix}0.6333\\0.3667\end{pmatrix}$	$\begin{pmatrix}0.5000\\0.5000\end{pmatrix}$	$\begin{pmatrix}0.8333\\0.1667\end{pmatrix}$	$\begin{pmatrix}0.5000\\0.5000\end{pmatrix}$	$\begin{pmatrix}0.4333\\0.5667\end{pmatrix}$	$\begin{pmatrix}0.3333\\0.6667\end{pmatrix}$

Table 11 Positive and Negative Ideal Solutions

Γ^+/Γ^-	Main Cost	Fuel Efficiency	Safety Features	Resale Value	Purchase Cost	Reliability	Driving Comfort	Good Warranty
Γ^+	$\begin{pmatrix}1.0000\\0.0000\end{pmatrix}$	$\begin{pmatrix}0.9667\\0.0333\end{pmatrix}$	$\begin{pmatrix}0.9333\\0.0667\end{pmatrix}$	$\begin{pmatrix}0.1333\\0.8667\end{pmatrix}$	$\begin{pmatrix}0.8667\\0.1333\end{pmatrix}$	$\begin{pmatrix}0.8667\\0.1333\end{pmatrix}$	$\begin{pmatrix}0.8667\\0.1333\end{pmatrix}$	$\begin{pmatrix}0.9333\\0.0667\end{pmatrix}$
Γ^-	$\begin{pmatrix}0.0667\\0.9333\end{pmatrix}$	$\begin{pmatrix}0.3333\\0.6667\end{pmatrix}$	$\begin{pmatrix}0.4333\\0.5667\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$	$\begin{pmatrix}0.5000\\0.5000\end{pmatrix}$	$\begin{pmatrix}0.2333\\0.7667\end{pmatrix}$	$\begin{pmatrix}0.3333\\0.6667\end{pmatrix}$

Table 12 Correlation Coefficients for the Positive Ideal Solution and the Vehicle Alternatives

q	$\Phi(\Gamma^+, \mathfrak{W}_1)$	$\Phi(\Gamma^+, \mathfrak{W}_2)$	$\Phi(\Gamma^+, \mathfrak{W}_3)$	$\Phi(\Gamma^+, \mathfrak{W}_4)$	$\Phi(\Gamma^+, \mathfrak{W}_5)$	$\Phi(\Gamma^+, \mathfrak{W}_6)$	$\Phi(\Gamma^+, \mathfrak{W}_7)$
1	0.9880	0.9907	0.9831	0.9876	0.9902	0.9839	0.9848
2	0.9939	0.996	0.9886	0.9949	0.9963	0.9915	0.9927
3	0.9962	0.998	0.9917	0.9966	0.9979	0.9944	0.9953
4	0.9977	0.9990	0.9937	0.9973	0.9988	0.9960	0.9967
5	0.9984	0.9995	0.9951	0.9976	0.9994	0.9969	0.9977
6	0.9987	0.9997	0.9960	0.9978	0.9998	0.9975	0.9983

Table 13 Correlation Coefficients for the Negative Ideal Solution and the Vehicle Alternatives

q	$\Phi(\Gamma^-, \mathfrak{W}_1)$	$\Phi(\Gamma^-, \mathfrak{W}_2)$	$\Phi(\Gamma^-, \mathfrak{W}_3)$	$\Phi(\Gamma^-, \mathfrak{W}_4)$	$\Phi(\Gamma^-, \mathfrak{W}_5)$	$\Phi(\Gamma^-, \mathfrak{W}_6)$	$\Phi(\Gamma^-, \mathfrak{W}_7)$
1	0.9843	0.9822	0.9917	0.9872	0.9854	0.9917	0.9907
2	0.992	0.9907	0.9969	0.9947	0.9907	0.9963	0.9959
3	0.9961	0.9957	0.9983	0.9980	0.9934	0.9981	0.9980
4	0.998	0.9981	0.9990	0.9990	0.9947	0.9989	0.9990
5	0.9990	0.9992	0.9995	0.9995	0.9956	0.9995	0.9995
6	0.9995	0.9997	0.9998	0.9998	0.9962	0.9998	0.9998

Table 14 Maximum and Minimum Correlation Values

q	$\Phi(\Gamma^+)$	$\Phi(\Gamma^-)$
1	0.9907	0.9822
2	0.9963	0.9907
3	0.9979	0.9934
4	0.9990	0.9947
5	0.9995	0.9956
6	0.9998	0.9962

as local road conditions, fuel efficiency, maintenance costs, and suitability for the Nigerian climate. Making an informed decision when selecting a vehicle ensures that one's vehicle meets both practical needs and personal preferences. Since the Q-ROFS theory is well suited to manage vagueness and fuzziness, it is integrated into the process via the Q-ROFCM technique for vehicle selection to enhance the reliability of outputs. The study utilizes real data collected from three randomly selected automobile maintenance experts at the Village, Old Otukpo Road, Makurdi, Benue State, Nigeria. The data are collected using linguistic variables, which are then transformed into q-ROFNs for analysis. The set,

$$\mathfrak{W} = \{\mathfrak{W}_1, \mathfrak{W}_2, \mathfrak{W}_3, \mathfrak{W}_4, \mathfrak{W}_5\mathfrak{W}_6, \mathfrak{W}_7\}$$

Represents the brands of vehicles to be evaluated for the selection analysis. The anonymity of the vehicle's brands is maintained since an investigation permit is not obtained. The following set:

$$\mathfrak{C} = \{\mathfrak{C}_1, \mathfrak{C}_2, \mathfrak{C}_3, \mathfrak{C}_4, \mathfrak{C}_5, \mathfrak{C}_6, \mathfrak{C}_7, \mathfrak{C}_8\}$$

Where $\mathfrak{C}_1$ represents maintenance cost, $\mathfrak{C}_2$ represents fuel efficiency, $\mathfrak{C}_3$ represents safety, $\mathfrak{C}_4$ represents resale value, $\mathfrak{C}_5$ represents the purchase cost, $\mathfrak{C}_6$ represents reliability, $\mathfrak{C}_7$ represents driving comfort, and $\mathfrak{C}_8$ represents a

Table 15 Computed Values for Closeness Coefficients

q	$\mathfrak{y}_1$	$\mathfrak{y}_2$	$\mathfrak{y}_3$	$\mathfrak{y}_4$	$\mathfrak{y}_5$	$\mathfrak{y}_6$	$\mathfrak{y}_7$
1	− 0.0049	0.0000	− 0.0173	− 0.0082	− 0.0037	− 0.0165	− 0.0146
2	− 0.0037	− 0.0003	− 0.0140	− 0.0128	0.0000	− 0.0105	− 0.0089
3	− 0.0044	− 0.0022	− 0.0111	− 0.0059	0.0000	− 0.0082	− 0.0072
4	− 0.0046	− 0.0034	− 0.0096	− 0.006	− 0.0002	− 0.0072	− 0.0066
5	− 0.0045	− 0.0036	− 0.0083	− 0.0058	− 0.0001	− 0.0065	− 0.0057
6	− 0.0044	− 0.0036	− 0.0074	− 0.0056	0.0000	− 0.0059	− 0.0051

Table 16 Computed Values for Preference Parameters

q	Ψ_1	Ψ_2	Ψ_3	Ψ_4	Ψ_5	Ψ_6	Ψ_7
1	0.2832	0.0000	1.0000	0.474	0.2139	0.9538	0.8439
2	0.2643	0.0214	1.0000	0.9143	0.0000	0.7500	**0**.6357
3	0.3964	0.1982	1.0000	0.5315	0.0000	0.7387	0.6486
4	0.4792	0.3542	1.0000	0.6250	0.0208	0.7500	0.6875
5	0.5422	0.4337	1.0000	0.6988	0.0120	0.7831	0.6867
6	0.5946	0.4865	1.0000	0.7568	0.0000	0.7972	0.6891

Table 17 Ranking of Preference Parameters for the Vehicle Brands

q	Preference Ordering	Suitable Vehicle
1	$\mathfrak{M}_2 \prec \mathfrak{M}_5 \prec \mathfrak{M}_1 \prec \mathfrak{M}_4 \prec \mathfrak{M}_7 \prec \mathfrak{M}_6 \prec \mathfrak{M}_3$	$\mathfrak{M}_2$
2	$\mathfrak{M}_5 \prec \mathfrak{M}_2 \prec \mathfrak{M}_1 \prec \mathfrak{M}_7 \prec \mathfrak{M}_6 \prec \mathfrak{M}_4 \prec \mathfrak{M}_3$	$\mathfrak{M}_5$
3	$\mathfrak{M}_5 \prec \mathfrak{M}_2 \prec \mathfrak{M}_1 \prec \mathfrak{M}_4 \prec \mathfrak{M}_7 \prec \mathfrak{M}_6 \prec \mathfrak{M}_3$	$\mathfrak{M}_5$
4	$\mathfrak{M}_5 \prec \mathfrak{M}_2 \prec \mathfrak{M}_1 \prec \mathfrak{M}_4 \prec \mathfrak{M}_7 \prec \mathfrak{M}_6 \prec \mathfrak{M}_3$	$\mathfrak{M}_5$
5	$\mathfrak{M}_5 \prec \mathfrak{M}_2 \prec \mathfrak{M}_1 \prec \mathfrak{M}_7 \prec \mathfrak{M}_4 \prec \mathfrak{M}_6 \prec \mathfrak{M}_3$	$\mathfrak{M}_5$
6	$\mathfrak{M}_5 \prec \mathfrak{M}_2 \prec \mathfrak{M}_1 \prec \mathfrak{M}_7 \prec \mathfrak{M}_4 \prec \mathfrak{M}_6 \prec \mathfrak{M}_3$	$\mathfrak{M}_5$

Table 20 Comparative Results for the Maximum and Minimum Correlation Values

Q-ROFCM	$\Phi(\Gamma^+)$	$\Phi(\Gamma^-)$
Φ_D[12]	0.9229	0.1235
Φ_{SG}[14]	0.9347	– 0.0997
Φ_{BE1}[15]	0.5618	0.0024
Φ_{BE2}[15]	0.812	0.0044
Φ_{LE}[13]	0.8416	– 0.1679
Φ_{ES}[16]	0.9286	0.9248
Φ	0.9998	0.9962

good warranty is the criterion set for selecting the most suitable vehicle brand to purchase.

Table 2 presents the Q-ROFN for the linguistic variables employed by the three vehicle maintenance experts (VME) to evaluate each vehicle brand according to the specified criteria. Tables 3, 4, and 5 are the linguistic variables for each vehicle brand provided by the VME I, VME II, and VME III, respectively. Tables 6, 7, and 8 are the q-ROFNs from Tables 3, 4, and 5, respectively, based on the q-ROFNs in Table 2. Table 9 gives the average q-ROFNs from Tables 6, 7, and 8.

4.1 Q-ROF MADM's algorithm

To determine the most convenient car brand considering the criteria, we use the MADM methodology via the following algorithm:

Step I: Obtain the q-rung orthopair fuzzy decision matrix (Q-ROFDM) represented by $\mathcal{M} = \left\{\mathfrak{C}_i(\mathfrak{W}_j)\right\}_{(k\times m)}$, where $\mathfrak{C}_i$ is the criteria for selecting the most suitable car to use in Nigeria and $\mathfrak{W}_j$ is the car brand, for $i = 1, 2, \ldots, k$ and $j = 1,2, \ldots, m$.

Step II: Find the cost criterion (ℓ), which is the criterion with the least MD, and the benefit criteria ($||$), which are the other criteria.

Step III: Formulate the normalized Q-ROFDM represented by $N = \left(\mathfrak{W}_{j\alpha}(\mathfrak{C}_i), \mathfrak{W}_{j\beta}(\mathfrak{C}_i)\right)$. The normalized Q-ROFDM is calculated using (9):

$$\left(\mathfrak{W}_{j\alpha}(\mathfrak{C}_i), \mathfrak{W}_{j\beta}(\mathfrak{C}_i)\right) = \begin{cases} \left(\mathfrak{W}_{j\alpha}(\mathfrak{C}_i), \mathfrak{W}_{j\beta}(\mathfrak{C}_i)\right), for\ k \\ \left(\mathfrak{W}_{j\beta}(\mathfrak{C}_i), \mathfrak{W}_{j\alpha}(\mathfrak{C}_i)\right), for\ l \end{cases} \tag{9}$$

Table 18 Comparative Results for Correlation Coefficients for the Positive Ideal Solution and the Vehicle Alternatives

Q-ROFCM	$\Phi(\Gamma^+,\mathfrak{M}_1)$	$\Phi(\Gamma^+,\mathfrak{M}_2)$	$\Phi(\Gamma^+,\mathfrak{M}_3)$	$\Phi(\Gamma^+,\mathfrak{M}_4)$	$\Phi(\Gamma^+,\mathfrak{M}_5)$	$\Phi(\Gamma^+,\mathfrak{M}_6)$	$\Phi(\Gamma^+,\mathfrak{M}_7)$
Φ_D[12]	0.7392	0.7946	0.7055	0.6195	0.9229	0.6127	0.6649
Φ_{SG}[14]	0.0962	0.2821	0.2257	0.0328	0.9347	0.0219	0.1487
Φ_{BE1}[15]	0.2842	0.3699	0.2379	0.1361	0.5618	0.1137	0.1325
Φ_{BE2}[15]	0.5958	0.812	0.3928	0.7236	0.7797	0.4128	0.5826
Φ_{LE}[13]	0.0377	0.1476	0.1427	0.1528	0.8416	0.1824	0.2566
Φ_{ES}[16]	0.8835	0.899	0.8976	0.8746	0.9286	0.857	0.8603
Φ	0.9987	0.9997	0.9960	0.9978	0.9998	0.9975	0.9983

Table 19 Comparative Results for Correlation Coefficients for the Negative Ideal Solution and the Vehicle Alternatives

Q-ROFCM	$\Phi(\Gamma^-,\mathfrak{M}_1)$	$\Phi(\Gamma^-,\mathfrak{M}_2)$	$\Phi(\Gamma^-,\mathfrak{M}_3)$	$\Phi(\Gamma^-,\mathfrak{M}_4)$	$\Phi(\Gamma^-,\mathfrak{M}_5)$	$\Phi(\Gamma^-,\mathfrak{M}_6)$	$\Phi(\Gamma^-,\mathfrak{M}_7)$
Φ_D[12]	0.2679	0.2370	0.7932	0.2162	0.1235	0.4157	0.3041
Φ_{SG}[14]	0.1341	0.3007	0.4712	– 0.0997	0.1419	– 0.0174	0.0516
Φ_{BE1}[15]	0.0319	0.0262	0.417	0.02	0.0024	0.1181	0.0561
Φ_{BE2}[15]	0.039	0.0305	0.6465	0.0415	0.0044	0.1675	0.0963
Φ_{LE}[13]	0.0952	– 0.1418	0.7615	– 0.0306	– 0.1679	0.2187	0.2755
Φ_{ES}[16]	0.9452	0.9248	0.9655	0.9693	0.9613	1.0006	0.9849
Φ	0.9995	0.9997	0.9998	0.9998	0.9962	0.9998	0.9998

Table 21 Comparative Results for the Closeness Coefficients

Q-ROFCM	$\mathfrak{y}_1$	$\mathfrak{y}_2$	$\mathfrak{y}_3$	$\mathfrak{y}_4$	$\mathfrak{y}_5$	$\mathfrak{y}_6$	$\mathfrak{y}_7$
Φ_D[12]	– 1.3683	– 1.0581	– 5.6582	– 1.0794	0.0000	– 2.7021	– 1.7419
Φ_{SG}[14]	1.4480	3.3179	4.9676	– 0.9649	2.4233	– 0.1511	0.6766
Φ_{BE1}[15]	– 12.7858	– 10.2582	– 173.3265	– 8.0911	0.0000	– 49.0059	– 23.1392
Φ_{BE2}[15]	– 8.1299	– 5.9318	– 146.4481	– 8.5407	– 0.0398	– 37.5598	– 21.1689
Φ_{LE}[13]	0.6118	– 0.6692	4.7050	– 0.0007	0.0000	1.5193	1.9458
Φ_{ES}[16]	– 0.0706	– 0.0319	– 0.0774	– 0.1063	– 0.0395	– 0.1591	– 0.1385
Φ	– 0.0044	– 0.0036	– 0.0074	– 0.0056	0.0000	– 0.0059	– 0.0051

Table 22 Comparative Results for the Preference Parameters for the Vehicle Brands

Q-ROFCM	Ψ_1	Ψ_2	Ψ_3	Ψ_4	Ψ_5	Ψ_6	Ψ_7
Φ_D[12]	0.2418	0.1870	1.0000	0.1908	0.0000	0.4776	0.3079
Φ_{SG}[14]	– 9.5831	– 21.9583	– 32.8762	6.3858	– 16.0377	1.0000	– 4.4778
Φ_{BE1}[15]	0.0738	0.0592	1.0000	0.0467	0.0000	0.2827	0.1335
Φ_{BE2}[15]	0.0555	0.0405	1.0000	0.0583	0.0003	0.2565	0.1445
Φ_{LE}[13]	– 0.9142	1.0000	– 7.0308	0.0010	0.0000	– 2.2703	– 2.9077
Φ_{ES}[16]	0.4437	0.2483	0.4865	0.6681	0.2005	1.0000	0.8705
Φ	0.5946	0.4865	1.0000	0.7568	0.0000	0.7972	0.6891

Table 23 Comparative Results for the Vehicles Ordering

Q– ROFCM	Preference Ordering	Suitable Vehicle
Φ_D[12]	$\mathfrak{M}_5 \prec \mathfrak{M}_2 \prec \mathfrak{M}_4 \prec \mathfrak{M}_1 \prec \mathfrak{M}_7 \prec \mathfrak{M}_6 \prec \mathfrak{M}_3$	$\mathfrak{M}_5$
Φ_{SG}[14]	$\mathfrak{M}_3 \prec \mathfrak{M}_2 \prec \mathfrak{M}_5 \prec \mathfrak{M}_1 \prec \mathfrak{M}_7 \prec \mathfrak{M}_6 \prec \mathfrak{M}_4$	$\mathfrak{M}_3$
Φ_{BE1}[15]	$\mathfrak{M}_5 \prec \mathfrak{M}_4 \prec \mathfrak{M}_2 \prec \mathfrak{M}_1 \prec \mathfrak{M}_7 \prec \mathfrak{M}_6 \prec \mathfrak{M}_3$	$\mathfrak{M}_5$
Φ_{BE2}[15]	$\mathfrak{M}_5 \prec \mathfrak{M}_2 \prec \mathfrak{M}_1 \prec \mathfrak{M}_4 \prec \mathfrak{M}_7 \prec \mathfrak{M}_6 \prec \mathfrak{M}_3$	$\mathfrak{M}_5$
Φ_{LE}[13]	$\mathfrak{M}_3 \prec \mathfrak{M}_7 \prec \mathfrak{M}_6 \prec \mathfrak{M}_1 \prec \mathfrak{M}_5 \prec \mathfrak{M}_4 \prec \mathfrak{M}_2$	$\mathfrak{M}_3$
Φ_{ES}[16]	$\mathfrak{M}_5 \prec \mathfrak{M}_2 \prec \mathfrak{M}_1 \prec \mathfrak{M}_3 \prec \mathfrak{M}_4 \prec \mathfrak{M}_7 \prec \mathfrak{M}_6$	$\mathfrak{M}_5$
Φ	$\mathfrak{M}_5 \prec \mathfrak{M}_2 \prec \mathfrak{M}_1 \prec \mathfrak{M}_7 \prec \mathfrak{M}_4 \prec \mathfrak{M}_6 \prec \mathfrak{M}_3$	$\mathfrak{M}_5$

Step IV: Compute the positive ideal solution (PIS), $\Gamma^+ = \{\Gamma_1^+, \ldots, \Gamma_m^+\}$ and a negative ideal solution (NIS), $\Gamma^- = \{\Gamma_1^-, \ldots, \Gamma_m^-\}$ by (10):

$$\left.\begin{array}{l} \Gamma^+ = \begin{cases} \left(max\{\mathfrak{W}_{j\alpha}(\mathfrak{C}_i)\}, min\{\mathfrak{W}_{j\beta}(\mathfrak{C}_i)\}\right), & if\, \mathfrak{C}_i \; is\; k \\ \left(min\{\mathfrak{W}_{j\alpha}(\mathfrak{C}_i)\}, max\{\mathfrak{W}_{j\beta}(\mathfrak{C}_i)\}\right), & if\; \mathfrak{C}_i \; is\; l \end{cases} \\ \Gamma^- = \begin{cases} \left(min\{\mathfrak{W}_{j\alpha}(\mathfrak{C}_i)\}, max\{\mathfrak{W}_{j\beta}(\mathfrak{C}_i)\}\right), & if\; \mathfrak{C}_i \; is\; k \\ \left(max\{\mathfrak{W}_{j\alpha}(\mathfrak{C}_i)\}, min\{\mathfrak{W}_{j\beta}(\mathfrak{C}_i)\}\right), & if\; \mathfrak{C}_i \; is\; l \end{cases} \end{array}\right\} \tag{10}$$

Step V: Find $\Phi(\Gamma^+, \mathfrak{W}_j)$ and $\Phi(\Gamma^-, \mathfrak{W}_j)$ through the Q-ROFCMs.

Step VI: Determine the maximum correlation value, $\Phi(\Gamma^+)$ and the minimum correlation value, $\Phi(\Gamma^-)$ using (11):

$$\left.\begin{array}{l} \Phi(\Gamma^+) = \max\limits_{1 \le i \le j}\{\Phi(\Gamma^+, \mathfrak{W}_j)\} \\ \Phi(\Gamma^-) = \min\limits_{1 \le i \le j}\{\Phi(\Gamma^-, \mathfrak{W}_j)\} \end{array}\right\} \tag{11}$$

Step VII: Compute the closeness coefficient by (12)

$$\mathfrak{y}_j = \left(\frac{\Phi(\Gamma^+, \mathfrak{W}_j)}{\Phi(\Gamma^+)} - \frac{\Phi(\Gamma^-, \mathfrak{W}_j)}{\Phi(\Gamma^-)}\right), \text{for } 1 \le j \le m \tag{12}$$

Step VIII: Calculate the preference parameter, Ψ_j using (13):

$$\Psi_j = \frac{\mathfrak{y}_j}{\min\limits_{1 \le j \le m}\{\mathfrak{y}_j\}} \tag{13}$$

Step IX: Rank the car brands in the ascending order of Ψ_j.

The algorithm can be represented by Fig. 1.

Using Steps I and II, the Q-ROFDM is Table 9, and resale value represented by $\mathfrak{C}_4$ is the cost criterion. The results for the normalized q-ROFDM are presented in Table 10.

Using Step IV, we obtain Table 11.

Using Step V via the new Q-ROFCM for $q = 6$, the correlation coefficients between the vehicle alternatives and the PIS/NIS on Tables 9 and 11, the following values are obtained:

$$\begin{aligned}\Phi\left(\Gamma^{+},\mathfrak{M}_1\right) &= 0.9987, \Phi\left(\Gamma^{+},\mathfrak{M}_2\right) = 0.9997, \Phi\left(\Gamma^{+},\mathfrak{M}_3\right)\\ &= 0.9960, \Phi\left(\Gamma^{+},\mathfrak{M}_4\right) = 0.9978, \Phi\left(\Gamma^{+},\mathfrak{M}_5\right)\\ &= 0.9998, \Phi\left(\Gamma^{+},\mathfrak{M}_6\right) = 0.9975, \Phi\left(\Gamma^{+},\mathfrak{M}_7\right)\\ &= 0.9983\end{aligned}$$

$$\begin{aligned}\Phi\left(\Gamma^{-},\mathfrak{M}_1\right) &= 0.9995, \Phi\left(\Gamma^{-},\mathfrak{M}_2\right) = 0.9997, \Phi\left(\Gamma^{-},\mathfrak{M}_3\right)\\ &= 0.9998, \Phi\left(\Gamma^{-},\mathfrak{M}_4\right) = 0.9998, \Phi\left(\Gamma^{-},\mathfrak{M}_5\right)\\ &= 0.9962, \Phi\left(\Gamma^{-},\mathfrak{M}_6\right) = 0.9998, \Phi\left(\Gamma^{-},\mathfrak{M}_7\right)\\ &= 0.9998\end{aligned}$$

Then, we obtain $\Phi\left(\Gamma^{+}\right) = 0.9998$ and $\Phi\left(\Gamma^{-}\right) = 0.9962$ by Step VI

Using VII, the closeness coefficients are: $\mathfrak{y}_1 = -0.0044$, $\mathfrak{y}_2 = -0.0036$, $\mathfrak{y}_3 = -0.0074$, $\mathfrak{y}_4 = -0.0056$, $\mathfrak{y}_5 = 0.0000$, $\mathfrak{y}_6 = -0.0059$, $\mathfrak{y}_7 = -0.0051$.

By Step VIII, the preference parameters are computed are: $\Psi_1 = 0.5946$,$\Psi_2 = 0.4865$,$\Psi_3 = 1.0000$, $\Psi_4 = 0.7568$, $\Psi_5 = 0.0000$, $\Psi_6 = 0.7972$, $\Psi_7 = 0.6891$.

The preference parameters are arranged in ascending order using Step IX, and we obtain the ordering:

$$\mathfrak{M}_5 \prec \mathfrak{M}_2 \prec \mathfrak{M}_1 \prec \mathfrak{M}_7 \prec \mathfrak{M}_4 \prec \mathfrak{M}_6 \prec \mathfrak{M}_3$$

The ordering shows that the suitable vehicle brand to purchase for effective service in Nigeria is $\mathfrak{M}_5$, considering maintenance cost, fuel efficiency, safety, resale value, purchase cost, reliability, driving comfort, and good warranty, respectively.

4.2 Sensitivity analysis

The sensitivity analysis to ascertain the influence $q \geq 1$ in the new Q-ROFCM technique is conducted in this subsection. Using Step V via the new Q-ROFCM method for $q = 1,2,\ldots,6$, the correlation coefficient values in Tables 12 and 13 are computed.

The results in Tables 12 and 13 show the sensitivity of q in the values of the correlation coefficient. The reliability of the correlation coefficient values increases as q increase. By Step VI, we obtain $\Phi\left(\Gamma^{+}\right)$ and $\Phi\left(\Gamma^{-}\right)$ in Table 14.

Using Steps VII and VIII, we get the $\mathfrak{y}_j$ and Ψ_j as seen in Tables 15 and 16.

By ranking the results in Table 16, we obtain Table 17.

From the results, the suitable vehicle to purchase based on maintenance cost, fuel efficiency, safety, resale value, purchase cost, reliability, driving comfort, and good warranty is $\mathfrak{M}_5$. The least vehicle to purchase is $\mathfrak{M}_3$. Interestingly, all the levels of the Q-ROFSs yield the same interpretation, with the exception of $q = 1$, because of the inability of IFS to curb fuzziness.

4.3 Comparison based on the MADM's algorithm

Next, we present a comparative investigation of the new Q-ROFCM methodology with existing Q-ROFCM approaches [12, 16] based on the MADM technique. By using the MADM algorithm for the new Q-ROFCM methodology and the existing Q-ROFCM methodologies for $q = 6$, we obtain $\Phi\left(\Gamma^{+},\mathfrak{M}_j\right)$ and $\Phi\left(\Gamma^{-},\mathfrak{M}_j\right)$ displayed in Tables 18 and 19.

A careful study of Table 18 shows that the new Q-ROFCM yields the most accurate outcomes compared to the extant Q-ROFCM methods. The accuracy of the new Q-ROFCM technique is attributed to its incorporation of all the properties of Q-ROFS and its foundation on Spearman's model.

Similarly, as shown in Table 19, the new Q-ROFCM method yields the most accurate outcomes with a reliable interpretation compared to the existing Q-ROFCM methods [12–16]. The accuracy of the new Q-ROFCM method justifies the essence of its formulation.

The maximum of $\Phi\left(\Gamma^{+},\mathfrak{M}_j\right)$ and the minimum of $\Phi\left(\Gamma^{-},\mathfrak{M}_j\right)$ are shown in Table 20, and the closeness coefficients are displayed in Table 21.

By using Step VIII, we get Table 22.

Using Step IX and the information in Table 22, the comparative preference orderings in Table 23 are obtained.

The ordering shows that the suitable vehicle brand to purchase for effective service in Nigeria is $\mathfrak{M}_5$, considering maintenance cost, fuel efficiency, safety, resale value, purchase cost, reliability, driving comfort, and good warranty, respectively. The opposing interpretations obtained using the methods in [13, 14] cannot be taken into account due to the defectiveness of the Q-ROFCM.

5 Conclusion

The idea of Q-ROFCM is an effective method of decision support systems [12, 14, 16] and clustering analysis [13, 15]. As seen in the literature review, several authors have developed techniques for Q-ROFCM to enhance decision-making in practical areas, such as the selection process. However, these methodologies lack credibility and precision, as some do not account for the hesitation margins of the Q-ROFS, and none are constructed using Spearman's scheme of correlation coefficient. Due to the weaknesses of existing methods and the limitations of Spearman's correlation coefficient, a new Q-ROFCM technique was developed to enhance decision-making within the Q-ROF framework. This technique has been validated to express precision and credence over the extant Q-ROFCM techniques. In addition, the novel scheme is used to decide the most suitable vehicle to purchase via the MADM method, where real data were collected from three randomly selected VME in Makurdi metropolis, Nigeria. It is worth mentioning that the new Q-ROFCM method yields more precise and consistent results because it considers the 3D parameters of Q-ROFS, unlike the techniques in [12, 13, 15]. Additionally, it employs Spearman's scheme of correlation coefficient, unlike the existing methods. Regarding the strength of the new scheme of Q-ROFCM, it can be commendably applied to address decision-making in miscellaneous areas under indeterminate conditions. In particular, it could be used to discuss the idea of partial correlation coefficient [47] and applied to energy source selection [48]. The new scheme of Q-ROFCM can be extended to encompass further fuzzy spheres and used to address complex decision-making challenges.

Appendix

Java code for the new correlation method

Java Code for the New Correlation Method

Code Fragment 1	***Code Fragment 2***	***Code Fragment 3***
public static void main(String[] args) {	double npA5D=0.0;	double nnpA2C=0.0;
double n = 8;	double hpA5D=0.0;	double hhpA2C=0.0;
double mA1C = 0.0;	double mpA6D=0.0;	double mmpA2D=0.0;
double nA1C = 0.0;	double npA6D=0.0;	double nnpA2D=0.0;
double hA1C = 0.0;	double hpA6D=0.0;	double hhpA2D=0.0;
double mA2C = 0.0;	double mpA7D=0.0;	double mmpA3C=0.0;
double nA2C = 0.0;	double npA7D=0.0;	double nnpA3C=0.0;
double hA2C = 0.0;	double hpA7D=0.0;	double hhpA3C=0.0;
double mA3C = 0.0;	double mcA1C=0.0;	double mmpA3D=0.0;
double nA3C = 0.0;	double ncA1C=0.0;	double nnpA3D=0.0;
double hA3C = 0.0;	double hcA1C=0.0;	double hhpA3D=0.0;
double mA4C = 0.0;	double mcA2C=0.0;	double mmpA4C=0.0;
double nA4C = 0.0;	double ncA2C=0.0;	double nnpA4C=0.0;
double hA4C = 0.0;	double hcA2C=0.0;	double hhpA4C=0.0;
double mA5C = 0.0;	double mcA3C=0.0;	double mmpA4D=0.0;
double nA5C = 0.0;	double ncA3C=0.0;	double nnpA4D=0.0;
double hA5C = 0.0;	double hcA3C=0.0;	double hhpA4D=0.0;
double mA6C = 0.0;	double mcA4C=0.0;	double mmpA5C=0.0;
double nA6C = 0.0;	double ncA4C=0.0;	double nnpA5C=0.0;
double hA6C = 0.0;	double hcA4C=0.0;	double hhpA5C=0.0;
double mA7C = 0.0;	double mcA5C=0.0;	double mmpA5D=0.0;
double nA7C = 0.0;	double ncA5C=0.0;	double nnpA5D=0.0;
double hA7C = 0.0;	double hcA5C=0.0;	double hhpA5D=0.0;
double mA1D = 0.0;	double mcA6C=0.0;	double mmpA6C=0.0;
double nA1D = 0.0;	double ncA6C=0.0;	double nnpA6C=0.0;
double hA1D = 0.0;	double hcA6C=0.0;	double hhpA6C=0.0;
double mA2D = 0.0;	double mcA7C=0.0;	double mmpA6D=0.0;
double nA2D = 0.0;	double ncA7C=0.0;	double nnpA6D=0.0;
double hA2D = 0.0;	double hcA7C=0.0;	double hhpA6D=0.0;
double mA3D = 0.0;	double mcA1D=0.0;	double mmpA7C=0.0;
double nA3D = 0.0;	double ncA1D=0.0;	double nnpA7C=0.0;
double hA3D = 0.0;	double hcA1D=0.0;	double hhpA7C=0.0;
double mA4D = 0.0;	double mcA2D=0.0;	double mmpA7D=0.0;
double nA4D = 0.0;	double ncA2D=0.0;	double nnpA7D=0.0;
double hA4D = 0.0;	double hcA2D=0.0;	double hhpA7D=0.0;
double mA5D = 0.0;	double mcA3D=0.0;	double mmcA1C=0.0;
double nA5D = 0.0;	double ncA3D=0.0;	double nncA1C=0.0;
double hA5D = 0.0;	double hcA3D=0.0;	double hhcA1C=0.0;
double mA6D = 0.0;	double mcA4D=0.0;	double mmcA1D=0.0;
double nA6D = 0.0;	double ncA4D=0.0;	double nncA1D=0.0;
double hA6D = 0.0;	double hcA4D=0.0;	double hhcA1D=0.0;
double mA7D = 0.0;	double mcA5D=0.0;	double mmcA2C=0.0;
double nA7D = 0.0;	double ncA5D=0.0;	double nncA2C=0.0;
double hA7D = 0.0;	double hcA5D=0.0;	double hhcA2C=0.0;
double mpA1C=0.0;	double mcA6D=0.0;	double mmcA2D=0.0;
double npA1C=0.0;	double ncA6D=0.0;	double nncA2D=0.0;
double hpA1C=0.0;	double hcA6D=0.0;	double hhcA2D=0.0;
double mpA2C=0.0;	double mcA7D=0.0;	double mmcA3C=0.0;
double npA2C=0.0;	double ncA7D=0.0;	double nncA3C=0.0;
double hpA2C=0.0;	double hcA7D=0.0;	double hhcA3C=0.0;
double mpA3C=0.0;	double CorA1C=0.0;	double mmcA3D=0.0;
double npA3C=0.0;	double CorA2C=0.0;	double nncA3D=0.0;
double hpA3C=0.0;	double CorA3C=0.0;	double hhcA3D=0.0;
double mpA4C=0.0;	double CorA4C=0.0;	double mmcA4C=0.0;
double npA4C=0.0;	double CorA5C=0.0;	double nncA4C=0.0;
double hpA4C=0.0;	double CorA6C=0.0;	double hhcA4C=0.0;

double mpA5C=0.0;
double npA5C=0.0;
double hpA5C=0.0;
double mpA6C=0.0;
double npA6C=0.0;
double hpA6C=0.0;
double mpA7C=0.0;
double npA7C=0.0;
double hpA7C=0.0;
double mpA1D=0.0;
double npA1D=0.0;
double hpA1D=0.0;
double mpA2D=0.0;
double npA2D=0.0;
double hpA2D=0.0;
double mpA3D=0.0;
double npA3D=0.0;
double hpA3D=0.0;
double mpA4D=0.0;
double npA4D=0.0;
double hpA4D=0.0;
double mpA5D=0.0;
	double CorA7C=0.0;
	double CorA1D=0.0;
	double CorA2D=0.0;
	double CorA3D=0.0;
	double CorA4D=0.0;
	double CorA5D=0.0;
	double CorA6D=0.0;
	double CorA7D=0.0;
	double A1=0.0;
	double A2=0.0;
	double A3=0.0;
	double A4=0.0;
	double A5=0.0;
	double A6=0.0;
	double A7=0.0;
	ouble mmpA1C=0.0;
	double nnpA1C=0.0;
	double hhpA1C=0.0;
	double mmpA1D=0.0;
	double nnpA1D=0.0;
	double hhpA1D=0.0;
	double mmpA2C=0.0;
		double mmcA4D=0.0;
		double nncA4D=0.0;
		double hhcA4D=0.0;
		double mmcA5C=0.0;
		double nncA5C=0.0;
		double hhcA5C=0.0;
		double mmcA5D=0.0;
		double nncA5D=0.0;
		double hhcA5D=0.0;
		double mmcA6C=0.0;
		double nncA6C=0.0;
		double hhcA6C=0.0;
		double mmcA6D=0.0;
		double nncA6D=0.0;
		double hhcA6D=0.0;
		double mmcA7C=0.0;
		double nncA7C=0.0;
		double hhcA7C=0.0;
		double mmcA7D=0.0;
		double nncA7D=0.0;
		double hhcA7D=0.0;

Code Fragment 4

```
double scale = Math.pow(10, 4);
double muA1[] = {0.7667, 0.7, 0.4333, 0.8667, 0.7, 0.7667, 0.7, 0.9333};
double nuA1[] = {0.2333, 0.3, 0.5667, 0.1333, 0.3, 0.2333, 0.3, 0.0667};
double piA1[] = new double[8];
double muA2[] = {0.9, 0.7667, 0.7667, 0.8, 0.7, 0.7667, 0.8667, 0.6667};
double nuA2[] = {0.1, 0.2333, 0.2333, 0.2, 0.3, 0.2333, 0.1333, 0.3333};
double piA2[] = new double[8];
double muA3[] = {0.0667, 0.4333, 0.9333, 0.6333, 0.8667, 0.8333, 0.2667, 0.5};
double nuA3[] = {0.9333, 0.5667, 0.0667, 0.3667, 0.1333, 0.1667, 0.7333, 0.5};
double piA3[] = new double[8];
double muA4[] = {0.6667, 0.6667, 0.7, 0.6333, 0.5667, 0.8, 0.6667, 0.6333};
double nuA4[] = {0.3333, 0.3333, 0.3, 0.3667, 0.4333, 0.2, 0.3333, 0.3667};
double piA4[] = new double[8];
double muA5[] = {1.0, 0.9667, 0.5667, 0.4333, 0.5667, 0.5, 0.7, 0.6667};
double nuA5[] = {0.0, 0.0333, 0.4333, 0.5667, 0.4333, 0.5, 0.3, 0.3333};
double piA5[] = new double[8];
double muA6[] = {0.7, 0.3333, 0.5667, 0.4333, 0.5667, 0.8667, 0.2333, 0.5667};
double nuA6[] = {0.3, 0.6667, 0.4333, 0.5667, 0.4333, 0.1333, 0.7667, 0.4333};
double piA6[] = new double[8];
double muA7[] = {0.7667, 0.5667, 0.6333, 0.5, 0.8333, 0.5, 0.4333, 0.3333};
double nuA7[] = {0.2333, 0.4333, 0.3667, 0.5, 0.1667, 0.5, 0.5667, 0.6667};
double piA7[] = new double[8];
double muC[] = {1.0, 0.9667, 0.9333, 0.1333, 0.8667, 0.8667, 0.8667, 0.9333};
double nuC[] = {0.0, 0.0333, 0.0667, 0.8667, 0.1333, 0.1333, 0.1333, 0.0667};
double piC[] = new double[8];
double muD[] = {0.0667, 0.3333, 0.4333, 0.5667, 0.5667, 0.5, 0.2333, 0.3333};
double nuD[] = {0.9333, 0.6667, 0.5667, 0.4333, 0.4333, 0.5, 0.7667, 0.6667};
double piD[] = new double[8];
for(int i = 0; i<=n-1; i++)
{
 piA1[i] = Math.pow(1- Math.pow(muA1[i], 6.0)-Math.pow(nuA1[i], 6.0), 1 / 6);
 piA2[i] = Math.pow(1- Math.pow(muA2[i], 6.0)-Math.pow(nuA2[i], 6.0), 1 / 6);
 piA3[i] = Math.pow(1- Math.pow(muA3[i], 6.0)-Math.pow(nuA3[i], 6.0), 1 / 6);
 piA4[i] = Math.pow(1- Math.pow(muA4[i], 6.0)-Math.pow(nuA4[i], 6.0), 1 / 6);
 piA5[i] = Math.pow(1- Math.pow(muA5[i], 6.0)-Math.pow(nuA5[i], 6.0), 1 / 6);
 piA6[i] = Math.pow(1- Math.pow(muA6[i], 6.0)-Math.pow(nuA6[i], 6.0), 1 / 6);
 piA7[i] = Math.pow(1- Math.pow(muA7[i], 6.0)-Math.pow(nuA7[i], 6.0), 1 / 6);
  piC[i] = Math.pow(1- Math.pow(muC[i],  6.0)-Math. pow(nuC[i], 6.0), 1 / 6);
  piD[i] = Math.pow(1- Math.pow(muD[i],  6.0)-Math.pow(nuD[i],  6.0), 1 / 6);
}
for(int i = 0; i<=n-1; i++)
{
   mA1C += Math.pow(Math.abs((Math.pow(muA1[i], 6.0)-Math.pow(muC[i], 6.0))), 6.0);
   nA1C += Math.pow(Math.abs((Math.pow(nuA1[i], 6.0)-Math.pow(nuC[i], 6.0))), 6.0);
   hA1C += Math.pow(Math.abs((Math.pow(piA1[i], 6.0)-Math.pow(piC[i], 6.0))), 6.0);
   mA2C += Math.pow(Math.abs((Math.pow(muA2[i], 6.0)-Math.pow(muC[i], 6.0))), 6.0);
   nA2C += Math.pow(Math.abs((Math.pow(nuA2[i], 6.0)-Math.pow(nuC[i], 6.0))), 6.0);
   hA2C += Math.pow(Math.abs((Math.pow(piA2[i], 6.0)-Math.pow(piC[i], 6.0))), 6.0);
   mA3C += Math.pow(Math.abs((Math.pow(muA3[i], 6.0)-Math.pow(muC[i], 6.0))), 6.0);
   nA3C += Math.pow(Math.abs((Math.pow(nuA3[i], 6.0)-Math.pow(nuC[i], 6.0))), 6.0);
   hA3C += Math.pow(Math.abs((Math.pow(piA3[i], 6.0)-Math.pow(piC[i], 6.0))), 6.0);
   mA4C += Math.pow(Math.abs((Math.pow(muA4[i], 6.0)-Math.pow(muC[i], 6.0))), 6.0);
   nA4C += Math.pow(Math.abs((Math.pow(nuA4[i], 6.0)-Math.pow(nuC[i], 6.0))), 6.0);
   hA4C += Math.pow(Math.abs((Math.pow(piA4[i], 6.0)-Math.pow(piC[i], 6.0))), 6.0);
   mA5C += Math.pow(Math.abs((Math.pow(muA5[i], 6.0)-Math.pow(muC[i], 6.0))), 6.0);
   nA5C += Math.pow(Math.abs((Math.pow(nuA5[i], 6.0)-Math.pow(nuC[i], 6.0))), 6.0);
   hA5C += Math.pow(Math.abs((Math.pow(piA5[i], 6.0)-Math.pow(piC[i], 6.0))), 6.0);
```

```
    mA6C += Math.pow(Math.abs((Math.pow(muA6[i], 6.0)-Math.pow(muC[i], 6.0))), 6.0);
    nA6C += Math.pow(Math.abs((Math.pow(nuA6[i], 6.0)-Math.pow(nuC[i], 6.0))), 6.0);
    hA6C += Math.pow(Math.abs((Math.pow(piA6[i], 6.0)-Math.pow(piC[i], 6.0))), 6.0);
    mA7C += Math.pow(Math.abs((Math.pow(muA7[i], 6.0)-Math.pow(muC[i], 6.0))), 6.0);
    nA7C += Math.pow(Math.abs((Math.pow(nuA7[i], 6.0)-Math.pow(nuC[i], 6.0))), 6.0);
    hA7C += Math.pow(Math.abs((Math.pow(piA7[i], 6.0)-Math.pow(piC[i], 6.0))), 6.0);
    mA1D += Math.pow(Math.abs((Math.pow(muA1[i], 6.0)-Math.pow(muD[i], 6.0))), 6.0);
    nA1D += Math.pow(Math.abs((Math.pow(nuA1[i], 6.0)-Math.pow(nuD[i], 6.0))), 6.0);
    hA1D += Math.pow(Math.abs((Math.pow(piA1[i], 6.0)-Math.pow(piD[i], 6.0))), 6.0);
    mA2D += Math.pow(Math.abs((Math.pow(muA2[i], 6.0)-Math.pow(muD[i], 6.0))), 6.0);
    nA2D += Math.pow(Math.abs((Math.pow(nuA2[i], 6.0)-Math.pow(nuD[i], 6.0))), 6.0);
    hA2D += Math.pow(Math.abs((Math.pow(piA2[i], 6.0)-Math.pow(piD[i], 6.0))), 6.0);
    mA3D += Math.pow(Math.abs((Math.pow(muA3[i], 6.0)-Math.pow(muD[i], 6.0))), 6.0);
    nA3D += Math.pow(Math.abs((Math.pow(nuA3[i], 6.0)-Math.pow(nuD[i], 6.0))), 6.0);
    hA3D += Math.pow(Math.abs((Math.pow(piA3[i], 6.0)-Math.pow(piD[i], 6.0))), 6.0);
    mA4D += Math.pow(Math.abs((Math.pow(muA4[i], 6.0)-Math.pow(muD[i], 6.0))), 6.0);
    nA4D += Math.pow(Math.abs((Math.pow(nuA4[i], 6.0)-Math.pow(nuD[i], 6.0))), 6.0);
    hA4D += Math.pow(Math.abs((Math.pow(piA4[i], 6.0)-Math.pow(piD[i], 6.0))), 6.0);
    mA5D += Math.pow(Math.abs((Math.pow(muA5[i], 6.0)-Math.pow(muD[i], 6.0))), 6.0);
    nA5D += Math.pow(Math.abs((Math.pow(nuA5[i], 6.0)-Math.pow(nuD[i], 6.0))), 6.0);
    hA5D += Math.pow(Math.abs((Math.pow(piA5[i], 6.0)-Math.pow(piD[i], 6.0))), 6.0);
    mA6D += Math.pow(Math.abs((Math.pow(muA6[i], 6.0)-Math.pow(muD[i], 6.0))), 6.0);
    nA6D += Math.pow(Math.abs((Math.pow(nuA6[i], 6.0)-Math.pow(nuD[i], 6.0))), 6.0);
    hA6D += Math.pow(Math.abs((Math.pow(piA6[i], 6.0)-Math.pow(piD[i], 6.0))), 6.0);
    mA7D += Math.pow(Math.abs((Math.pow(muA7[i], 6.0)-Math.pow(muD[i], 6.0))), 6.0);
    nA7D += Math.pow(Math.abs((Math.pow(nuA7[i], 6.0)-Math.pow(nuD[i], 6.0))), 6.0);
    hA7D += Math.pow(Math.abs((Math.pow(piA7[i], 6.0)-Math.pow(piD[i], 6.0))), 6.0);
  }
  mpA1C=mA1C/(Math.pow((n+1), 3.0)-(n+1));
  npA1C=nA1C/(Math.pow((n+1), 3.0)-(n+1));
  hpA1C=hA1C/(Math.pow((n+1), 3.0)-(n+1));
  mpA2C=mA2C/(Math.pow((n+1), 3.0)-(n+1));
  npA2C=nA2C/(Math.pow((n+1), 3.0)-(n+1));
  hpA2C=hA2C/(Math.pow((n+1), 3.0)-(n+1));
  mpA3C=mA3C/(Math.pow((n+1), 3.0)-(n+1));
  npA3C=nA3C/(Math.pow((n+1), 3.0)-(n+1));
  hpA3C=hA3C/(Math.pow((n+1), 3.0)-(n+1));
  mpA4C=mA4C/(Math.pow((n+1), 3.0)-(n+1));
  npA4C=nA4C/(Math.pow((n+1), 3.0)-(n+1));
  hpA4C=hA4C/(Math.pow((n+1), 3.0)-(n+1));
  mpA5C=mA5C/(Math.pow((n+1), 3.0)-(n+1));
  npA5C=nA5C/(Math.pow((n+1), 3.0)-(n+1));
  hpA5C=hA5C/(Math.pow((n+1), 3.0)-(n+1));
  mpA6C=mA6C/(Math.pow((n+1), 3.0)-(n+1));
  npA6C=nA6C/(Math.pow((n+1), 3.0)-(n+1));
  hpA6C=hA6C/(Math.pow((n+1), 3.0)-(n+1));
  mpA7C=mA7C/(Math.pow((n+1), 3.0)-(n+1));
  npA7C=nA7C/(Math.pow((n+1), 3.0)-(n+1));
  hpA7C=hA7C/(Math.pow((n+1), 3.0)-(n+1));
  mpA1D=mA1D/(Math.pow((n+1), 3.0)-(n+1));
  npA1D=nA1D/(Math.pow((n+1), 3.0)-(n+1));
  hpA1D=hA1D/(Math.pow((n+1), 3.0)-(n+1));
  mpA2D=mA2D/(Math.pow((n+1), 3.0)-(n+1));
  npA2D=nA2D/(Math.pow((n+1), 3.0)-(n+1));
  hpA2D=hA2D/(Math.pow((n+1), 3.0)-(n+1));
  mpA3D=mA3D/(Math.pow((n+1), 3.0)-(n+1));
  npA3D=nA3D/(Math.pow((n+1), 3.0)-(n+1));
  hpA3D=hA3D/(Math.pow((n+1), 3.0)-(n+1));
```

```
mpA4D=mA4D/(Math.pow((n+1), 3.0)-(n+1));
npA4D=nA4D/(Math.pow((n+1), 3.0)-(n+1));
hpA4D=hA4D/(Math.pow((n+1), 3.0)-(n+1));
mpA5D=mA5D/(Math.pow((n+1), 3.0)-(n+1));
npA5D=nA5D/(Math.pow((n+1), 3.0)-(n+1));
hpA5D=hA5D/(Math.pow((n+1), 3.0)-(n+1));
mpA6D=mA6D/(Math.pow((n+1), 3.0)-(n+1));
npA6D=nA6D/(Math.pow((n+1), 3.0)-(n+1));
hpA6D=hA6D/(Math.pow((n+1), 3.0)-(n+1));
mpA7D=mA7D/(Math.pow((n+1), 3.0)-(n+1));
npA7D=nA7D/(Math.pow((n+1), 3.0)-(n+1));
hpA7D=hA7D/(Math.pow((n+1), 3.0)-(n+1));
mcA1C=1-(6*mpA1C);
ncA1C=1-(6*npA1C);
hcA1C=1-(6*hpA1C);
mcA2C=1-(6*mpA2C);
ncA2C=1-(6*npA2C);
hcA2C=1-(6*hpA2C);
mcA3C=1-(6*mpA3C);
ncA3C=1-(6*npA3C);
hcA3C=1-(6*hpA3C);
mcA4C=1-(6*mpA4C);
ncA4C=1-(6*npA4C);
hcA4C=1-(6*hpA4C);
mcA5C=1-(6*mpA5C);
ncA5C=1-(6*npA5C);
hcA5C=1-(6*hpA5C);
mcA6C=1-(6*mpA6C);
ncA6C=1-(6*npA6C);
hcA6C=1-(6*hpA6C);
mcA7C=1-(6*mpA7C);
ncA7C=1-(6*npA7C);
hcA7C=1-(6*hpA7C);
mcA1D=1-(6*mpA1D);
ncA1D=1-(6*npA1D);
hcA1D=1-(6*hpA1D);
mcA2D=1-(6*mpA2D);
ncA2D=1-(6*npA2D);
hcA2D=1-(6*hpA2D);
mcA3D=1-(6*mpA3D);
ncA3D=1-(6*npA3D);
hcA3D=1-(6*hpA3D);
mcA4D=1-(6*mpA4D);
ncA4D=1-(6*npA4D);
hcA4D=1-(6*hpA4D);
mcA5D=1-(6*mpA5D);
ncA5D=1-(6*npA5D);
hcA5D=1-(6*hpA5D);
mcA6D=1-(6*mpA6D);
ncA6D=1-(6*npA6D);
hcA6D=1-(6*hpA6D);
mcA7D=1-(6*mpA7D);
ncA7D=1-(6*npA7D);
hcA7D=1-(6*hpA7D);
CorA1C = (double) Math.round( (mcA1C+ncA1C+hcA1C)/3  * scale) / scale;
CorA2C = (double) Math.round( (mcA2C+ncA2C+hcA2C)/3  * scale) / scale;
```

```
CorA3C = (double) Math.round( (mcA3C+ncA3C+hcA3C)/3  * scale) / scale;
CorA4C = (double) Math.round( (mcA4C+ncA4C+hcA4C)/3  * scale) / scale;
CorA5C = (double) Math.round( (mcA5C+ncA5C+hcA5C)/3  * scale) / scale;
CorA6C = (double) Math.round( (mcA6C+ncA6C+hcA6C)/3  * scale) / scale;
CorA7C = (double) Math.round( (mcA7C+ncA7C+hcA7C)/3  * scale) / scale;
CorA1D = (double) Math.round( (mcA1D+ncA1D+hcA1D)/3  * scale) / scale;
CorA2D = (double) Math.round( (mcA2D+ncA2D+hcA2D)/3  * scale) / scale;
CorA3D = (double) Math.round( (mcA3D+ncA3D+hcA3D)/3  * scale) / scale;
CorA4D = (double) Math.round( (mcA4D+ncA4D+hcA4D)/3  * scale) / scale;
CorA5D = (double) Math.round( (mcA5D+ncA5D+hcA5D)/3  * scale) / scale;
CorA6D = (double) Math.round( (mcA6D+ncA6D+hcA6D)/3  * scale) / scale;
CorA7D = (double) Math.round( (mcA7D+ncA7D+hcA7D)/3  * scale) / scale;
A1 = (double) Math.round(CorA1C/(CorA1C+CorA1D) * scale) / scale;
A2 = (double) Math.round(CorA2C/(CorA2C+CorA2D) * scale) / scale;
A3 = (double) Math.round(CorA3C/(CorA3C+CorA3D) * scale) / scale;
A4 = (double) Math.round(CorA4C/(CorA4C+CorA4D) * scale) / scale;
A5 = (double) Math.round(CorA5C/(CorA5C+CorA5D) * scale) / scale;
A6 = (double) Math.round(CorA6C/(CorA6C+CorA6D) * scale) / scale;
A7 = (double) Math.round(CorA7C/(CorA7C+CorA7D) * scale) / scale;
System.out.println("");
System.out.println("");
System.out.println("A1: " + A1);
System.out.println("A2: " + A2);
System.out.println("A3: " + A3);
System.out.println("A4: " + A4);
System.out.println("A5: " + A5);
System.out.println("A6: " + A6);
System.out.println("A7: " + A7);
System.out.println("CorA1C: " + CorA1C);
System.out.println("CorA2C: " + CorA2C);
System.out.println("CorA3C: " + CorA3C);
System.out.println("CorA4C: " + CorA4C);
System.out.println("CorA5C: " + CorA5C);
System.out.println("CorA6C: " + CorA6C);
System.out.println("CorA7C: " + CorA7C);
System.out.println("");
System.out.println("");
System.out.println("CorA1D: " + CorA1D);
System.out.println("CorA2D: " + CorA2D);
System.out.println("CorA3D: " + CorA3D);
System.out.println("CorA4D: " + CorA4D);
System.out.println("CorA5D: " + CorA5D);
System.out.println("CorA6D: " + CorA6D);
System.out.println("CorA7D: " + CorA7D);
```

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All authors contributed equally.

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Data availability
No datasets were generated or analysed during the current study.

Declarations

Ethics approval and consent to participate
Not applicable.

Competing interests
The authors declare no competing interests.

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